Chapter 46 Solutions

46.1 Assuming that the proton and antiproton are left nearly at rest after they are produced, the energy of the photon E, must be

$$E = 2E_0 = 2(938.3 \text{ MeV}) = 1876.6 \text{ MeV} = 3.00 \times 10^{-10} \text{ J}$$

Thus, $E = hf = 3.00 \times 10^{-10}$ J

$$f = \frac{3.00 \times 10^{-10} \text{ J}}{6.626 \times 10^{-34} \text{ J} \cdot \text{s}} = \boxed{4.53 \times 10^{23} \text{ Hz}}$$

$$\lambda = \frac{c}{f} = \frac{3.00 \times 10^{8} \text{ m/s}}{4.53 \times 10^{23} \text{ Hz}} = \boxed{6.62 \times 10^{-16} \text{ m}}$$

46.2 The minimum energy is released, and hence the minimum frequency photons are produced, when the proton and antiproton are at rest when they annihilate.

That is, $E = E_0$ and $K = 0$. To conserve momentum, each photon must carry away one-half the energy. Thus,

$$E_{\min} = hf_{\min} = \frac{(2E_0)}{2} = E_0 = 938.3 \text{ MeV}$$

Thus, $f_{\min} = \dfrac{(938.3 \text{ MeV})(1.60 \times 10^{-13} \text{ J/MeV})}{6.626 \times 10^{-34} \text{ J} \cdot \text{s}} = \boxed{2.27 \times 10^{23} \text{ Hz}}$

$$\lambda = \frac{c}{f_{\min}} = \frac{3.00 \times 10^{8} \text{ m/s}}{2.27 \times 10^{23} \text{ Hz}} = \boxed{1.32 \times 10^{-15} \text{ m}}$$

***46.3** In $\gamma \to p^+ + p^-$, we start with energy 2.09 GeV

we end with energy 938.3 MeV + 938.3 MeV + 95.0 MeV + K_2

where K_2 is the kinetic energy of the second proton.

Conservation of energy gives $\boxed{K_2 = 118 \text{ MeV}}$

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Goal Solution

A photon with an energy $E_\gamma = 2.09$ GeV creates a proton-antiproton pair in which the proton has a kinetic energy of 95.0 MeV. What is the kinetic energy of the antiproton? ($m_p c^2 = 938.3$ MeV).

G: An antiproton has the same mass as a proton, so it seems reasonable to expect that both particles will have similar kinetic energies.

O: The total energy of each particle is the sum of its rest energy and its kinetic energy. Conservation of energy requires that the total energy before this pair production event equal the total energy after.

A: $E_\gamma = (E_{Rp} + K_p) + (E_{R\bar{p}} + K_{\bar{p}})$

The energy of the photon is given as $E_\gamma = 2.09 \text{ GeV} = 2.09 \times 10^3$ MeV. From Table 46.2, we see that the rest energy of both the proton and the antiproton is

$$E_{Rp} = E_{R\bar{p}} = m_p c^2 = 938.3 \text{ MeV}$$

If the kinetic energy of the proton is observed to be 95.0 MeV, the kinetic energy of the antiproton is

$$K_{\bar{p}} = E_\gamma - E_{R\bar{p}} - E_{Rp} - K_p = 2.09 \times 10^3 \text{ MeV} - 2(938.5 \text{ MeV}) - 95.0 \text{ MeV} = 118 \text{ MeV}$$

L: The kinetic energy of the antiproton is slightly (~20%) greater than the proton. The two particles most likely have different shares in momentum of the gamma ray, and therefore will not have equal energies, either.

***46.4** The reaction is $\mu^+ + e^- \to \nu + \nu$

muon-lepton number before reaction: (–1) + (0) = –1

electron-lepon number before reaction: (0) + (1) = 1

Therefore, after the reaction, the muon-lepton number must be –1. Thus, one of the neutrinos must be the anti-neutrino associated with muons, and one of the neutrinos must be the neutrino associated with electrons:

$$\boxed{\bar{\nu}_\mu \quad \text{and} \quad \nu_e}$$

Then $\mu^+ + e^- \to \bar{\nu}_\mu + \nu_e$

46.5 The creation of a virtual Z^0 boson is an energy fluctuation $\Delta E = 93 \times 10^9$ eV. It can last no longer than $\Delta t = \hbar/2\Delta E$ and move no farther than

$$c(\Delta t) = \frac{hc}{4\pi\,\Delta E} = \frac{(6.626 \times 10^{-34} \text{ J}\cdot\text{s})(3.00 \times 10^8 \text{ m/s})}{4\pi(93 \times 10^9 \text{ eV})}\left(\frac{1 \text{ eV}}{1.60 \times 10^{-19} \text{ J}}\right) = 1.06 \times 10^{-18} \text{ m} = \boxed{\sim 10^{-18} \text{ m}}$$

46.6 (a) $\Delta E = (m_n - m_p - m_e)c^2$

From Table A-3, $\Delta E = (1.008\,665 - 1.007\,825)931.5 = \boxed{0.782 \text{ MeV}}$

(b) Assuming the neutron at rest, momentum is conserved, $p_p = p_e$

relativistic energy is conserved, $\sqrt{(m_p c^2)^2 + p_p^2 c^2} + \sqrt{(m_e c^2)^2 + p_e^2 c^2} = m_n c^2$

Since $p_p = p_e$, $\sqrt{(938.3)^2 + (pc)^2} + \sqrt{(0.511)^2 + (pc)^2} = 939.6 \text{ MeV}$

Solving the algebra $pc = 1.19 \text{ MeV}$

If $p_e c = \gamma m_e v_e c = 1.19$ MeV, then $\dfrac{\gamma v_e}{c} = \dfrac{1.19 \text{ MeV}}{0.511 \text{ MeV}} = \dfrac{x}{\sqrt{1 - x^2}} = 2.33$ where $x = \dfrac{v_e}{c}$

Solving, $x^2 = (1 - x^2)5.43$ and $x = v_e / c = 0.919$

$\boxed{v_e = 0.919c}$

Then $m_p v_p = \gamma_e m_e v_e$:

$$v_p = \frac{\gamma_e m_e v_e c}{m_p c} = \frac{(1.19 \text{ MeV})(1.60 \times 10^{-13} \text{ J/MeV})}{(1.67 \times 10^{-27})(3.00 \times 10^8)} = 3.80 \times 10^5 \text{ m/s} = \boxed{380 \text{ km/s}}$$

(c) $\boxed{\text{The electron is relativistic, the proton is not.}}$

***46.7** The time for a particle traveling with the speed of light to travel a distance of 3×10^{-15} m is

$$\Delta t = \frac{d}{v} = \frac{3 \times 10^{-15} \text{ m}}{3 \times 10^8 \text{ m/s}} = \boxed{\sim 10^{-23} \text{ s}}$$

***46.8** With energy 938.3 MeV, the time that a virtual proton could last is at most Δt in $\Delta E \Delta t \sim \hbar$.

