Chapter 44 Solutions

***44.1** An iron nucleus (in hemoglobin) has a few more neutrons than protons, but in a typical water molecule there are eight neutrons and ten protons.

So protons and neutrons are nearly equally numerous in your body, each contributing mass (say) 35 kg:

$$35\text{ kg}\left(\frac{1\text{ nucleon}}{1.67\times10^{-27}\text{ kg}}\right)\boxed{\sim 10^{28}\text{ protons}}\quad\text{and}\quad\boxed{\sim 10^{28}\text{ neutrons}}$$

The electron number is precisely equal to the proton number, $\boxed{\sim 10^{28}\text{ electrons}}$

44.2 $\frac{1}{2}mv^2 = q(\Delta V)$ and $\frac{mv^2}{r} = qvB \;\Rightarrow\; 2m(\Delta V) = qr^2B^2$

$$r = \sqrt{\frac{2m(\Delta V)}{qB^2}} = \left[\frac{2(1000\text{ V})}{(1.60\times10^{-19}\text{ C})(0.200\text{ T})^2}\right]^{1/2}\sqrt{m}$$

$$r = \left(5.59\times10^{11}\ \frac{\text{m}}{\sqrt{\text{kg}}}\right)\sqrt{m}$$

(a) For ^{12}C: $m = 12\text{ u}$ and

$$r = \left(5.59\times10^{11}\ \frac{\text{m}}{\sqrt{\text{kg}}}\right)\sqrt{12(1.66\times10^{-27}\text{ kg})} = 0.0789\text{ m} = \boxed{7.89\text{ cm}}$$

For ^{13}C: $$r = \left(5.59\times10^{11}\ \frac{\text{m}}{\sqrt{\text{kg}}}\right)\sqrt{13(1.66\times10^{-27}\text{ kg})} = 0.0821\text{ m} = \boxed{8.21\text{ cm}}$$

(b) With $r_1 = \sqrt{\frac{2m_1(\Delta V)}{qB^2}}$ and $r_2 = \sqrt{\frac{2m_2(\Delta V)}{qB^2}}$,

the ratio gives $\boxed{\frac{r_1}{r_2} = \sqrt{\frac{m_1}{m_2}}}$.

$\frac{r_1}{r_2} = \frac{7.89\text{ cm}}{8.21\text{ cm}} = 0.961$ and $\sqrt{\frac{m_1}{m_2}} = \sqrt{\frac{12\text{ u}}{13\text{ u}}} = 0.961$ so they do agree.

***44.3** (a) $F = k_e \dfrac{Q_1 Q_2}{r^2} = (8.99 \times 10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)\dfrac{(2)(6)(1.60\times 10^{-19}\ \text{C})^2}{(1.00\times 10^{-14}\ \text{m})^2} = \boxed{27.6\ \text{N}}$

(b) $a = \dfrac{F}{m} = \dfrac{27.6\ \text{N}}{6.64\times 10^{-27}\ \text{kg}} = \boxed{4.17\times 10^{27}\ \text{m/s}^2}$ away from the nucleus..

(c) $U = k_e \dfrac{Q_1 Q_2}{r} = (8.99\times 10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)\dfrac{(2)(6)(1.60\times 10^{-19}\ \text{C})^2}{(1.00\times 10^{-14}\ \text{m})} = 2.76\times 10^{-13}\ \text{J} = \boxed{1.73\ \text{MeV}}$

44.4 $E_\alpha = 7.70\ \text{MeV}$

(a) $d_{\min} = \dfrac{4k_e Ze^2}{mv^2} = \dfrac{2k_e Ze^2}{E_\alpha} = \dfrac{2(8.99\times 10^9)(79)(1.60\times 10^{-19})^2}{7.70(1.60\times 10^{-13})} = 29.5\times 10^{-15}\ \text{m} = \boxed{29.5\ \text{fm}}$

(b) The de Broglie wavelength of the α is

$$\lambda = \frac{h}{m_\alpha v_\alpha} = \frac{h}{\sqrt{2m_\alpha E_\alpha}} = \frac{6.626\times 10^{-34}}{\sqrt{2(6.64\times 10^{-27})7.70(1.60\times 10^{-13})}} = 5.18\times 10^{-15}\ \text{m} = \boxed{5.18\ \text{fm}}$$

(c) Since $\boxed{\lambda \text{ is much less than the distance of closest approach}}$, the α may be considered a particle.

44.5 (a) The initial kinetic energy of the alpha particle must equal the electrostatic potential energy at the distance of closest approach.

$$K_i = U_f = \frac{k_e qQ}{r_{\min}}$$

$$r_{\min} = \frac{k_e qQ}{K_i} = \frac{\left(8.99\times 10^9\ \text{N}\cdot\text{m}^2/\text{C}^2\right)(2)(79)\left(1.60\times 10^{-19}\ \text{C}\right)^2}{(0.500\ \text{MeV})\left(1.60\times 10^{-13}\ \text{J/MeV}\right)} = \boxed{4.55\times 10^{-13}\ \text{m}}$$

(b) Since $K_i = \frac{1}{2} m_\alpha v_i^2 = \dfrac{k_e qQ}{r_{\min}}$,

$$v_i = \sqrt{\frac{2k_e qQ}{m_\alpha r_{\min}}} = \sqrt{\frac{2\left(8.99\times 10^9\ \text{N}\cdot\text{m}^2/\text{C}^2\right)(2)(79)\left(1.60\times 10^{-19}\ \text{C}\right)^2}{(4.00\ \text{u})\left(1.66\times 10^{-27}\ \text{kg/u}\right)\left(3.00\times 10^{-13}\ \text{m}\right)}} = \boxed{6.04\times 10^6\ \text{m/s}}$$

Goal Solution

(a) Use energy methods to calculate the distance of closest approach for a head-on collision between an alpha particle having an initial energy of 0.500 MeV and a gold nucleus (^{197}Au) at rest. (Assume the gold nucleus remains at rest during the collision.) (b) What minimum initial speed must the alpha particle have in order to get as close as 300 fm?

G: The positively charged alpha particle ($q = +2e$) will be repelled by the positive gold nucleus ($Q = +79e$), so that the particles probably will not touch each other in this electrostatic "collision." Therefore, the closest the alpha particle can get to the gold nucleus would be if the two nuclei did touch, in which case the distance between their centers would be about 6 fm (using $r = r_0 A^{1/3}$ for the radius of each nucleus). To get this close, or even within 300 fm, the alpha particle must be traveling very fast, probably close to the speed of light (but of course v must be less than c).

O: At the distance of closest approach, $r_{\min}$, the initial kinetic energy will equal the electrostatic potential energy between the alpha particle and gold nucleus.

A: (a) $K_\alpha = U = k_e \dfrac{qQ}{r_{\min}}$ and $r_{\min} = k_e \dfrac{qQ}{K_\alpha} = \dfrac{(8.99 \times 10^9 \text{ N}\cdot\text{m}^2/\text{C}^2)(2)(79)(1.60 \times 10^{-19} \text{ C})^2}{(0.500 \text{ MeV})(1.60 \times 10^{-13} \text{ J/MeV})} = 455 \text{ fm}$

(b) Since $K_\alpha = \frac{1}{2} m v^2 = k_e \dfrac{qQ}{r_{\min}}$

$$v = \sqrt{\frac{2k_e qQ}{m r_{\min}}} = \sqrt{\frac{2(8.99 \times 10^9 \text{ N}\cdot\text{m}^2/\text{C}^2)(2)(79)(1.60 \times 10^{-19} \text{ C})^2}{4(1.66 \times 10^{-27} \text{ kg})(3.00 \times 10^{-13} \text{ m})}} = 6.04 \times 10^6 \text{ m/s}$$

L: The minimum distance in part (a) is about 100 times greater than the combined radii of the particles. For part (b), the alpha particle must have more than 0.5 MeV of energy since it gets closer to the nucleus than the 455 fm found in part (a). Even so, the speed of the alpha particle in part (b) is only about 2% of the speed of light, so we are justified in not using a relativistic approach. In solving this problem, we ignored the effect of the electrons around the gold nucleus that tend to "screen" the nucleus so that the alpha particle sees a reduced positive charge. If this screening effect were considered, the potential energy would be slightly reduced and the alpha particle could get closer to the gold nucleus for the same initial energy.