The distance it could move is at most

$$c\,\Delta t \sim \frac{\hbar c}{\Delta E} = \frac{(1.055 \times 10^{-34} \text{ J} \cdot \text{s})(3 \times 10^8 \text{ m/s})}{(938.3)(1.6 \times 10^{-13} \text{ J})} = \boxed{\sim 10^{-16} \text{ m}}$$

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46.9 By Table 46.2, $M_{\pi^0} = 135\ \text{MeV}/c^2$

Therefore, $E_\gamma = \boxed{67.5\ \text{MeV}}$ for each photon

$$p = \frac{E_\gamma}{c} = \boxed{67.5\ \frac{\text{MeV}}{c}} \quad \text{and} \quad f = \frac{E_\gamma}{h} = \boxed{1.63 \times 10^{22}\ \text{Hz}}$$

***46.10** In $? + p^+ \to n + \mu^+$, charge conservation requires the unknown particle to be neutral. Baryon number conservation requires baryon number = 0. The muon-lepton number of ? must be –1.

So the unknown particle must be $\boxed{\bar{\nu}_\mu}$.

46.11 $\Omega^+ \to \overline{\Lambda}^0 + K^+$

$\overline{K}_S^0 \to \pi^+ + \pi^-$ (or $\pi^0 + \pi^0$)

$\overline{\Lambda}^0 \to \overline{p} + \pi^+$

$\overline{n} \to \overline{p} + e^+ + \nu_e$

46.12

(a) $p + \overline{p} \to \mu^+ + e^-$ $\boxed{L_e}$ $0 + 0 \to 0 + 1$ and $\boxed{L_\mu}$ $0 + 0 \to -1 + 0$

(b) $\pi^- + p \to p + \pi^+$ $\boxed{\text{charge}}$ $-1 + 1 \to +1 + 1$

(c) $p + p \to p + \pi^+$ $\boxed{\text{baryon number}}$ $1 + 1 \to 1 + 0$

(d) $p + p \to p + p + n$ $\boxed{\text{baryon number}}$ $1 + 1 \to 1 + 1 + 1$

(e) $\gamma + p \to n + \pi^0$ $\boxed{\text{charge}}$ $0 + 1 \to 0 + 0$

***46.13**

(a) Baryon number and charge are conserved, with values of 0 + 1 = 0 + 1 and 1 + 1 = 1 + 1 in both reactions.

(b) $\boxed{\text{Strangeness is } \textit{not} \text{ conserved}}$ in the second reaction.

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46.14 Baryon number conservation $\boxed{\text{allows the first and forbids the second}}$.

46.15 (a) $\pi^- \to \mu^- + \boxed{\bar{\nu}_\mu}$ L_μ: $0 \to 1 - 1$

(b) $K^+ \to \mu^+ + \boxed{\nu_\mu}$ L_μ: $0 \to -1 + 1$

(c) $\boxed{\bar{\nu}_e} + p^+ \to n + e^+$ L_e: $-1 + 0 \to 0 - 1$

(d) $\boxed{\nu_e} + n \to p^+ + e^-$ L_e: $1 + 0 \to 0 + 1$

(e) $\boxed{\nu_\mu} + n \to p^+ + \mu^-$ L_μ: $1 + 0 \to 0 + 1$

(f) $\mu^- \to e^- + \boxed{\bar{\nu}_e} + \boxed{\nu_\mu}$ L_μ: $1 \to 0 + 0 + 1$ and L_e: $0 \to 1 - 1 + 0$

***46.16** Momentum conservation requires the pions to have equal speeds.

The total energy of each is 497.7 MeV/2

so $E^2 = p^2c^2 + (mc^2)^2$ gives $(248.8\text{ MeV})^2 = (pc)^2 + (139.6\text{ MeV})^2$

Solving, $pc = 206\text{ MeV} = \gamma mvc = \dfrac{mc^2}{\sqrt{1-(v/c)^2}}\left(\dfrac{v}{c}\right)$

$$\frac{pc}{mc^2} = \frac{206\text{ MeV}}{139.6\text{ MeV}} = \frac{1}{\sqrt{1-(v/c)^2}}\left(\frac{v}{c}\right) = 1.48$$

$(v/c) = 1.48\sqrt{1-(v/c)^2}$ and $(v/c)^2 = 2.18\left[1-(v/c)^2\right] = 2.18 - 2.18(v/c)^2$

$3.18(v/c)^2 = 2.18$ so $\dfrac{v}{c} = \sqrt{\dfrac{2.18}{3.18}} = 0.828$ and $\boxed{v = 0.828c}$

46.17 (a) $p^+ \to \pi^+ + \pi^0$ $\boxed{\text{Baryon number}}$ is violated: $1 \to 0 + 0$

(b) $p^+ + p^+ \to p^+ + p^+ + \pi^0$ This reaction $\boxed{\text{can occur}}$.

(c) $p^+ + p^+ \to p^+ + \pi^+$ $\boxed{\text{Baryon number}}$ is violated: $1 + 1 \to 1 + 0$

(d) $\pi^+ \to \mu^+ + \nu_\mu$ This reaction $\boxed{\text{can occur}}$.

(e) $n^0 \to p^+ + e^- + \bar{\nu}_e$ This reaction $\boxed{\text{can occur}}$.

(f) $\pi^+ \to \mu^+ + n$ Violates $\boxed{\text{baryon number}}$: $0 \to 0 + 1$

Violates **muon-lepton number** : $0 \rightarrow -1 + 0$

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46.18 (a) $p \to e^+ + \gamma$ Baryon number: $+1 \to 0 + 0$ $\Delta B \neq 0$, so baryon number is violated.

(b) From conservation of momentum: $p_e = p_\gamma$

Then, for the positron, $E_e^2 = (p_e c)^2 + E_{0,e}^2$ becomes $E_e^2 = (p_\gamma c)^2 + E_{0,e}^2 = E_\gamma^2 + E_{0,e}^2$

From conservation of energy: $E_{0,p} = E_e + E_\gamma$ or $E_e = E_{0,p} - E_\gamma$

so $E_e^2 = E_{0,p}^2 - 2E_{0,p}E_\gamma + E_\gamma^2$.

Equating this to the result from above gives $E_\gamma^2 + E_{0,e}^2 = E_{0,p}^2 - 2E_{0,p}E_\gamma + E_\gamma^2$,

or $$E_\gamma = \frac{E_{0,p}^2 - E_{0,e}^2}{2E_{0,p}} = \frac{(938.3 \text{ MeV})^2 - (0.511 \text{ MeV})^2}{2(938.3 \text{ MeV})} = \boxed{469 \text{ MeV}}$$

Thus, $E_e = E_{0,p} - E_\gamma = 938.3 \text{ MeV} - 469 \text{ MeV} = \boxed{469 \text{ MeV}}$

Also, $p_\gamma = \dfrac{E_\gamma}{c} = \boxed{469 \text{ MeV/c}}$ and $p_e = p_\gamma = \boxed{469 \text{ MeV}/c}$

(c) The total energy of the positron is $E_e = 469$ MeV.

But, $E_e = \gamma E_{0,e} = \dfrac{E_{0,e}}{\sqrt{1-(v/c)^2}}$ so $\sqrt{1-(v/c)^2} = \dfrac{E_{0,e}}{E_e} = \dfrac{0.511 \text{ MeV}}{469 \text{ MeV}} = 1.09 \times 10^{-3}$

which yields: $\boxed{v = 0.999\,999\,4c}$

***46.19** The relevant conservation laws are: $\Delta L_e = 0$, $\Delta L_\mu = 0$, and $\Delta L_\tau = 0$.

(a) $\pi^+ \to \pi^0 + e^+ + ?$ $L_e: 0 \to 0 - 1 + L_e \Rightarrow L_e = 1$ and we have a $\boxed{\nu_e}$

(b) $? + p \to \mu^- + p + \pi^+$ $L_\mu: L_\mu + 0 \to +1 + 0 + 0 \Rightarrow L_\mu = 1$ and we have a $\boxed{\nu_\mu}$

(c) $\Lambda^0 \to p + \mu^- + ?$ $L_\mu: 0 \to 0 + 1 + L_\mu \Rightarrow L_\mu = -1$ and we have a $\boxed{\bar{\nu}_\mu}$

(d) $\tau^+ \to \mu^+ + ? + ?$ $L_\mu: 0 \to -1 + L_\mu \Rightarrow L_\mu = 1$ and we have a $\boxed{\nu_\mu}$

$L_\tau: +1 \to 0 + L_\tau \Rightarrow L_\tau = 1$ and we have a $\boxed{\bar{\nu}_\tau}$

Conclusion for (d): $L_\mu = 1$ for one particle, and $L_\tau = 1$ for the other particle.

We have $\boxed{\nu_\mu}$ and $\boxed{\bar{\nu}_\tau}$.

46.20 The $\rho^0 \to \pi^+ + \pi^-$ decay must occur via the strong interaction.

The $K_S^0 \to \pi^+ + \pi^-$ decay must occur via the weak interaction.

46.21 (a) $\Lambda^0 \to p + \pi^-$ Strangeness: $-1 \to 0 + 0$ (strangeness is $\boxed{\text{not conserved}}$)

(b) $\pi^- + p \to \Lambda^0 + K^0$ Strangeness: $0 + 0 \to -1 + 1$ (0 = 0 and strangeness is $\boxed{\text{conserved}}$)

(c) $\bar{p} + p \to \bar{\Lambda}^0 + \Lambda^0$ Strangeness: $0 + 0 \to +1 - 1$ (0 = 0 and strangeness is $\boxed{\text{conserved}}$)

(d) $\pi^- + p \to \pi^- + \Sigma^+$ Strangeness: $0 + 0 \to 0 - 1$ ($0 \neq -1$: strangeness is $\boxed{\text{not conserved}}$)

(e) $\Xi^- \to \Lambda^0 + \pi^-$ Strangeness: $-2 \to -1 + 0$ ($-2 \neq -1$ so strangeness is $\boxed{\text{not conserved}}$
)

(f) $\Xi^0 \to p + \pi^-$ Strangeness: $-2 \to 0 + 0$ ($-2 \neq 0$ so strangeness is $\boxed{\text{not conserved}}$)

46.22 (a) $\mu^- \to e^- + \gamma$ L_e: $0 \to 1 + 0$, and L_μ: $1 \to 0$

(b) $n \to p + e^- + \nu_e$ L_e: $0 \to 0 + 1 + 1$

(c) $\Lambda^0 \to p + \pi^0$ Strangeness: $-1 \to 0 + 0$, and charge: $0 \to +1 + 0$

(d) $p \to e^+ + \pi^0$ Baryon number: $+1 \to 0 + 0$

(e) $\Xi^0 \to n + \pi^0$ Strangeness: $-2 \to 0 + 0$

***46.23** (a) $\pi^- + p \to 2\eta$ violates conservation of baryon number as $0 + 1 \to 0$. $\boxed{\text{not allowed}}$

(b) $K^- + n \to \Lambda^0 + \pi^-$
Baryon number = $0 + 1 \to 1 + 0$ Charge = $-1 + 0 \to 0 - 1$
Strangeness, $-1 + 0 \to -1 + 0$ Lepton number, $0 \to 0$
The interaction may occur via the $\boxed{\text{strong interaction}}$ since all are conserved.

(c) $K^- \to \pi^- + \pi^0$
Strangeness, $-1 \to 0 + 0$ Baryon number, $0 \to 0$
Lepton number, $0 \to 0$ Charge, $-1 \to -1 + 0$
Strangeness is violated by one unit, but everything else is conserved. Thus, the reaction can occur via the $\boxed{\text{weak interaction}}$, but not the strong or electromagnetic interaction.

(d) $\Omega^- \to \Xi^- + \pi^0$
Baryon number, $1 \to 1 + 0$ Lepton number, $0 \to 0$
Charge, $-1 \to -1 + 0$ Strangeness, $-3 \to -2 + 0$
May occur by $\boxed{\text{weak interaction}}$, but not by strong or electromagnetic.

(e) $\eta \to 2\gamma$
Baryon number, $0 \to 0$ Lepton number, $0 \to 0$
Charge, $0 \to 0$ Strangeness, $0 \to 0$

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No conservation laws are violated, but photons are the mediators of the electromagnetic interaction. Also, the lifetime of the η is consistent with the $\boxed{\text{electromagnetic interaction}}$.

***46.24** (a) $\Xi^- \to \Lambda^0 + \mu^- + \nu_\mu$

Baryon number: $+1 \to +1 + 0 + 0$ Charge: $-1 \to 0 - 1 + 0$

L_e: $0 \to 0 + 0 + 0$ L_μ: $0 \to 0 + 1 + 1$

L_τ: $0 \to 0 + 0 + 0$ Strangeness: $-2 \to -1 + 0 + 0$

Conserved quantities are: $\boxed{B\text{, charge, } L_e\text{, and } L_\tau}$

(b) $K_S^0 \to 2\pi^0$

Baryon number: $0 \to 0$ Charge: $0 \to 0$

L_e: $0 \to 0$ L_μ: $0 \to 0$

L_τ: $0 \to 0$ Strangeness: $+1 \to 0$

Conserved quantities are: $\boxed{B\text{, charge, } L_e\text{, } L_\mu\text{, and } L_\tau}$

(c) $K^- + p \to \Sigma^0 + n$

Baryon number: $0 + 1 \to 1 + 1$ Charge: $-1 + 1 \to 0 + 0$

L_e: $0 + 0 \to 0 + 0$ L_μ: $0 + 0 \to 0 + 0$

L_τ: $0 + 0 \to 0 + 0$ Strangeness: $-1 + 0 \to -1 + 0$

Conserved quantities are: $\boxed{S\text{, charge, } L_e\text{, } L_\mu\text{, and } L_\tau}$