***44.6** It must start with kinetic energy equal to $K_i = U_f = k_e qQ/r_f$. Here r_f stands for the sum of the radii of the $^4_2$He and $^{197}_{79}$Au nuclei, computed as

$$r_f = r_0 A_1^{1/3} + r_0 A_2^{1/3} = \left(1.20 \times 10^{-15} \text{ m}\right)\left(4^{1/3} + 197^{1/3}\right) = 8.89 \times 10^{-15} \text{ m}$$

Thus, $$K_i = U_f = \frac{\left(8.99 \times 10^9 \text{ N}\cdot\text{m}^2/\text{C}^2\right)(2)(79)\left(1.60 \times 10^{-19} \text{ C}\right)^2}{8.89 \times 10^{-15} \text{ m}} = 4.09 \times 10^{-12} \text{ J} = \boxed{25.6 \text{ MeV}}$$

44.7 (a) $r = r_0 A^{1/3} = (1.20 \times 10^{-15}\text{ m})(4)^{1/3} = \boxed{1.90 \times 10^{-15}\text{ m}}$

(b) $r = r_0 A^{1/3} = (1.20 \times 10^{-15}\text{ m})(238)^{1/3} = \boxed{7.44 \times 10^{-15}\text{ m}}$

***44.8** From $r = r_0 A^{1/3}$, the radius of uranium is $r_U = r_0 (238)^{1/3}$.

Thus, if $r = \frac{1}{2} r_U$ then $r_0 A^{1/3} = \frac{1}{2} r_0 (238)^{1/3}$

from which $\boxed{A = 30}$

44.9 The number of nucleons in a star of two solar masses is

$$A = \frac{2\left(1.99 \times 10^{30}\text{ kg}\right)}{1.67 \times 10^{-27}\text{ kg/nucleon}} = 2.38 \times 10^{57}\text{ nucleons}$$

Therefore $r = r_0 A^{1/3} = \left(1.20 \times 10^{-15}\text{ m}\right)\left(2.38 \times 10^{57}\right)^{1/3} = \boxed{16.0\text{ km}}$

***44.10** $V = \frac{4}{3}\pi r^3 = 4.16 \times 10^{-5}\text{ m}^3$

$m = \rho V = (2.31 \times 10^{17}\text{ kg / m}^3)(4.16 \times 10^{-5}\text{ m}^3) = 9.61 \times 10^{12}\text{ kg}$ and

$F = G\frac{m_1 m_2}{r^2} = (6.67 \times 10^{-11}\text{ Nm}^2\text{ / kg}^2)\frac{(9.61 \times 10^{12}\text{ kg})^2}{(1.00\text{ m})^2} = \boxed{6.16 \times 10^{15}\text{ N}}$ toward the other ball.

44.11 The stable nuclei that correspond to magic numbers are:

Z magic: ^{2}He ^{8}O ^{20}Ca ^{28}Ni ^{50}Sn ^{82}Pb 126

N magic: T_1^3, He_2^4, N_7^{15}, O_8^{16}, Cl_{17}^{37}, K_{19}^{39}, Ca_{20}^{40}, V_{23}^{51}, Cr_{24}^{52}, Sr_{38}^{88}, Y_{39}^{89},

Zr_{40}^{90}, Xe_{54}^{136}, Ba_{56}^{138}, La_{57}^{139}, Ce_{58}^{140}, Pr_{59}^{141}, Nd_{60}^{142}, Pb_{82}^{208}, Bi_{83}^{209}, Po_{84}^{210}

44.12 Of the 102 stable nuclei listed in Table A.3,

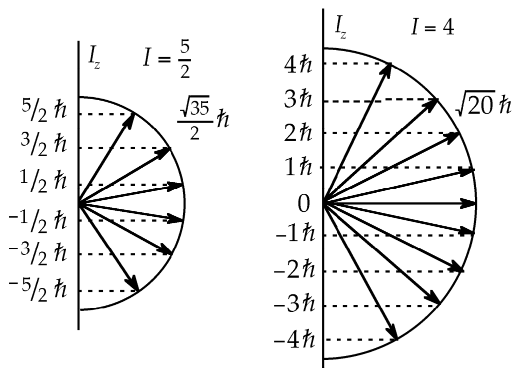
(a) Even Z, Even N $\boxed{48}$

(b) Even Z, Odd N $\boxed{6}$

(c) Odd Z, Even N $\boxed{44}$

(d) Odd Z, Odd N $\boxed{4}$

44.13



(a) (b)

44.14 (a) $f_n = \frac{|2\mu B|}{h} = \frac{2(1.9135)(5.05\times10^{-27}\text{ J / T})(1.00\text{ T})}{6.626\times10^{-34}\text{ J}\cdot\text{s}} = \boxed{29.2\text{ MHz}}$

(b) $f_p = \frac{2(2.7928)(5.05\times10^{-27}\text{ J / T})(1.00\text{ T})}{6.626\times10^{-34}\text{ J}\cdot\text{s}} = \boxed{42.6\text{ MHz}}$

(c) In the Earth's magnetic field, $f_p = \frac{2(2.7928)(5.05\times10^{-27})(50.0\times10^{-6})}{6.626\times10^{-34}} = \boxed{2.13\text{ kHz}}$

44.15 Using atomic masses as given in Table A.3,

(a) For $^2H_1$, $\frac{-2.014\,102 + 1(1.008\,665) + 1(1.007\,825)}{2}$

$$E_b = (0.001194\text{ u})\left(\frac{931.5\text{ MeV}}{\text{u}}\right) = \boxed{1.11\text{ MeV/nucleon}}$$

(b) For ^{4}He, $\frac{2(1.008\,665) + 2(1.007\,825) - 4.002\,602}{4}$

$E_b = 0.00759 \text{ u} = \boxed{7.07 \text{ MeV/nucleon}}$

(c) For $^{56}\text{Fe}_{26}$, $30(1.008\,665) + 26(1.007\,825) - 55.934\,940 = 0.528 \text{ u}$

$E_b = \frac{0.528}{56} = 0.00944 \text{ u} = \boxed{8.79 \text{ MeV/nucleon}}$

(d) For $^{238}\text{U}_{92}$, $146(1.008\,665) + 92(1.007\,825) - 238.050\,784 = 1.934\,2 \text{ u}$

$E_b = \frac{1.934\,2}{238} = 0.00813 \text{ u} = \boxed{7.57 \text{ MeV/nucleon}}$

44.16 $\Delta M = Zm_H + Nm_n - M \qquad \frac{\text{BE}}{A} = \frac{\Delta M(931.5)}{A}$

Nuclei	Z	N	M in u	ΔM in u	BE/A in MeV
^{55}Mn	25	30	54.938048	0.517527	8.765
^{56}Fe	26	30	55.934940	0.528460	8.786
^{59}Co	27	32	58.933198	0.555357	8.768

$\therefore$ ^{56}Fe has a greater BE/A than its neighbors. This tells us finer detail than is shown in Figure 44.8.

44.17 (a) The neutron-to-proton ratio, $(A - Z)/Z$ is greatest for $\boxed{^{139}_{55}\text{Cs}}$ and is equal to 1.53.

(b) $\boxed{^{139}\text{La}}$ has the largest binding energy per nucleon of 8.378 MeV.

(c) ^{139}Cs with a mass of 138.913 u. We locate the nuclei carefully on Figure 44.3, the neutron-proton plot of stable nuclei. $\boxed{\text{Cesium}}$ appears to be farther from the center of the zone of stability. Its instability means extra energy and extra mass.

44.18 Use Equation 44.4.

The $^{23}_{11}$Na, $\frac{E_b}{A} = 8.11 \text{ MeV/nucleon}$

and for $^{23}_{12}$Mg, $\frac{E_b}{A} = 7.90 \text{ MeV/nucleon}$

The binding energy per nucleon is greater for $^{23}_{11}$Na by $\boxed{0.210 \text{ MeV}}$. (There is less proton repulsion in Na^{23}.)

44.19 The binding energy of a nucleus is $E_b(\text{MeV}) = \left[Z\,M(\text{H}) + Nm_n - M\left({}^A_Z\text{X}\right)\right](931.494\ \text{MeV/u})$

For ${}^{15}_{8}\text{O}$: $E_b = \left[8(1.007\ 825\ \text{u}) + 7(1.008\ 665\ \text{u}) - 15.003\ 065\ \text{u}\right](931.494\ \text{MeV/u}) = 111.96\ \text{MeV}$

For ${}^{15}_{7}\text{N}$: $E_b = \left[7(1.007\ 825\ \text{u}) + 8(1.008\ 665\ \text{u}) - 15.000\ 108\ \text{u}\right](931.494\ \text{MeV/u}) = 115.49\ \text{MeV}$

Therefore, $\boxed{\text{the binding energy of } {}^{15}_{7}\text{N is larger by 3.54 MeV}}$.

44.20 (a) The radius of the ${}^{40}\text{Ca}$ nucleus is: $R = r_0 A^{1/3} = \left(1.20 \times 10^{-15}\ \text{m}\right)(40)^{1/3} = 4.10 \times 10^{-15}\ \text{m}$

The energy required to overcome electrostatic repulsion is

$$U = \frac{3k_eQ^2}{5R} = \frac{3\left(8.99 \times 10^9\ \text{N}\cdot\text{m}^2/\text{C}^2\right)\left[20\left(1.60 \times 10^{-19}\ \text{C}\right)\right]^2}{5\left(4.10 \times 10^{-15}\ \text{m}\right)} = 1.35 \times 10^{-11}\ \text{J} = \boxed{84.1\ \text{MeV}}$$

(b) The binding energy of ${}^{40}_{20}\text{Ca}$ is

$$E_b = \left[20(1.007\ 825\ \text{u}) + 20(1.008\ 665\ \text{u}) - 39.962\ 591\ \text{u}\right](931.5\ \text{MeV/u}) = \boxed{342\ \text{MeV}}$$

(c) The nuclear force is so strong that the binding energy greatly exceeds the minimum energy needed to overcome electrostatic repulsion.

44.21 Removal of a neutron from ${}^{43}_{20}\text{Ca}$ would result in the residual nucleus, ${}^{42}_{20}\text{Ca}$. If the required separation energy is S_n, the overall process can be described by

$$\text{mass}\left({}^{43}_{20}\text{Ca}\right) + S_n = \text{mass}\left({}^{42}_{20}\text{Ca}\right) + \text{mass(n)}$$

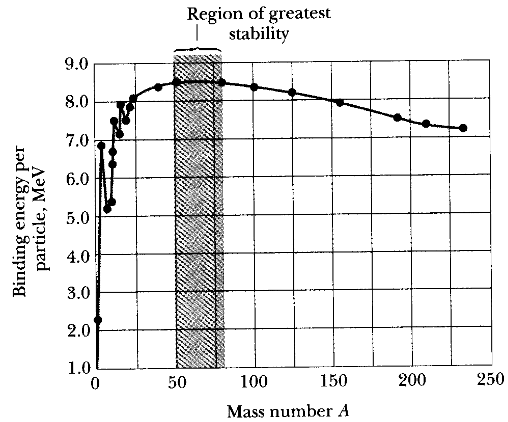
$$S_n = (41.958\ 618 + 1.008\ 665 - 42.958\ 767)\ \text{u} = (0.008\ 516\ \text{u})(931.5\ \text{MeV/u}) = \boxed{7.93\ \text{MeV}}$$

44.22 (a) The first term overstates the importance of volume and the second term *subtracts* this overstatement.

(b) For spherical volume $\dfrac{\frac{4}{3}\pi R^3}{4\pi R^2} = \boxed{\dfrac{R}{3}}$ For cubical volume $\dfrac{R^3}{6R^2} = \boxed{\dfrac{R}{6}}$

The maximum binding energy or lowest state of energy is achieved by building "nearly" spherical nuclei.

44.23 $\Delta E_b = E_{bf} - E_{bi}$



For $A = 200$, $\dfrac{E_b}{A} = 7.4$ MeV

so $E_{bi} = 200(7.4 \text{ MeV}) = 1480$ MeV

For $A \approx 100$, $E_b/A \approx 8.4$ MeV

so $E_{bf} = 2(100)(8.4 \text{ MeV}) = 1680$ MeV

$\Delta E_b = E_{bf} - E_{bi}$

$E_b = 1680 \text{ MeV} - 1480 \text{ MeV} = \boxed{200 \text{ MeV}}$

44.24 (a) "Volume" term: $E_1 = C_1 A = (15.7 \text{ MeV})(56) = 879$ MeV

"Surface" term: $E_2 = -C_2 A^{2/3} = -(17.8 \text{ MeV})(56)^{2/3} = -260$ MeV

"Coulomb" term: $E_3 = -C_3 \dfrac{Z(Z-1)}{A^{1/3}} = -(0.71 \text{ MeV}) \dfrac{(26)(25)}{(56)^{1/3}} = -121$ MeV

"Asymmetry" term: $E_4 = C_4 \dfrac{(A-2Z)^2}{A} = -(23.6 \text{ MeV}) \dfrac{(56-52)^2}{56} = -6.74$ MeV

$\boxed{E_b = 491 \text{ MeV}}$

(b) $\dfrac{E_1}{E_b} = 179\%$; $\dfrac{E_2}{E_b} = -53.0\%$, $\dfrac{E_3}{E_b} = -24.6\%$; $\dfrac{E_4}{E_b} = -1.37\%$

44.25 $\dfrac{dN}{dt} = -\lambda N$ so $\lambda = \dfrac{1}{N}\left(-\dfrac{dN}{dt}\right) = (1.00 \times 10^{-15})(6.00 \times 10^{11}) = 6.00 \times 10^{-4} \text{ s}^{-1}$

$T_{1/2} = \dfrac{\ln 2}{\lambda} = \boxed{1.16 \times 10^3 \text{ s}}$ (= 19.3 min)

***44.26** $R = R_0 e^{-\lambda t} = (6.40 \text{ mCi}) e^{-\left(\frac{\ln 2}{8.04 \text{ d}}\right)(40.2 \text{ d})} = (6.40 \text{ mCi})\left(e^{-\ln 2}\right)^5 = (6.40 \text{ mCi})\left(\dfrac{1}{2^5}\right) = \boxed{0.200 \text{ mCi}}$

44.27 (a) From $R = R_0 e^{-\lambda t}$,

$$\lambda = \frac{1}{t}\ln\left(\frac{R_0}{R}\right) = \left(\frac{1}{4.00\text{ h}}\right)\ln\left(\frac{10.0}{8.00}\right) = 5.58\times10^{-2}\text{ h}^{-1} = \boxed{1.55\times10^{-5}\text{ s}^{-1}} \qquad T_{1/2} = \frac{\ln 2}{\lambda} = \boxed{12.4\text{ h}}$$

(b) $$N_0 = \frac{R_0}{\lambda} = \frac{10.0\times10^{-3}\text{ Ci}}{1.55\times10^{-5}/\text{s}}\left(\frac{3.70\times10^{10}/\text{s}}{1\text{ Ci}}\right) = \boxed{2.39\times10^{13}\text{ atoms}}$$

(c) $$R = R_0 e^{-\lambda t} = (10.0\text{ mCi})\exp(-5.58\times10^{-2}\times30.0) = \boxed{1.87\text{ mCi}}$$

Goal Solution

A freshly prepared sample of a certain radioactive isotope has an activity of 10.0 mCi. After 4.00 h, its activity is 8.00 mCi. (a) Find the decay constant and half-life. (b) How many atoms of the isotope were contained in the freshly prepared sample? (c) What is the sample's activity 30.0 h after it is prepared?

G: Over the course of 4 hours, this isotope lost 20% of its activity, so its half-life appears to be around 10 hours, which means that its activity after 30 hours (~3 half-lives) will be about 1 mCi. The decay constant and number of atoms are not so easy to estimate.

O: From the rate equation, $R = R_0 e^{-\lambda t}$, we can find the decay constant λ, which can then be used to find the half life, the original number of atoms, and the activity at any other time, t.

A: (a) $$\lambda = \frac{1}{t}\ln\left(\frac{R_o}{R}\right) = \left(\frac{1}{(4.00\text{ h})(60.0\text{ s/h})}\right)\ln\left(\frac{10.0\text{ mCi}}{8.00\text{ mCi}}\right) = 1.55\times10^{-5}\text{ s}^{-1}$$

$$T_{1/2} = \frac{\ln 2}{\lambda} = \frac{0.693}{0.0558\text{ h}^{-1}} = 12.4\text{ h}$$

(b) The number of original atoms can be found if we convert the initial activity from curies into becquerels (decays per second): $1\text{ Ci} \equiv 3.7\times10^{10}\text{ Bq}$

$$R_0 = 10.0\text{ mCi} = \left(10.0\times10^{-3}\text{ Ci}\right)\left(3.70\times10^{10}\text{ Bq/Ci}\right) = 3.70\times10^{8}\text{ Bq}$$

Since $R_0 = \lambda N_0$, $$N_0 = \frac{R_0}{\lambda} = \frac{3.70\times10^{8}\text{ decays/s}}{1.55\times10^{-5}\text{ s}} = 2.39\times10^{13}\text{ atoms}$$

(c) $$R = R_o e^{-\lambda t} = (10.0\text{ mCi})e^{-(5.58\times10^{-2}\text{ h}^{-1})(30.0\text{ h})} = 1.87\text{ mCi}$$

L: Our estimate of the half life was about 20% short because we did not account for the non-linearity of the decay rate. Consequently, our estimate of the final activity also fell short, but both of these calculated results are close enough to be reasonable.

The number of atoms is much less than one mole, so this appears to be a very small sample. To get a sense of how small, we can assume that the molar mass is about 100 g/mol, so the sample has a mass of only $m \approx \left(2.4\times10^{13}\text{ atoms}\right)(100\text{ g/mol})\big/\left(6.02\times10^{23}\text{ atoms/mol}\right) \approx 0.004\ \mu\text{g}$

This sample is so small it cannot be measured by a commercial mass balance! The problem states that this sample was "freshly prepared," from which we assumed that **all** the atoms within the sample are initially radioactive. Generally this is not true, so that N_0 only accounts for the formerly radioactive atoms, and does not include additional atoms in the sample that were not radioactive. Realistically then, the sample mass should be significantly greater than our above estimate.