(d) $\Sigma^0 \to \Lambda^0 + \gamma$

Baryon number: $+1 \to 1 + 0$ Charge: $0 \to 0$

L_e: $0 \to 0 + 0$ L_μ: $0 \to 0 + 0$

L_τ: $0 \to 0 + 0$ Strangeness: $-1 \to -1 + 0$

Conserved quantities are: $\boxed{B\text{, } S\text{, charge, } L_e\text{, } L_\mu\text{, and } L_\tau}$

(e) $e^+ + e^- \to \mu^+ + \mu^-$

Baryon number: $0 + 0 \to 0 + 0$ Charge: $+1 - 1 \to +1 - 1$

L_e: $-1 + 1 \to 0 + 0$ L_μ: $0 + 0 \to +1 - 1$

L_τ: $0 + 0 \to 0 + 0$ Strangeness: $0 + 0 \to 0 + 0$

Conserved quantities are: $\boxed{B\text{, } S\text{, charge, } L_e\text{, } L_\mu\text{, and } L_\tau}$

(f) $\bar{p} + n \to \bar{\Lambda}^0 + \Sigma^-$

Baryon number: $-1 + 1 \to -1 + 1$ Charge: $-1 + 0 \to 0 - 1$

L_e: $0 + 0 \to 0 + 0$ L_μ: $0 + 0 \to 0 + 0$

L_τ: $0 + 0 \to 0 + 0$ Strangeness: $0 + 0 \to +1 - 1$

Conserved quantities are: $\boxed{B\text{, } S\text{, charge, } L_e\text{, } L_\mu\text{, and } L_\tau}$

***46.25** (a) $K^+ + p \to \underline{?} + p$

The strong interaction conserves everything.

Baryon number,	$0+1 \to B+1$	so	$B = 0$
Charge,	$+1+1 \to Q+1$	so	$Q = +1$
Lepton numbers,	$0+0 \to L+0$	so	$L_e = L_\mu = L_\tau = 0$
Strangeness,	$+1+0 \to S+0$	so	$S = 1$

The conclusion is that the particle must be positively charged, a non-baryon, with strangeness of +1. Of particles in Table 46.2, it can only be the $\boxed{K^+}$. Thus, this is an elastic scattering process.

The weak interaction conserves all but strangeness, and $\Delta S = \pm 1$.

(b) $\Omega^- \to \underline{?} + \pi^-$

Baryon number,	$+1 \to B+0$	so	$B = 1$
Charge,	$-1 \to Q-1$	so	$Q = 0$
Lepton numbers,	$0 \to L+0$	so	$L_e = L_\mu = L_\tau = 0$
Strangeness,	$-3 \to S+0$	so $\Delta S = 1$:	$S = -2$

The particle must be a neutral baryon with strangeness of –2. Thus, it is the $\boxed{\Xi^0}$.

(c) $K^+ \to \underline{?} + \mu^+ + \nu_\mu$:

Baryon number,	$0 \to B+0+0$	so	$B = 0$
Charge,	$+1 \to Q+1+0$	so	$Q = 0$
Lepton Numbers	L_e, $0 \to L_e+0+0$	so	$L_e = 0$
	L_μ, $0 \to L_\mu - 1 + 1$	so	$L_\mu = 0$
	L_τ, $0 \to L_\tau + 0 + 0$	so	$L_\tau = 0$
Strangeness:	$1 \to S+0+0$	so	$\Delta S = \pm 1$ (for weak interaction): $S = 0$

The particle must be a neutral meson with strangeness = 0 $\Rightarrow$ $\boxed{\pi^0}$.

***46.26** (a)

	proton	u	u	d	total
strangeness	0	0	0	0	0
baryon number	1	1/3	1/3	1/3	1
charge	e	$2e/3$	$2e/3$	$-e/3$	e

(b)

	neutron	u	d	d	total
strangeness	0	0	0	0	0
baryon number	1	1/3	1/3	1/3	1
charge	0	$2e/3$	$-e/3$	$-e/3$	0

***46.27** (a) The number of protons $N_p = 1000\text{ g}\left(\dfrac{6.02\times10^{23}\text{ molecules}}{18.0\text{ g}}\right)\left(\dfrac{10\text{ protons}}{\text{molecule}}\right) = 3.34\times10^{26}\text{ protons}$

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and there are $N_n = (1000\text{ g})\left(\dfrac{6.02\times 10^{23}\text{ molecules}}{18.0\text{ g}}\right)\left(\dfrac{8\text{ neutrons}}{\text{molecule}}\right) = 2.68\times 10^{26}\text{ neutrons}$

So there are for electric neutrality $\boxed{3.34\times 10^{26}\text{ electrons}}$

The up quarks have number $2\times 3.34\times 10^{26} + 2.68\times 10^{26} = \boxed{9.36\times 10^{26}\text{ up quarks}}$

and there are $2\times 2.68\times 10^{26} + 3.34\times 10^{26} = \boxed{8.70\times 10^{26}\text{ down quarks}}$

(b) Model yourself as 65 kg of water. Then you contain $65\times 3.34\times 10^{26}$ $\boxed{\sim 10^{28}\text{ electrons}}$

$65\times 9.36\times 10^{26}$ $\boxed{\sim 10^{29}\text{ up quarks}}$

$65\times 8.70\times 10^{26}$ $\boxed{\sim 10^{29}\text{ down quarks}}$

Only these fundamental particles form your body. You have no strangeness, charm, topness or bottomness.

46.28 (a)

	K^0	d	$\bar{s}$	total
strangeness	1	0	1	1
baryon number	0	1/3	–1/3	0
charge	0	$-e/3$	$e/3$	0

(b)

	Λ^0	u	d	s	total
strangeness	–1	0	0	–1	–1
baryon number	1	1/3	1/3	1/3	1
charge	0	$2e/3$	$-e/3$	$-e/3$	0

46.29 Quark composition of proton = uud and of neutron = udd.

Thus, if we neglect binding energies, we may write $m_p = 2m_u + m_d$ (1)

and $m_n = m_u + 2m_d$ (2)

Solving simultaneously, we find

$$m_u = \frac{1}{3}(2m_p - m_n) = \frac{1}{3}\left[2(938.3\text{ MeV}/c^2) - 939.6\text{ MeV}/c^2\right] = \boxed{312\text{ MeV}/c^2}$$

and from either (1) or (2), $m_d = \boxed{314\text{ MeV}/c^2}$

***46.30** In the first reaction, $\pi^- + p \to K^0 + \Lambda^0$, the quarks in the particles are: $\bar{u}d + uud \to d\bar{s} + uds$. There is a net of 1 up quark both before and after the reaction, a net of 2 down quarks both before and after, and a net of zero strange quarks both before and after. Thus, the reaction conserves the net number of each type of quark.

In the second reaction, $\pi^- + \text{p} \to \text{K}^0 + \text{n}$, the quarks in the particles are: $\bar{\text{u}}\text{d} + \text{uud} \to \text{d}\bar{\text{s}} + \text{udd}$. In this case, there is a net of 1 up and 2 down quarks before the reaction but a net of 1 up, 3 down, and 1 anti-strange quark after the reaction. Thus, the reaction does not conserve the net number of each type of quark.