44.28 $R = R_0 e^{-\lambda t}$ where $\lambda = \dfrac{\ln 2}{26.0\text{ h}} = 0.0266/\text{h}$

$\dfrac{R}{R_0} = 0.100 = e^{-\lambda t}$ so $\ln(0.100) = -\lambda t$

$2.30 = \left(\dfrac{0.0266}{\text{h}}\right) t$ $t = \boxed{86.4\text{ h}}$

44.29 The number of nuclei which decay during the interval will be $N_1 - N_2 = N_0\left(e^{-\lambda t_1} - e^{-\lambda t_2}\right)$

First we find λ: $\lambda = \dfrac{\ln 2}{T_{1/2}} = \dfrac{0.693}{64.8\text{ h}} = 0.0107\text{ h}^{-1} = 2.97\times 10^{-6}\text{ s}^{-1}$

and $N_0 = \dfrac{R_0}{\lambda} = \dfrac{(40.0\ \mu\text{Ci})(3.70\times 10^4\text{ cps}/\mu\text{Ci})}{2.97\times 10^{-6}\text{ s}^{-1}} = 4.98\times 10^{11}$ nuclei

Substituting these values, $N_1 - N_2 = \left(4.98\times 10^{11}\right)\left[e^{-(0.0107\text{ h}^{-1})(10.0\text{ h})} - e^{-(0.0107\text{ h}^{-1})(12.0\text{ h})}\right]$

Hence, the number of nuclei decaying during the interval is $N_1 - N_2 = \boxed{9.47\times 10^9\text{ nuclei}}$

44.30 The number of nuclei which decay during the interval will be $N_1 - N_2 = N_0\left(e^{-\lambda t_1} - e^{-\lambda t_2}\right)$

First we find λ: $\lambda = \dfrac{\ln 2}{T_{1/2}}$

so $e^{-\lambda t} = e^{\ln 2(-t/T_{1/2})} = 2^{-t/T_{1/2}}$ and $N_0 = \dfrac{R_0}{\lambda} = \dfrac{R_0 T_{1/2}}{\ln 2}$

Substituting in these values $N_1 - N_2 = \dfrac{R_0 T_{1/2}}{\ln 2}\left(e^{-\lambda t_1} - e^{-\lambda t_2}\right) = \boxed{\dfrac{R_0 T_{1/2}}{\ln 2}\left(2^{-t_1/T_{1/2}} - 2^{-t_2/T_{1/2}}\right)}$

44.31 $R = \lambda N = \left(\dfrac{\ln 2}{5.27\text{ yr}}\right)\left(\dfrac{1.00\text{ g}}{59.93\text{ g/mol}}\right)\left(6.02\times 10^{23}\right)$

$R = \left(1.32\times 10^{21}\ \dfrac{\text{decays}}{\text{yr}}\right)\left(\dfrac{1\text{ yr}}{3.16\times 10^7\text{ s}}\right) = 4.18\times 10^{13}\text{ Bq}$

44.32 (a) $^{65}_{28}\text{Ni}^*$

(b) $^{211}_{82}\text{Pb}$

(c) $^{55}_{27}\text{Co}$

(d) $^{0}_{-1}\text{e}$

(e) $^{1}_{1}\text{H}$ (or p)

44.33 $Q = \left(M_{^{238}\text{U}} - M_{^{234}\text{Th}} - M_{^{4}\text{He}}\right)(931.5\ \text{MeV/u})$

$Q = (238.050\,784 - 234.043\,593 - 4.002\,602)\,\text{u}\,(931.5\ \text{MeV/u}) = \boxed{4.27\ \text{MeV}}$

44.34 $N_C = \left(\dfrac{0.0210\ \text{g}}{12.0\ \text{g/mol}}\right)\left(6.02\times10^{23}\ \text{molecules/mol}\right)$

$\left(N_C = 1.05\times10^{21}\ \text{carbon atoms}\right)$ of which 1 in 7.70×10^{11} is a ^{14}C atom

$(N_0)_{^{14}\text{C}} = 1.37\times10^{9}$, $\lambda_{^{14}\text{C}} = \dfrac{\ln 2}{5730\ \text{yr}} = 1.21\times10^{-4}\ \text{yr}^{-1} = 3.83\times10^{-12}\ \text{s}^{-1}$

$R = \lambda N = \lambda N_0 e^{-\lambda t}$

At $t = 0$, $R_0 = \lambda N_0 = \left(3.83\times10^{-12}\ \text{s}^{-1}\right)\left(1.37\times10^{9}\right)\left[\dfrac{7(86400\ \text{s})}{1\ \text{week}}\right] = 3.17\times10^{3}\ \dfrac{\text{decays}}{\text{week}}$

At time t, $R = \dfrac{837}{0.88} = 951\ \text{decays/week}$

Taking logarithms, $\ln\dfrac{R}{R_0} = -\lambda t$ so $t = \dfrac{-1}{\lambda}\ln\left(\dfrac{R}{R_0}\right)$

$t = \dfrac{-1}{1.21\times10^{-4}\ \text{yr}^{-1}}\ln\left(\dfrac{951}{3.17\times10^{3}}\right) = \boxed{9.96\times10^{3}\ \text{yr}}$

44.35 In the decay $^{3}_{1}\text{H} \to {}^{3}_{2}\text{He} + {}^{0}_{-1}\text{e} + \bar{\nu}$, the energy released is $E = (\Delta m)c^2 = \left[M_{^{3}_{1}\text{H}} - M_{^{3}_{2}\text{He}}\right]c^2$ since the antineutrino is massless and the mass of the electron is accounted for in the masses of $^{3}_{1}\text{H}$ and $^{3}_{2}\text{He}$.

Thus, $E = [3.016\,049\,\text{u} - 3.016\,029\,\text{u}](931.5\ \text{MeV/u}) = 0.0186\ \text{MeV} = \boxed{18.6\ \text{keV}}$

44.36 (a) For e^+ decay,

$$Q = \left(M_X - M_Y - 2m_e\right)c^2 = \left[39.962\ 591\ \text{u} - 39.964\ 000\ \text{u} - 2\left(0.0000\ 549\ \text{u}\right)\right]\left(931.5\ \text{MeV/u}\right)$$

$$Q = -2.34\ \text{MeV}$$

Since $Q < 0$, the decay $\boxed{\text{cannot occur}}$ spontaneously.

(b) For alpha decay,

$$Q = \left(M_X - M_\alpha - M_Y\right)c^2 = \left[97.905\ 287\ \text{u} - 4.002\ 602\ \text{u} - 93.905\ 085\ \text{u}\right]\left(931.5\ \text{MeV/u}\right)$$

$$Q = -2.24\ \text{MeV}$$

Since $Q < 0$, the decay $\boxed{\text{cannot occur}}$ spontaneously.

(c) For alpha decay,

$$Q = \left(M_X - M_\alpha - M_Y\right)c^2 = \left[143.910\ 082\ \text{u} - 4.002\ 602\ \text{u} - 139.905\ 434\ \text{u}\right]\left(931.5\ \text{MeV/u}\right)$$

$$Q = 1.91\ \text{MeV}$$

Since $Q > 0$, the decay $\boxed{\text{can occur}}$ spontaneously.

44.37 (a) $\boxed{e^- + p \to n + \nu}$

(b) For nuclei, $^{15}O + e^- \to {}^{15}N + \nu$.

Add seven electrons to both sides to obtain $\boxed{{}^{15}_{8}O \text{ atom} \to {}^{15}_{7}N \text{ atom} + \nu}$.

(c) From Table A.3, $m\left({}^{15}O\right) = m\left({}^{15}N\right) + \dfrac{Q}{c^2}$

Δm = 15.003 065 u – 15.000 108 u = 0.002 957 u

Q = (931.5 MeV / u)(0.002 957 u) = $\boxed{2.75\ \text{MeV}}$

44.38 (a) Let N be the number of ^{238}U nuclei and N' be ^{206}Pb nuclei.

Then $N = N_0 e^{-\lambda t}$ and $N_0 = N + N'$ so $N = (N + N')e^{-\lambda t}$ or $e^{\lambda t} = 1 + \frac{N'}{N}$

Taking logarithms, $\lambda t = \ln\left(1 + \frac{N'}{N}\right)$ where $\lambda = (\ln 2)/T_{1/2}$.

Thus, $$t = \left(\frac{T_{1/2}}{\ln 2}\right)\ln\left(1 + \frac{N'}{N}\right)$$

If $\frac{N}{N'} = 1.164$ for the $^{238}\text{U} \to {}^{206}\text{Pb}$ chain with $T_{1/2} = 4.47 \times 10^9$ yr, the age is:

$$t = \left(\frac{4.47 \times 10^9 \text{ yr}}{\ln 2}\right)\ln\left(1 + \frac{1}{1.164}\right) = \boxed{4.00 \times 10^9 \text{ yr}}$$

(b) From above, $e^{\lambda t} = 1 + \frac{N'}{N}$. Solving for $\frac{N}{N'}$ gives $\frac{N}{N'} = \frac{e^{-\lambda t}}{1 - e^{-\lambda t}}$

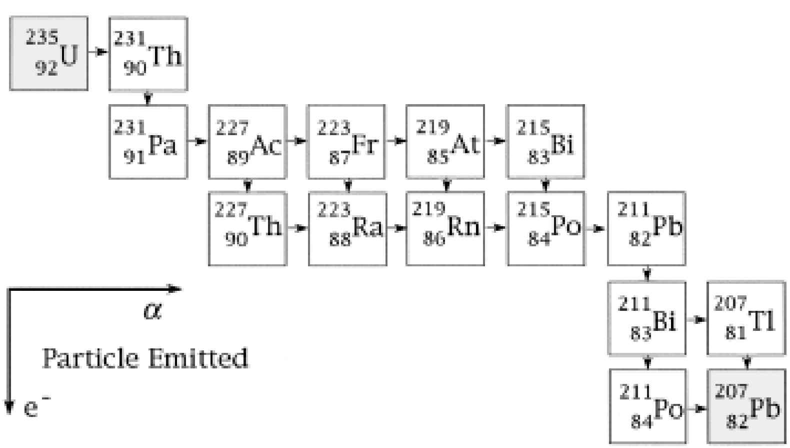
With $t = 4.00 \times 10^9$ yr and $T_{1/2} = 7.04 \times 10^8$ yr for the $^{235}\text{U} \to {}^{207}\text{Pb}$ chain,

$$\lambda t = \left(\frac{\ln 2}{T_{1/2}}\right)t = \frac{(\ln 2)(4.00 \times 10^9 \text{ yr})}{7.04 \times 10^8 \text{ yr}} = 3.938 \quad \text{and} \quad \boxed{\frac{N}{N'} = 0.0199}$$

With $t = 4.00 \times 10^9$ yr and $T_{1/2} = 1.41 \times 10^{10}$ yr for the $^{232}\text{Th} \to {}^{208}\text{Pb}$ chain,

$$\lambda t = \frac{(\ln 2)(4.00 \times 10^9 \text{ yr})}{1.41 \times 10^{10} \text{ yr}} = 0.1966 \quad \text{and} \quad \boxed{\frac{N}{N'} = 4.60}$$

44.39



***44.40** (a) $4.00\ \text{pCi/L} = \left(\frac{4.00\times10^{-12}\ \text{Ci}}{1\ \text{L}}\right)\left(\frac{3.70\times10^{10}\ \text{Bq}}{1\ \text{Ci}}\right)\left(\frac{1.00\times10^{3}\ \text{L}}{1\ \text{m}^3}\right) = \boxed{148\ \text{Bq/m}^3}$

(b) $N = \frac{R}{\lambda} = R\left(\frac{T_{1/2}}{\ln 2}\right) = \left(148\ \frac{\text{Bq}}{\text{m}^3}\right)\left(\frac{3.82\ \text{d}}{\ln 2}\right)\left(\frac{86\ 400\ \text{s}}{1\ \text{d}}\right) = \boxed{7.05\times10^{7}\ \text{atoms/m}^3}$

(c) $\text{mass} = \left(7.05\times10^{7}\ \frac{\text{atoms}}{\text{m}^3}\right)\left(\frac{1\ \text{mol}}{6.02\times10^{23}\ \text{atoms}}\right)\left(\frac{222\ \text{g}}{1\ \text{mol}}\right) = 2.60\times10^{-14}\ \frac{\text{g}}{\text{m}^3}$

Since air has a density of $1.20\ \text{kg/m}^3$, the fraction consisting of radon is

$\text{fraction} = \frac{2.60\times10^{-14}\ \text{g/m}^3}{1.20\ \text{kg/m}^3} = \boxed{2.17\times10^{-17}}$

***44.41** Number remaining: $N = N_0 e^{-(\ln 2)t/T_{1/2}}$

Fraction remaining: $\frac{N}{N_0} = e^{-\lambda t} = e^{-(\ln 2)t/T_{1/2}}$

(a) With $T_{1/2} = 3.82$ d and $t = 7.00$ d, $\frac{N}{N_0} = e^{-(\ln 2)(7.00)/(3.82)} = \boxed{0.281}$

(b) When $t = 1.00\ \text{yr} = 365.25$ d, $\frac{N}{N_0} = e^{-(\ln 2)(365.25)/(3.82)} = \boxed{1.65\times10^{-29}}$

(c) $\boxed{\text{Radon is continuously created}}$ as one daughter in the series of decays starting from the long-lived isotope ^{238}U.

44.42 $Q = \left[M_{^{27}\text{Al}} + M_{\alpha} - M_{^{30}\text{P}} - m_n\right]c^2$

$Q = [26.981\ 538 + 4.002\ 602 - 29.978\ 307 - 1.008\ 665]\ \text{u}\ (931.5\ \text{MeV/u}) = \boxed{-2.64\ \text{MeV}}$

44.43 (a) ${}^{197}_{79}\text{Au} + {}^{1}_{0}\text{n} \rightarrow {}^{197}_{79}\text{Au}^{*} \rightarrow {}^{198}_{80}\text{Hg} + {}^{0}_{-1}\text{e} + \bar{\nu}$

(b) Consider adding 79 electrons: ${}^{197}_{79}\text{Au atom} + {}^{1}_{0}\text{n} \rightarrow {}^{198}_{80}\text{Hg atom} + \bar{\nu} + Q$

$Q = \left[M_{^{197}\text{Au}} + m_{\text{n}} - M_{^{198}\text{Hg}}\right]c^2$

$Q = [196.966\ 543 + 1.008\ 665 - 197.966\ 743]\ \text{u}\ (931.5\ \text{MeV/u}) = \boxed{7.89\ \text{MeV}}$

***44.44** (a) For X, $A = 24 + 1 - 4 = 21$ and $Z = 12 + 0 - 2 = 10$, so X is $\boxed{{}^{21}_{10}\text{Ne}}$

(b) $A = 235 + 1 - 90 - 2 = 144$ and $Z = 92 + 0 - 38 - 0 = 54$, so X is $\boxed{{}^{144}_{54}\text{Xe}}$

(c) $A = 2 - 2 = 0$ and $Z = 2 - 1 = +1$, so X must be a positron.

As it is ejected, so is a neutrino: $\boxed{X = {}^{0}_{1}\text{e}^{+}}$ and $\boxed{X' = {}^{0}_{0}\nu}$

***44.45** Neglect recoil of product nucleus, (i.e., do not require momentum conservation). The energy balance gives $K_{\text{emerging}} = K_{\text{incident}} + Q$. To find Q:

$$Q = \left[\left(M_{\text{H}} + M_{\text{Al}}\right) - \left(M_{\text{Si}} + m_n\right)\right]c^2$$

$$Q = \left[(1.007\ 825 + 26.981\ 528) - (26.986\ 721 + 1.008\ 665)\right]\text{u}\ (931.5\ \text{MeV/u}) = -5.61\ \text{MeV}$$

Thus, $K_{\text{emerging}} = 6.61\ \text{MeV} - 5.61\ \text{MeV} = \boxed{1.00\ \text{MeV}}$

***44.46** (a) ${}^{10}_{5}\text{B} + {}^{4}_{2}\text{He} \rightarrow {}^{13}_{6}\text{C} + {}^{1}_{1}\text{H}$

The product nucleus is $\boxed{{}^{13}_{6}\text{C}}$

(b) ${}^{13}_{6}\text{C} + {}^{1}_{1}\text{H} \rightarrow {}^{10}_{5}\text{B} + {}^{4}_{2}\text{He}$

The product nucleus is $\boxed{{}^{10}_{5}\text{B}}$

44.47 ${}^{9}_{4}\text{Be} + 1.666\ \text{MeV} \rightarrow {}^{8}_{4}\text{Be} + {}^{1}_{0}\text{n}$, so $M_{{}^{8}_{4}\text{Be}} = M_{{}^{9}_{4}\text{Be}} - \dfrac{Q}{c^2} - m_n$

$$M_{{}^{8}_{4}\text{Be}} = 9.012\ 174\ \text{u} - \frac{(-1.666\ \text{MeV})}{931.5\ \text{MeV/u}} - 1.008\ 665\ \text{u} = \boxed{8.005\ 3\ \text{u}}$$

${}^{9}_{4}\text{Be} + {}^{1}_{0}\text{n} \rightarrow {}^{10}_{4}\text{Be} + 6.810\ \text{MeV}$, so $M_{{}^{10}_{4}\text{Be}} = M_{{}^{9}_{4}\text{Be}} + m_n - \dfrac{Q}{c^2}$

$$M_{{}^{10}_{4}\text{Be}} = 9.012\ 174\ \text{u} + 1.008\ 665\ \text{u} - \frac{6.810\ \text{MeV}}{931.5\ \text{MeV/u}} = \boxed{10.013\ 5\ \text{u}}$$

Goal Solution

Using the Q values of appropriate reactions and from Table 44.5, calculate the masses of ^{8}Be and ^{10}Be in atomic mass units to four decimal places.

G: The mass of each isotope in atomic mass units will be approximately the number of nucleons (8 or 10), also called the mass number. The electrons are much less massive and contribute only about 0.03% to the total mass.

O: In addition to summing the mass of the subatomic particles, the net mass of the isotopes must account for the binding energy that holds the atom together. Table 44.5 includes the energy released for each nuclear reaction. Precise atomic masses values are found in Table A.3.