46.31 (a) $\pi^- + \text{p} \to \text{K}^0 + \Lambda^0$

In terms of constituent quarks: $\boxed{\bar{\text{u}}\text{d} + \text{uud} \to \text{d}\bar{\text{s}} + \text{uds}}$.

up quarks:	$-1 + 2 \to 0 + 1$, or $1 \to 1$
down quarks:	$1 + 1 \to 1 + 1$, or $2 \to 2$
strange quarks:	$0 + 0 \to -1 + 1$, or $0 \to 0$

(b) $\pi^+ + \text{p} \to \text{K}^+ + \Sigma^+ \Rightarrow$ $\boxed{\text{u}\bar{\text{d}} + \text{uud} \to \text{u}\bar{\text{s}} + \text{uus}}$

up quarks:	$1 + 2 \to 1 + 2$, or $3 \to 3$
down quarks:	$-1 + 1 \to 0 + 0$, or $0 \to 0$
strange quarks:	$0 + 0 \to -1 + 1$, or $0 \to 0$

(c) $\text{K}^- + \text{p} \to \text{K}^+ + \text{K}^0 + \Omega^- \Rightarrow$ $\boxed{\bar{\text{u}}\text{s} + \text{uud} \to \text{u}\bar{\text{s}} + \text{d}\bar{\text{s}} + \text{sss}}$

up quarks:	$-1 + 2 \to 1 + 0 + 0$, or $1 \to 1$
down quarks:	$0 + 1 \to 0 + 1 + 0$, or $1 \to 1$
strange quarks:	$1 + 0 \to -1 - 1 + 3$, or $1 \to 1$

(d) $\text{p} + \text{p} \to \text{K}^0 + \text{p} + \pi^+ + \underline{?} \Rightarrow$ $\text{uud} + \text{uud} \to \text{d}\bar{\text{s}} + \text{uud} + \text{u}\bar{\text{d}} + \underline{?}$

The quark combination of ? must be such as to balance the last equation for up, down, and strange quarks.

up quarks:	$2 + 2 = 0 + 2 + 1 + ?$	(has 1 u quark)
down quarks:	$1 + 1 = 1 + 1 - 1 + ?$	(has 1 d quark)
strange quarks:	$0 + 0 = -1 + 0 + 0 + ?$	(has 1 s quark)

quark composite = uds = $\boxed{\Lambda^0 \text{ or } \Sigma^0}$

46.32 $\Sigma^0 + \text{p} \to \Sigma^+ + \gamma + X$ dds + uud → uds + 0 + ?

The left side has a net 3d, 2u and 1s. The right-hand side has 1d, 1u, and 1s leaving 2d and 1u missing.

$\boxed{\text{The unknown particle is a neutron, udd.}}$

Baryon and strangeness numbers are conserved.

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***46.33** Compare the given quark states to the entries in Tables 46.4 and 46.5.

(a) $\mathrm{suu} = \boxed{\Sigma^+}$

(b) $\bar{\mathrm{u}}\mathrm{d} = \boxed{\pi^-}$

(c) $\bar{\mathrm{s}}\mathrm{d} = \boxed{\mathrm{K}^0}$

(d) $\mathrm{ssd} = \boxed{\Xi^-}$

***46.34** (a) $\bar{\mathrm{u}}\bar{\mathrm{u}}\bar{\mathrm{d}}$: charge $= \left(-\frac{2}{3}e\right) + \left(-\frac{2}{3}e\right) + \left(\frac{1}{3}e\right) = \boxed{-e}$. This is the $\boxed{\text{antiproton}}$.

(b) $\bar{\mathrm{u}}\bar{\mathrm{d}}\bar{\mathrm{d}}$: charge $= \left(-\frac{2}{3}e\right) + \left(\frac{1}{3}e\right) + \left(\frac{1}{3}e\right) = \boxed{0}$. This is the $\boxed{\text{antineutron}}$.

***46.35** Section 39.4 says $\quad f_{\text{observer}} = f_{\text{source}}\sqrt{\dfrac{1+v_a/c}{1-v_a/c}}$

The velocity of approach, v_a, is the negative of the velocity of mutual recession: $v_a = -v$.

Then, $\quad \dfrac{c}{\lambda'} = \dfrac{c}{\lambda}\sqrt{\dfrac{1-v/c}{1+v/c}}$ and $\boxed{\lambda' = \lambda\sqrt{\dfrac{1+v/c}{1-v/c}}}$

46.36 $v = HR$ (Equation 46.7) $\qquad H = \dfrac{(1.7\times 10^{-2}\ \mathrm{m/s})}{\mathrm{ly}}$

(a) $v\,(2.00\times 10^6\ \mathrm{ly}) = 3.4\times 10^4\ \mathrm{m/s}$ $\qquad \lambda' = \lambda\sqrt{\dfrac{1+v/c}{1-v/c}} = 590(1.0001133) = \boxed{590.07\ \mathrm{nm}}$

(b) $v\,(2.00\times 10^8\ \mathrm{ly}) = 3.4\times 10^6\ \mathrm{m/s}$ $\qquad \lambda' = 590\sqrt{\dfrac{1+0.01133}{1-0.01133}} = \boxed{597\ \mathrm{nm}}$

(c) $v\,(2.00\times 10^9\ \mathrm{ly}) = 3.4\times 10^7\ \mathrm{m/s}$ $\qquad \lambda' = 590\sqrt{\dfrac{1+0.1133}{1-0.1133}} = \boxed{661\ \mathrm{nm}}$

46.37 (a) $\dfrac{\lambda'}{\lambda} = \dfrac{650\ \mathrm{nm}}{434\ \mathrm{nm}} = 1.50 = \sqrt{\dfrac{1+v/c}{1-v/c}}$ $\qquad \dfrac{1+v/c}{1-v/c} = 2.24$

$v = 0.383c,$ $\qquad \boxed{\text{38.3\% the speed of light}}$

(b) Equation 46.7, $v = HR$ $\qquad R = \dfrac{v}{H} = \dfrac{(0.383)(3.00\times 10^8)}{(1.7\times 10^{-2})} = \boxed{6.76\times 10^9\ \text{light years}}$

Goal Solution

A distant quasar is moving away from Earth at such high speed that the blue 434-nm hydrogen line is observed at 650 nm, in the red portion of the spectrum. (a) How fast is the quasar receding? You may use the result of Problem 35. (b) Using Hubble's law, determine the distance from Earth to this quasar.

G: The problem states that the quasar is moving very fast, and since there is a significant red shift of the light, the quasar must be moving away from Earth at a relativistic speed ($v > 0.1c$). Quasars are very distant astronomical objects, and since our universe is estimated to be about 15 billion years old, we should expect this quasar to be ~10^9 light-years away.

O: As suggested, we can use the equation in Problem 35 to find the speed of the quasar from the Doppler red shift, and this speed can then be used to find the distance using Hubble's law.