A: The notation $^{9}\text{Be}\,(\gamma,\ \text{n})\ ^{8}\text{Be}$ with $Q = -1.666\ \text{MeV}$

means $^{9}\text{Be} + \gamma \to\ ^{8}\text{Be} + \text{n} - 1.666\ \text{MeV}$

Therefore $m\left(^{8}\text{Be}\right) = m\left(^{9}\text{Be}\right) - m_n + \dfrac{1.666\ \text{MeV}}{931.5\ \text{MeV / u}}$

$$m\left(^{8}\text{Be}\right) = 9.012174 - 1.008665 + 0.001789 = 8.0053\ \text{u}$$

The notation $^{9}\text{Be}\,(\text{n},\ \gamma)\ ^{10}\text{Be}$ with $Q = 6.810\ \text{MeV}$

means $^{9}\text{Be} + \text{n} \to\ ^{10}\text{Be} + \gamma + 6.810\ \text{MeV}$

$$m\left(^{10}\text{Be}\right) = m\left(^{9}\text{Be}\right) + m_n + \frac{6.810\ \text{MeV}}{931.5\ \text{MeV / u}}$$

$$m\left(^{10}\text{Be}\right) = 9.012174 + 1.008665 - 0.001789 = 10.0135\ \text{u}$$

L: As expected, both isotopes have masses slightly greater than their mass numbers. We were asked to calculate the masses to four decimal places, but with the available data, the results could be reported accurately to as many as six decimal places.

44.48 $^{236}_{92}\text{U} \to\ ^{90}_{37}\text{Rb} +\ ^{143}_{55}\text{Cs} + 3\,^{1}_{0}\text{n}$,

so $Q = \left[M_{^{236}_{92}\text{U}} - M_{^{90}_{37}\text{Rb}} - M_{^{143}_{55}\text{Cs}} - 3m_n\right]c^2$

From Table A.3,

$$Q = \left[236.045\,562 - 89.914\,811 - 142.927\,220 - 3(1.008\,665)\right]\text{u}\ (931.5\ \text{MeV/u}) = \boxed{165\ \text{MeV}}$$

44.49 $$\frac{N_1}{N_2} = \frac{N_0 - N_0 e^{-\lambda T_h/2}}{N_0 e^{-\lambda T_h/2} - N_0 e^{-\lambda T_h}} = \frac{1 - e^{-\ln 2/2}}{e^{-\ln 2/2} - e^{-\ln 2}} = \frac{1 - 2^{-1/2}}{2^{-1/2} - 2^{-1}} = \boxed{\sqrt{2}}$$

***44.50** $\;{}^{1}_{1}\text{H} + {}^{7}_{3}\text{Li} \to {}^{7}_{4}\text{Be} + {}^{1}_{0}\text{n}$

$$Q = \left[(M_{\text{H}} + M_{\text{Li}}) - (M_{\text{Be}} + M_{\text{n}})\right](931.5 \text{ MeV / u})$$

$$Q = \left[(1.007\ 825 \text{ u} + 7.016\ 003 \text{ u}) - (7.016\ 928 \text{ u} + 1.008\ 665 \text{ u})\right](931.5 \text{ MeV / u})$$

$$Q = (-1.765 \times 10^{-3} \text{ u})(931.5 \text{ MeV / u}) = -1.644 \text{ MeV}$$

Thus, $$KE_{\min} = \left(1 + \frac{m_{\text{incident projectile}}}{m_{\text{target nucleus}}}\right)|Q| = \left(1 + \frac{1.007\ 825}{7.016\ 003}\right)(1.644 \text{ MeV}) = \boxed{1.88 \text{ MeV}}$$

44.51 (a) $$N_0 = \frac{\text{mass}}{\text{mass per atom}} = \frac{1.00 \text{ kg}}{(239.05 \text{ u})\left(1.66 \times 10^{-27} \text{ kg/u}\right)} = \boxed{2.52 \times 10^{24}}$$

(b) $$\lambda = \frac{\ln 2}{T_{1/2}} = \frac{\ln 2}{\left(2.412 \times 10^{4} \text{ yr}\right)\left(3.156 \times 10^{7} \text{ s/yr}\right)} = 9.106 \times 10^{-13} \text{ s}^{-1}$$

$$R_0 = \lambda N_0 = \left(9.106 \times 10^{-13} \text{ s}^{-1}\right)\left(2.52 \times 10^{24}\right) = \boxed{2.29 \times 10^{12} \text{ Bq}}$$

(c) $$R = R_0 e^{-\lambda t}, \quad \text{so} \quad t = \frac{-1}{\lambda} \ln\left(\frac{R}{R_0}\right) = \frac{1}{\lambda} \ln\left(\frac{R_0}{R}\right)$$

$$t = \frac{1}{9.106 \times 10^{-13} \text{ s}^{-1}} \ln\left(\frac{2.29 \times 10^{12} \text{ Bq}}{0.100 \text{ Bq}}\right) = 3.38 \times 10^{13} \text{ s}\left(\frac{1 \text{ yr}}{3.156 \times 10^{7} \text{ s}}\right) = \boxed{1.07 \times 10^{6} \text{ yr}}$$

44.52 (a) $\;{}^{57}_{27}\text{Co} \to {}^{57}_{26}\text{Fe} + {}^{0}_{+1}\text{e} + {}^{0}_{0}\nu$

The Q – value for this positron emission is $Q = \left[M_{^{57}\text{Co}} - M_{^{57}\text{Fe}} - 2m_e\right]c^2$

$$Q = \left[56.936\ 294 - 56.935\ 396 - 2\left(0.000\ 549\right)\right]\text{u}\ \left(931.5 \text{ MeV/u}\right) = -0.186 \text{ MeV}$$

Since $Q < 0$, this reaction $\boxed{\text{cannot spontaneously occur}}$.

(b) $\;{}^{14}_{6}\text{C} \to {}^{14}_{7}\text{N} + {}^{0}_{-1}\text{e} + {}^{0}_{0}\bar{\nu}$

The Q – value for this e^- decay is $Q = \left[M_{^{14}\text{C}} - M_{^{14}\text{N}}\right]c^2$.

$$Q = \left[14.003\ 242 - 14.003\ 074\right]\text{u}\ \left(931.5 \text{ MeV/u}\right) = 0.156 \text{ MeV} = 156 \text{ keV}$$

Since $Q > 0$, the decay $\boxed{\text{can spontaneously occur}}$.

(c) The energy released in the reaction of (b) is shared by the electron and neutrino. Thus, $\boxed{K_e \text{ can range from zero to 156 keV}}$.

44.53 (a) $r = r_0 A^{1/3} = 1.20 \times 10^{-15} A^{1/3}$ m. When $A = 12$, $r = \boxed{2.75 \times 10^{-15}\ \text{m}}$

(b) $$F = \frac{k_e (Z-1)e^2}{r^2} = \frac{(8.99 \times 10^9\ \text{N} \cdot \text{m}^2 / \text{C}^2)(Z-1)(1.60 \times 10^{-19}\ \text{C})^2}{r^2}$$

When $Z = 6$ and $r = 2.75 \times 10^{-15}$ m, $F = \boxed{152\ \text{N}}$

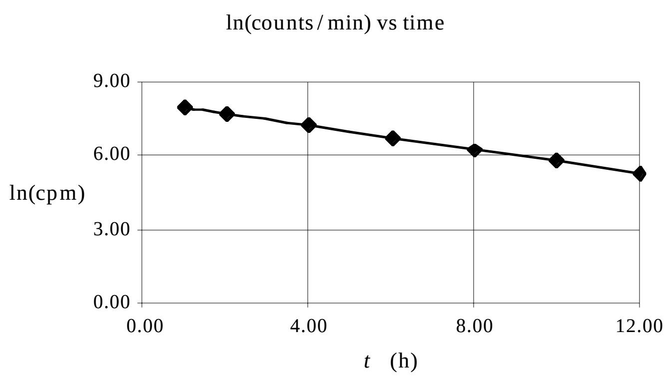
(c) $$U = \frac{k_e q_1 q_2}{r} = \frac{k_e (Z-1)e^2}{r} = \frac{(8.99 \times 10^9)(1.6 \times 10^{-19})^2 (Z-1)}{r}$$

When $Z = 6$ and $r = 2.75 \times 10^{-15}$ m, $U = 4.19 \times 10^{-13}\ \text{J} = \boxed{2.62\ \text{MeV}}$

(d) $A = 238$; $Z = 92$, $r = \boxed{7.44 \times 10^{-15}\ \text{m}}$ $F = \boxed{379\ \text{N}}$

and $U = 2.82 \times 10^{-12}\ \text{J} = \boxed{17.6\ \text{MeV}}$

44.54 (a)



A least-square fit to the graph yields: $\lambda = -\text{slope} = -\left(-0.250\ \text{h}^{-1}\right) = 0.250\ \text{h}^{-1}$,

and $\ln(\text{cpm})\big|_{t=0} = \text{intercept} = 8.30$

(b) $$\lambda = 0.250\ \text{h}^{-1}\left(\frac{1\ \text{h}}{60.0\ \text{min}}\right) = \boxed{4.17 \times 10^{-3}\ \text{min}^{-1}}$$

$$T_{1/2} = \frac{\ln 2}{\lambda} = \frac{\ln 2}{4.17 \times 10^{-3}\ \text{min}^{-1}} = 166\ \text{min} = \boxed{2.77\ \text{h}}$$

(c) From (a), intercept = $\ln(\text{cpm})_0 = 8.30$.

Thus, $(\text{cpm})_0 = e^{8.30}$ counts/min = $\boxed{4.02 \times 10^3\ \text{counts/min}}$

(d) $$N_0 = \frac{R_0}{\lambda} = \frac{1}{\lambda}\frac{(\text{cpm})_0}{\text{Eff}} = \frac{4.02 \times 10^3\ \text{counts/min}}{\left(4.17 \times 10^{-3}\ \text{min}^{-1}\right)(0.100)} = \boxed{9.65 \times 10^6\ \text{atoms}}$$

44.55 (a) Because the reaction $p \to n + e^+ + \nu$ would violate the law of $\boxed{\text{conservation of energy}}$,

$m_p = 1.007\ 276$ u $\quad m_n = 1.008\ 665$ u $\quad m_{e^+} = 5.49 \times 10^{-4}$ u $\qquad$ Note that $\quad m_n + m_{e^+} > m_p$

(b) The $\boxed{\text{required energy can come from the electrostatic repulsion}}$ of protons in the nucleus.

(c) Add seven electrons to both sides of the reaction for nuclei $\quad {}^{13}_{7}\text{N} \to {}^{13}_{6}\text{C} + e^+ + \nu$

to obtain the reaction for neutral atoms $\quad {}^{13}_{7}\text{N atom} \to {}^{13}_{6}\text{C atom} + e^+ + e^- + \nu$

$$Q = c^2\left[m\left({}^{13}\text{N}\right) - m\left({}^{13}\text{C}\right) - m_{e^+} - m_{e^-} - m_\nu\right]$$

$$Q = (931.5\ \text{MeV / u})\left[13.005\ 738 - 13.003\ 355 - 2\left(5.49 \times 10^{-4}\right) - 0\right]\text{u}$$

$$Q = (931.5\ \text{MeV/u})(1.285 \times 10^{-3}\ \text{u}) = \boxed{1.20\ \text{MeV}}$$

44.56 (a) If we assume all the ^{87}Sr came from ^{87}Rb, then $N = N_0 e^{-\lambda t}$ yields

$$t = \frac{-1}{\lambda}\ln\left(\frac{N}{N_0}\right) = \frac{T_{1/2}}{\ln 2}\ln\left(\frac{N_0}{N}\right), \quad \text{where} \quad N = N_{^{87}\text{Rr}} \text{ and } N_0 = N_{^{87}\text{Sr}} + N_{^{87}\text{Rb}}.$$

$$t = \frac{\left(4.75 \times 10^{10}\ \text{yr}\right)}{\ln 2}\ln\left(\frac{1.82 \times 10^{10} + 1.07 \times 10^{9}}{1.82 \times 10^{10}}\right) = \boxed{3.91 \times 10^9\ \text{yr}}$$

(b) It could be $\boxed{\text{no longer}}$. The rock could be younger if some ^{87}Sr were originally present.

44.57 (a) Let us assume that the parent nucleus (mass M_p) is initially at rest, and let us denote the masses of the daughter nucleus and alpha particle by M_d and M_α, respectively. Applying the equations of conservation of momentum and energy for the alpha decay process gives

$$M_d v_d = M_\alpha v_\alpha \tag{1}$$

$$M_p c^2 = M_d c^2 + M_\alpha c^2 + \tfrac{1}{2} M_\alpha v_\alpha^2 + \tfrac{1}{2} M_d v_d^2 \tag{2}$$

The disintegration energy Q is given by $\quad Q = (M_p - M_d - M_\alpha)c^2 = \tfrac{1}{2} M_\alpha v_\alpha^2 + \tfrac{1}{2} M_d v_d^2 \qquad (3)$

Eliminating v_d from Equations (1) and (3) gives

$$Q = \tfrac{1}{2} M_d\left(\frac{M_\alpha}{M_d} v_\alpha\right)^2 + \tfrac{1}{2} M_\alpha v_\alpha^2 = \frac{1}{2}\frac{M_\alpha^2}{M_d} v_\alpha^2 + \tfrac{1}{2} M_\alpha v_\alpha^2 = \tfrac{1}{2} M_\alpha v_\alpha^2\left(1 + \frac{M_\alpha}{M_d}\right) = \boxed{K_\alpha\left(1 + \frac{M_\alpha}{M_d}\right)}$$

(b) $$K_\alpha = \frac{Q}{1 + \left(M_\alpha / M_d\right)} = \frac{4.87\ \text{MeV}}{1 + (4 / 222)} = \boxed{4.78\ \text{MeV}}$$

44.58 (a) The reaction is $^{145}_{61}\text{Pm} \to {}^{141}_{59}\text{Pr} + \alpha$

(b) $Q = (M_{\text{Pm}} - M_\alpha - M_{\text{Pr}})931.5 = (144.912\,745 - 4.002\,602 - 140.907\,647)931.5 = \boxed{2.32 \text{ MeV}}$

(c) The alpha and daughter have equal and opposite momenta $p_\alpha = p_d$

$$E_\alpha = \frac{p_\alpha^2}{2m_\alpha} \qquad E_d = \frac{p_d^2}{2m_d}$$

$$\frac{E_\alpha}{E_{\text{tot}}} = \frac{E_\alpha}{E_\alpha + E_d} = \frac{\frac{p_\alpha^2}{2m_\alpha}}{\frac{p_\alpha^2}{2m_a} + \frac{p_\alpha^2}{2m_d}} = \frac{\frac{1}{2m_\alpha}}{\frac{1}{2m_\alpha} + \frac{1}{2m_d}} = \frac{m_d}{m_d + m_\alpha} = \frac{141}{141+4} = \boxed{97.2\%} \text{ or } 2.26 \text{ MeV}$$

This is carried away by the alpha

44.59 (a) If ΔE is the energy difference between the excited and ground states of the nucleus of mass M, and hf is the energy of the emitted photon, conservation of energy gives

$$\Delta E = hf + E_r \qquad (1)$$

Where E_r is the recoil energy of the nucleus, which can be expressed as

$$E_r = \frac{Mv^2}{2} = \frac{(Mv)^2}{2M} \qquad (2)$$

Since momentum must also be conserved, we have

$$Mv = \frac{hf}{c} \qquad (3)$$

Hence, E_r can be expressed as $E_r = \dfrac{(hf)^2}{2Mc^2}$.

When $hf \ll Mc^2$, we can make the approximation that $hf \approx \Delta E$, so $E_r \approx \boxed{\dfrac{(\Delta E)^2}{2Mc^2}}$

(b) $E_r = \dfrac{(\Delta E)^2}{2Mc^2}$ where $\Delta E = 0.0144$ MeV and $Mc^2 = (57 \text{ u})(931.5 \text{ MeV/u}) = 5.31 \times 10^4$ MeV

Therefore, $$E_r = \frac{(1.44 \times 10^{-2} \text{ MeV})^2}{(2)(5.31 \times 10^4 \text{ MeV})} = \boxed{1.94 \times 10^{-3} \text{ eV}}$$

***44.60** (a) One liter of milk contains this many ^{40}K nuclei:

$$N = (2.00\text{ g})\left(\frac{6.02\times10^{23}\text{ nuclei/mol}}{39.1\text{ g/mol}}\right)\left(\frac{0.0117}{100}\right) = 3.60\times10^{18}\text{ nuclei}$$

$$\lambda = \frac{\ln 2}{T_{1/2}} = \frac{\ln 2}{1.28\times10^{9}\text{ yr}}\left(\frac{1\text{ yr}}{3.156\times10^{7}\text{ s}}\right) = 1.72\times10^{-17}\text{ s}^{-1}$$

$$R = \lambda N = \left(1.72\times10^{-17}\text{ s}^{-1}\right)\left(3.60\times10^{18}\right) = \boxed{61.8\text{ Bq}}$$

(b) For the iodine, $R = R_0 e^{-\lambda t}$ with $\lambda = \dfrac{\ln 2}{8.04\text{ d}}$.

$$t = \frac{1}{\lambda}\ln\left(\frac{R_0}{R}\right) = \frac{8.04\text{ d}}{\ln 2}\ln\left(\frac{2000}{61.8}\right) = \boxed{40.3\text{ d}}$$

***44.61** (a) For cobalt-56, $\lambda = \dfrac{\ln 2}{T_{1/2}} = \dfrac{\ln 2}{77.1\text{ d}}\left(\dfrac{365.25\text{ d}}{1\text{ yr}}\right) = 3.28\text{ yr}^{-1}$.

The elapsed time from July 1054 to July 2000 is 946 yr.