A: (a) $\dfrac{\lambda'}{\lambda} = \dfrac{650 \text{ nm}}{434 \text{ nm}} = 1.498 = \sqrt{\dfrac{1+v/c}{1-v/c}}$ or squared, $\dfrac{1+v/c}{1-v/c} = 2.243$

Therefore, $v = 0.383c$ or 38.3% the speed of light

(b) Hubble's law asserts that the universe is expanding at a constant rate so that the speeds of galaxies are proportional to their distance R from Earth, $v = HR$

so, $$R = \frac{v}{H} = \frac{(0.383)\left(3.00\times 10^8 \text{ m / s}\right)}{\left(1.70\times 10^{-2} \text{ m / s}\cdot\text{ly}\right)} = 6.76\times 10^9 \text{ ly}$$

L: The speed and distance of this quasar are consistent with our predictions. It appears that this quasar is quite far from Earth but not the most distant object in the visible universe.

***46.38** (a) $\lambda_n' = \lambda_n\sqrt{\dfrac{1+v/c}{1-v/c}} = (Z+1)\lambda_n$ $\qquad \dfrac{1+v/c}{1-v/c} = (Z+1)^2$

$$1 + \frac{v}{c} = (Z+1)^2 - \left(\frac{v}{c}\right)(Z+1)^2$$

$$\left(\frac{v}{c}\right)(Z^2 + 2Z + 2) = Z^2 + 2Z$$

$$v = \boxed{c\left(\frac{Z^2 + 2Z}{Z^2 + 2Z + 2}\right)}$$

(b) $$R = \frac{v}{H} = \boxed{\frac{c}{H}\left(\frac{Z^2 + 2Z}{Z^2 + 2Z + 2}\right)}$$

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***46.39** The density of the Universe is $\rho = 1.20\rho_c = 1.20\left(3H^2/8\pi G\right)$.

Consider a remote galaxy at distance r. The mass interior to the sphere below it is

$$M = \rho\left(\frac{4\pi r^3}{3}\right) = 1.20\left(\frac{3H^2}{8\pi G}\right)\left(\frac{4\pi r^3}{3}\right) = \frac{0.600H^2r^3}{G}$$

both now and in the future when it has slowed to rest from its current speed $v = Hr$. The energy of this galaxy is constant as it moves to apogee distance R:

$$\tfrac{1}{2}mv^2 - \frac{GmM}{r} = 0 - \frac{GmM}{R} \qquad \text{so} \qquad \tfrac{1}{2}mH^2r^2 - \frac{Gm}{r}\left(\frac{0.600H^2r^3}{G}\right) = 0 - \frac{Gm}{R}\left(\frac{0.600H^2r^3}{G}\right)$$

$$-0.100 = -0.600\frac{r}{R} \qquad \text{so} \qquad R = 6.00r$$

The Universe will expand by a factor of $\boxed{6.00}$ from its current dimensions.

***46.40** (a) $k_B T \approx 2m_p c^2$ so $T \approx \dfrac{2m_p c^2}{k_B} = \dfrac{2(938.3\text{ MeV})}{\left(1.38\times 10^{-23}\text{ J/K}\right)}\left(\dfrac{1.60\times 10^{-13}\text{ J}}{1\text{ MeV}}\right)$ $\boxed{\sim 10^{13}\text{ K}}$

(b) $k_B T \approx 2m_e c^2$ $T \approx \dfrac{2m_e c^2}{k_B} = \dfrac{2(0.511\text{ MeV})}{\left(1.38\times 10^{-23}\text{ J/K}\right)}\left(\dfrac{1.60\times 10^{-13}\text{ J}}{1\text{ MeV}}\right)$ $\boxed{\sim 10^{10}\text{ K}}$

***46.41** (a) Wien's law: $\lambda_{max}T = 2.898\times 10^{-3}\text{ m}\cdot\text{K}$

Thus, $\lambda_{max} = \dfrac{2.898\times 10^{-3}\text{ m}\cdot\text{K}}{T} = \dfrac{2.898\times 10^{-3}\text{ m}\cdot\text{K}}{2.73\text{ K}} = 1.06\times 10^{-3}\text{ m} = \boxed{1.06\text{ mm}}$

(b) This is a $\boxed{\text{microwave}}$.

***46.42** (a) $L = \sqrt{\dfrac{\hbar G}{c^3}} = \sqrt{\dfrac{\left(1.055\times 10^{-34}\text{ J}\cdot\text{s}\right)\left(6.67\times 10^{-11}\text{ N}\cdot\text{m}^2/\text{kg}^2\right)}{\left(3.00\times 10^8\text{ m/s}\right)^3}} = \boxed{1.61\times 10^{-35}\text{ m}}$

(b) This time is given as $T = \dfrac{L}{c} = \dfrac{1.61\times 10^{-35}\text{ m}}{3.00\times 10^8\text{ m/s}} = \boxed{5.38\times 10^{-44}\text{ s}}$,

which is approximately equal to the ultra-hot epoch.

46.43 (a) $\Delta E \Delta t \approx \hbar$, and $\Delta t \approx \frac{r}{c} = \frac{1.4 \times 10^{-15}\ \text{m}}{3.0 \times 10^{8}\ \text{m/s}} = 4.7 \times 10^{-24}\ \text{s}$

$$\Delta E \approx \frac{\hbar}{\Delta t} = \frac{1.055 \times 10^{-34}\ \text{J} \cdot \text{s}}{4.7 \times 10^{-24}\ \text{s}} = 2.3 \times 10^{-11}\ \text{J}\left(\frac{1\ \text{MeV}}{1.60 \times 10^{-13}\ \text{J}}\right) = 1.4 \times 10^{2}\ \text{MeV}$$

$$m = \frac{\Delta E}{c^2} \approx 1.4 \times 10^{2}\ \text{MeV}/c^2 \quad \boxed{\sim 10^{2}\ \text{MeV}/c^2}$$

(b) From Table 46.2, $m_\pi c^2 = 139.6\ \text{MeV}$, $\boxed{\text{a pi-meson}}$

***46.44** (a) $\pi^- + \text{p} \to \Sigma^+ + \pi^0$ is forbidden by $\boxed{\text{charge conservation}}$

(b) $\mu^- \to \pi^- + \nu_e$ is forbidden by $\boxed{\text{energy conservation}}$

(c) $\text{p} \to \pi^+ + \pi^+ + \pi^-$ is forbidden by $\boxed{\text{baryon number conservation}}$

46.45 The total energy in neutrinos emitted per second by the Sun is:

$(0.4)(4\pi)(1.5 \times 10^{11})^2 = 1.1 \times 10^{23}\ \text{W}$

Over 10^9 years, the Sun emits 3.6×10^{39} J in neutrinos. This represents an annihilated mass

$mc^2 = 3.6 \times 10^{39}\ \text{J}$

$m = 4.0 \times 10^{22}\ \text{kg}$

About $\boxed{\text{1 part in 50,000,000}}$ of the Sun's mass, over 10^9 years, has been lost to neutrinos.

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Goal Solution

The energy flux carried by neutrinos from the Sun is estimated to be on the order of 0.4 W/m^2 at Earth's surface. Estimate the fractional mass loss of the Sun over 10^9 years due to the radiation of neutrinos. (The mass of the Sun is 2×10^{30} kg. The Earth-Sun distance is 1.5×10^{11} m.)

G: Our Sun is estimated to have a life span of about 10 billion years, so in this problem, we are examining the radiation of neutrinos over a considerable fraction of the Sun's life. However, the mass carried away by the neutrinos is a very small fraction of the total mass involved in the Sun's nuclear fusion process, so even over this long time, the mass of the Sun may not change significantly (probably less than 1%).

O: The change in mass of the Sun can be found from the energy flux received by the Earth and Einstein's famous equation, $E = mc^2$.

A: Since the neutrino flux from the Sun reaching the Earth is 0.4 W/m^2, the total energy emitted per second by the Sun in neutrinos in all directions is

$$\left(0.4 \text{ W / m}^2\right)\left(4\pi r^2\right) = \left(0.4 \text{ W / m}^2\right)(4\pi)\left(1.5 \times 10^{11} \text{ m}\right)^2 = 1.13 \times 10^{23} \text{ W}$$

In a period of 10^9 yr, the Sun emits a total energy of

$$\left(1.13 \times 10^{23} \text{ J / s}\right)\left(10^9 \text{ yr}\right)\left(3.156 \times 10^7 \text{ s / yr}\right) = 3.57 \times 10^{39} \text{ J}$$

in the form of neutrinos. This energy corresponds to an annihilated mass of

$$E = m_\nu c^2 = 3.57 \times 10^{39} \text{ J} \qquad \text{so} \qquad m_\nu = 3.97 \times 10^{22} \text{ kg}$$

Since the Sun has a mass of about 2×10^3 kg, this corresponds to a loss of only about 1 part in 50 000 000 of the Sun's mass over 10^9 yr in the form of neutrinos.

L: It appears that the neutrino flux changes the mass of the Sun by so little that it would be difficult to measure the difference in mass, even over its lifetime!

46.46 $\text{p} + \text{p} \rightarrow \text{p} + \pi^+ + X$

We suppose the protons each have 70.4 MeV of kinetic energy. From conservation of momentum, particle X has zero momentum and thus zero kinetic energy. Conservation of energy then requires

$$M_\text{p}c^2 + M_\pi c^2 + M_X c^2 = \left(M_\text{p}c^2 + K_\text{p}\right) + \left(M_\text{p}c^2 + K_\text{p}\right)$$

$$M_X c^2 = M_\text{p}c^2 + 2K_\text{p} - M_\pi c^2 = 938.3 \text{ MeV} + 2(70.4 \text{ MeV}) - 139.6 \text{ MeV} = 939.5 \text{ MeV}$$

X must be a neutral baryon of rest energy 939.5 MeV. Thus X is a $\boxed{\text{neutron}}$.

***46.47** We find the number N of neutrinos: $10^{46}\text{ J} = N(6\text{ MeV}) = N(6 \times 1.6 \times 10^{-13}\text{ J})$

$$N = 1.0 \times 10^{58}\text{ neutrinos}$$

The intensity at our location is

$$\frac{N}{A} = \frac{N}{4\pi r^2} = \frac{1.0\times10^{58}}{4\pi(1.7\times10^5\text{ ly})^2}\left(\frac{1\text{ ly}}{(3.0\times10^8\text{ m / s})(3.16\times10^7\text{ s})}\right)^2 = 3.1\times10^{14}\text{ / m}^2$$

The number passing through a body presenting $5000\text{ cm}^2 = 0.50\text{ m}^2$ is then

$$\left(3.1\times10^{14}\frac{1}{\text{m}^2}\right)\left(0.50\text{ m}^2\right) = 1.5\times10^{14}\quad\text{or}\quad\boxed{\sim 10^{14}}$$

***46.48** By relativistic energy conservation, $E_\gamma + m_e c^2 = \dfrac{3m_e c^2}{\sqrt{1-v^2/c^2}}$ (1)

By relativistic momentum conservation, $\dfrac{E_\gamma}{c} = \dfrac{3m_e v}{\sqrt{1-v^2/c^2}}$ (2)

Dividing (2) by (1), $X = \dfrac{E_\gamma}{E_\gamma + m_e c^2} = \dfrac{v}{c}$

Subtracting (2) from (1), $m_e c^2 = \dfrac{3m_e c^2}{\sqrt{1-X^2}} - \dfrac{3m_e c^2 X}{\sqrt{1-X^2}}$

Solving, $1 = \dfrac{3-3X}{\sqrt{1-X^2}}$ and $X = \dfrac{4}{5}$ so $E_\gamma = 4m_e c^2 = \boxed{2.04\text{ MeV}}$

46.49 $m_\Lambda c^2 = 1115.6\text{ MeV}$ $\Lambda^0 \to \text{p} + \pi^-$

$m_p c^2 = 938.3\text{ MeV}$ $m_\pi c^2 = 139.6\text{ MeV}$

The difference between starting mass-energy and final mass-energy is the kinetic energy of the products.

$$K_p + K_\pi = 37.7\text{ MeV}\quad\text{and}\quad p_p = p_\pi = p$$

Applying conservation of relativistic energy,

$$\left[\sqrt{(938.3)^2 + p^2c^2} - 938.3\right] + \left[\sqrt{(139.6)^2 + p^2c^2} - 139.6\right] = 37.7\text{ MeV}$$

Solving the algebra yields $p_\pi c = p_p c = 100.4\text{ MeV}$

Then, $K_p = \sqrt{(m_p c^2)^2 + (100.4)^2} - m_p c^2 = \boxed{5.35\text{ MeV}}$

$$K_\pi = \sqrt{(139.6)^2 + (100.4)^2} - 139.6 = \boxed{32.3\text{ MeV}}$$

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46.50 Momentum of proton is $qBr = \left(1.60\times10^{-19}\ \text{C}\right)\left(0.250\ \text{kg / C}\cdot\text{s}\right)\left(1.33\ \text{m}\right)$

$p_p = 5.32\times10^{-20}\ \dfrac{\text{kg}\cdot\text{m}}{\text{s}}$ $\qquad cp_p = 1.60\times10^{-11}\ \dfrac{\text{kg m}^2}{\text{s}^2} = 1.60\times10^{-11}\ \text{J} = 99.8\ \text{MeV}$

Therefore, $p_p = 99.8\ \dfrac{\text{MeV}}{c}$

The total energy of the proton is $E_p = \sqrt{E_0^2 + (cp)^2} = \sqrt{(938.3)^2 + (99.8)^2} = 944\ \text{MeV}$

For pion, the momentum qBr is the same (as it must be from conservation of momentum in a 2-particle decay).

$p_\pi = 99.8\ \dfrac{\text{MeV}}{c}$ $\qquad E_{0\pi} = 139.6\ \text{MeV}$

$$E_\pi = \sqrt{E_0^2 + (cp)^2} = \sqrt{(139.6)^2 + (99.8)^2} = 172\ \text{MeV}$$

Thus, $E_{\text{Total after}} = E_{\text{Total before}} = \text{Rest Energy}$

Rest Energy of unknown particle = 944 MeV + 172 MeV = 1116 MeV (This is a Λ^0 particle!)

Mass = $\boxed{1116\ \text{MeV}/c^2}$

46.51 $\Sigma^0 \to \Lambda^0 + \gamma$

From Table 46.2, $m_\Sigma = 1192.5\ \text{MeV}/c^2$ and $m_\Lambda = 1115.6\ \text{MeV}/c^2$

Conservation of energy requires $E_{0,\Sigma} = \left(E_{0,\Lambda} + K_\Lambda\right) + E_\gamma$,

or $$1192.5\ \text{MeV} = \left(1115.6\ \text{MeV} + \frac{p_\Lambda^2}{2m_\Lambda}\right) + E_\gamma$$

Momentum conservation gives $|p_\Lambda| = |p_\gamma|$, so the last result may be written as

$$1192.5\ \text{MeV} = \left(1115.6\ \text{MeV} + \frac{p_\gamma^2}{2m_\Lambda}\right) + E_\gamma$$

or $$1192.5\ \text{MeV} = \left(1115.6\ \text{MeV} + \frac{p_\gamma^2 c^2}{2m_\Lambda c^2}\right) + E_\gamma$$

Recognizing that $m_\Lambda c^2 = 1115.6\ \text{MeV}$ and $p_\gamma c = E_\gamma$,

we now have $$1192.5\ \text{MeV} = 1115.6\ \text{MeV} + \frac{E_\gamma^2}{2(1115.6\ \text{MeV})} + E_\gamma$$

Solving this quadratic equation, $E_\gamma \approx \boxed{74.4\ \text{MeV}}$

46.52 $p + p \to p + n + \pi^+$

The total momentum is zero before the reaction. Thus, all three particles present after the reaction may be at rest and still conserve momentum. This will be the case when the incident protons have minimum kinetic energy. Under these conditions, conservation of energy gives

$$2\left(m_p c^2 + K_p\right) = m_p c^2 + m_n c^2 + m_\pi c^2$$

so the kinetic energy of each of the incident protons is

$$K_p = \frac{m_n c^2 + m_\pi c^2 - m_p c^2}{2} = \frac{(939.6 + 139.6 - 938.3)\text{ MeV}}{2} = \boxed{70.4\text{ MeV}}$$

46.53 Time-dilated lifetime: $T = \gamma T_0 = \dfrac{0.900 \times 10^{-10}\text{ s}}{\sqrt{1 - v^2/c^2}} = \dfrac{0.900 \times 10^{-10}\text{ s}}{\sqrt{1 - (0.960)^2}} = 3.214 \times 10^{-10}\text{ s}$

distance $= (0.960)(3.00 \times 10^8\text{ m/s})(3.214 \times 10^{-10}\text{ s}) = \boxed{9.26\text{ cm}}$

46.54 $\pi^- \to \mu^- + \bar{\nu}_\mu$

From the conservation laws, $m_\pi c^2 = 139.5\text{ MeV} = E_\mu + E_\nu$ [1]

and $p_\mu = p_\nu, \quad E_\nu = p_\nu c$

Thus, $E_\mu^2 = \left(p_\mu c\right)^2 + (105.7\text{ MeV})^2 = \left(p_\nu c\right)^2 + (105.7\text{ MeV})^2$

or $E_\mu^2 - E_\nu^2 = (105.7\text{ MeV})^2$ [2]

Since $E_\mu + E_\nu = 139.5\text{ MeV}$ [1]

and $\left(E_\mu + E_\nu\right)\left(E_\mu - E_\nu\right) = (105.7\text{ MeV})^2$ [2]

then $E_\mu - E_\nu = \dfrac{(105.7\text{ MeV})^2}{139.5\text{ MeV}} = 80.1$ [3]

Subtracting [3] from [1], $2E_\nu = 59.4\text{ MeV}$ and $\boxed{E_\nu = 29.7\text{ MeV}}$

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***46.55** The expression $e^{-E/k_BT}\,dE$ gives the fraction of the photons that have energy between E and $E+dE$. The fraction that have energy between E and infinity is

$$\frac{\int_E^\infty e^{-E/k_BT}\,dE}{\int_0^\infty e^{-E/k_BT}\,dE}=\frac{\int_E^\infty e^{-E/k_BT}\left(-dE/k_BT\right)}{\int_0^\infty e^{-E/k_BT}\left(-dE/k_BT\right)}=\frac{e^{-E/k_BT}\Big|_E^\infty}{e^{-E/k_BT}\Big|_0^\infty}=e^{-E/k_BT}$$

We require T when this fraction has a value of 0.0100 (i.e., 1.00%)

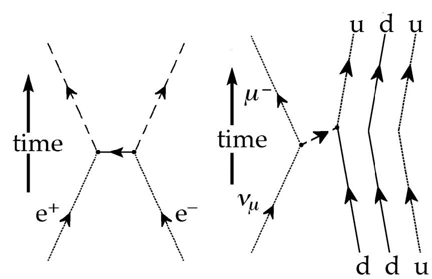
and $E = 1.00\text{ eV} = 1.60\times10^{-19}\text{ J}$

Thus, $0.0100 = e^{-\left(1.60\times10^{-19}\text{ J}\right)/\left(1.38\times10^{-23}\text{ J/K}\right)T}$

or $\ln(0.0100) = -\dfrac{1.60\times10^{-19}\text{ J}}{\left(1.38\times10^{-23}\text{ J/K}\right)T} = -\dfrac{1.16\times10^{4}\text{ K}}{T}$ giving $T = \boxed{2.52\times10^{3}\text{ K}}$

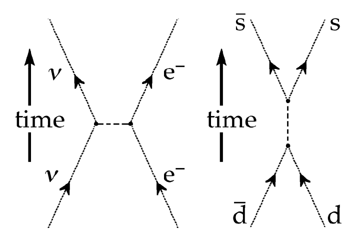
46.56 (a) This diagram represents the annihilation of an electron and an antielectron. From charge and lepton-number conservation at either vertex, the exchanged particle must be an electron, $\boxed{e^-}$.

(b) This is the tough one. A neutrino collides with a neutron, changing it into a proton with release of a muon. This is a weak interaction. The exchanged particle has charge $+1e$ and is a $\boxed{W^+}$.



(a) (b)

46.57 (a) The mediator of this weak interaction is a $\boxed{Z^0\text{ boson}}$.



(a) **(b)**

(b) The Feynman diagram shows a down quark and its antiparticle annihilating each other. They can produce a particle carrying energy, momentum, and angular momentum, but zero charge, zero baryon number, and, it may be, no color charge. In this case the product particle is a $\boxed{\text{photon}}$. For conservation of both energy and momentum, we would expect two photons; but momentum need not be strictly conserved, according to the uncertainty principle, if the photon travels a sufficiently short distance before producing another matter-antimatter pair of particles, as shown in Figure P46.57. Depending on the color charges of the d and $\bar{d}$ quarks, the ephemeral particle could also be a gluon , as suggested in the discussion of Figure 46.13(b).

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