$R = R_0 e^{-\lambda t}$ implies $\dfrac{R}{R_0} = e^{-\lambda t} = e^{-\left(3.28\text{ yr}^{-1}\right)(946\text{ yr})} = e^{-3106} = e^{-(\ln 10)1349} = \boxed{\sim 10^{-1349}}$

(b) For carbon-14, $\lambda = \dfrac{\ln 2}{5730\text{ yr}} = 1.21\times10^{-4}\text{ yr}^{-1}$

$$\frac{R}{R_0} = e^{-\lambda t} = e^{-\left(1.21\times10^{-4}\text{ yr}^{-1}\right)(946\text{ yr})} = e^{-0.114} = \boxed{0.892}$$

***44.62** We have $N_{235} = N_{0,235}e^{-\lambda_{235}t}$ and $N_{238} = N_{0,238}e^{-\lambda_{238}t}$

$$\frac{N_{235}}{N_{238}} = 0.00725 = e^{\left(-(\ln 2)t/T_{h,235} + (\ln 2)t/T_{h,238}\right)}$$

Taking logarithms, $-4.93 = \left(-\dfrac{\ln 2}{0.704\times10^{9}\text{ yr}} + \dfrac{\ln 2}{4.47\times10^{9}\text{ yr}}\right)t$

or $-4.93 = \left(-\dfrac{1}{0.704\times10^{9}\text{ yr}} + \dfrac{1}{4.47\times10^{9}\text{ yr}}\right)(\ln 2)t$

$$t = \frac{-4.93}{\left(-1.20\times10^{-9}\text{ yr}^{-1}\right)\ln 2} = \boxed{5.94\times10^{9}\text{ yr}}$$

44.63 (a) Add two electrons to both sides of the reaction to have it in energy terms: $4\,{}^1_1\text{H atom} \rightarrow {}^4_2\text{He atom} + Q$

$$Q = \Delta mc^2 = \left[4\,M_{{}^1_1\text{H}} - M_{{}^4_2\text{He}}\right]c^2$$

$$Q = \left[4(1.007\ 825\ \text{u}) - 4.002\ 602\ \text{u}\right](931.5\ \text{MeV/u})\left(\frac{1.60\times10^{-13}\ \text{J}}{1\ \text{MeV}}\right) = \boxed{4.28\times10^{-12}\ \text{J}}$$

(b) $$N = \frac{1.99\times10^{30}\ \text{kg}}{1.67\times10^{-27}\ \text{kg/atom}} = \boxed{1.19\times10^{57}\ \text{atoms}} = 1.19\times10^{57}\ \text{protons}$$

(c) The energy that could be created by this many protons in this reaction is:

$$\left(1.19\times10^{57}\ \text{protons}\right)\left(\frac{4.28\times10^{-12}\ \text{J}}{4\ \text{protons}}\right) = 1.27\times10^{45}\ \text{J}$$

$$\mathcal{P} = \frac{E}{t} \qquad \text{so} \qquad t = \frac{E}{\mathcal{P}} = \frac{1.27\times10^{45}\ \text{J}}{3.77\times10^{26}\ \text{W}} = 3.38\times10^{18}\ \text{s} = \boxed{107\ \text{billion years}}$$

44.64 (a) $$Q = \left[M_{{}^9\text{Be}} + M_{{}^4\text{He}} - M_{{}^{12}\text{C}} - m_n\right]c^2$$

$$Q = \left[9.012\ 174\ \text{u} + 4.002\ 602\ \text{u} - 12.000\ 000\ \text{u} - 1.008\ 665\ \text{u}\right](931.5\ \text{MeV/u}) = \boxed{5.69\ \text{MeV}}$$

(b) $$Q = \left[2\,M_{{}^2\text{H}} - M_{{}^3\text{He}} - m_n\right]$$

$$Q = \left[2(2.014\ 102) - 3.016\ 029 - 1.008\ 665\right]\text{u}\ (931.5\ \text{MeV/u}) = \boxed{3.27\ \text{MeV (exothermic)}}$$

44.65 $E = -\boldsymbol{\mu}\cdot\mathbf{B}$ so the energies are $E_1 = +\mu B$ and $E_2 = -\mu B$

$\mu = 2.7928\mu_n$ and $\mu_n = 5.05\times10^{-27}\ \text{J/T}$

$$\Delta E = 2\mu B = 2(2.7928)(5.05\times10^{-27}\ \text{J/T})(12.5\ \text{T}) = 3.53\times10^{-25}\ \text{J} = \boxed{2.20\times10^{-6}\ \text{eV}}$$

44.66 (a) $\lambda = \frac{\ln 2}{T_{1/2}} = \frac{\ln 2}{5.27\text{ yr}}\left(\frac{1\text{ yr}}{3.156\times 10^{7}\text{ s}}\right) = 4.17\times 10^{-9}\text{ s}^{-1}$

$$t = 30.0\text{ months} = (2.50\text{ yr})\left(\frac{3.156\times 10^{7}\text{ s}}{1\text{ yr}}\right) = 7.89\times 10^{7}\text{ s}$$

$$R = R_0 e^{-\lambda t} = (\lambda N_0)e^{-\lambda t}$$

so $N_0 = \left(\frac{R}{\lambda}\right)e^{\lambda t} = \left[\frac{(10.0\text{ Ci})(3.70\times 10^{10}\text{ Bq/Ci})}{4.17\times 10^{-9}\text{ s}^{-1}}\right]e^{(4.17\times 10^{-9}\text{ s}^{-1})(7.89\times 10^{7}\text{ s})}$

$$N_0 = 1.23\times 10^{20}\text{ nuclei}$$

$$\text{Mass} = (1.23\times 10^{20}\text{ atoms})\left(\frac{59.93\text{ g/mol}}{6.02\times 10^{23}\text{ atoms/mol}}\right) = 1.23\times 10^{-2}\text{ g} = \boxed{12.3\text{ mg}}$$

(b) We suppose that each decaying nucleus promptly puts out both a beta particle and two gamma rays, for

$$Q = (0.310 + 1.17 + 1.33)\text{ MeV} = 2.81\text{ Mev}$$

$$\mathcal{P} = QR = (2.81\text{ Mev})(1.6\times 10^{-13}\text{ J/MeV})(3.70\times 10^{11}\text{ s}^{-1}) = \boxed{0.166\text{ W}}$$

44.67 For an electric charge density $\rho = \frac{Ze}{\frac{4}{3}\pi R^3}$

Using Gauss's Law inside the sphere,

$$E\cdot 4\pi r^2 = \frac{\frac{4}{3}\pi r^3}{\epsilon_0}\frac{Ze}{\frac{4}{3}\pi R^3}: \qquad E = \frac{1}{4\pi\epsilon_0}\frac{Zer}{R^3} \quad (r \le R)$$

$$E = \frac{1}{4\pi\epsilon_0}\frac{Ze}{r^2} \quad (r \ge R)$$

We now find the electrostatic energy: $U = \int_{r=0}^{\infty}\frac{1}{2}\epsilon_0 E^2 4\pi r^2\,dr$

$$U = \frac{1}{2}\epsilon_0\int_0^R\left(\frac{1}{4\pi\epsilon_0}\right)^2\frac{Z^2e^2r^2}{R^6}4\pi r^2\,dr + \frac{1}{2}\epsilon_0\int_R^{\infty}\left(\frac{1}{4\pi\epsilon_0}\right)^2\frac{Z^2e^2}{r^4}4\pi r^2\,dr = \frac{Z^2e^2}{8\pi\epsilon_0}\left[\frac{R^5}{5R^6} + \frac{1}{R}\right] = \boxed{\frac{3}{20}\frac{Z^2e^2}{\pi\epsilon_0 R}}$$

44.68 (a) For the electron capture, $^{93}_{43}\mathrm{Tc} + {}^{0}_{-1}\mathrm{e} \rightarrow {}^{93}_{42}\mathrm{Mo} + \gamma$

The disintegration energy is $Q = \left[M_{^{93}\mathrm{Tc}} - M_{^{93}\mathrm{Mo}}\right]c^2$.

$$Q = [92.910\ 2 - 92.906\ 8]\,\mathrm{u}\,(931.5\ \mathrm{MeV/u}) = 3.17\ \mathrm{MeV} > 2.44\ \mathrm{MeV}$$

Electron capture is allowed $\boxed{\text{to all specified excited states}}$ in $^{93}_{42}\mathrm{Mo}$.

For positron emission, $^{93}_{43}\mathrm{Tc} \rightarrow {}^{93}_{42}\mathrm{Mo} + {}^{0}_{+1}\mathrm{e} + \gamma$

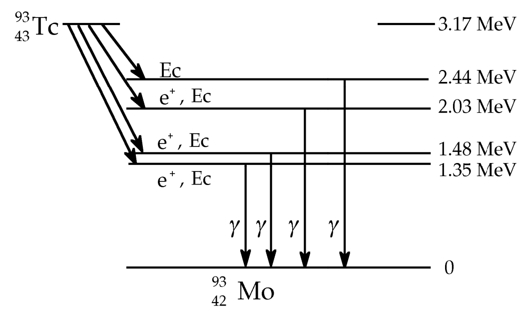
The disintegration energy is $Q' = \left[M_{^{93}\mathrm{Tc}} - M_{^{93}\mathrm{Mo}} - 2m_e\right]c^2$.

$$Q' = [92.910\ 2 - 92.906\ 8 - 2(0.000\ 549)]\,\mathrm{u}\,(931.5\ \mathrm{MeV/u}) = 2.14\ \mathrm{MeV}$$

Positron emission can reach

$\boxed{\text{the 1.35, 1.48, and 2.03 MeV states}}$

but there is insufficient energy to reach the 2.44 MeV state.



(b) The daughter nucleus in both forms of decay is $\boxed{^{93}_{42}\mathrm{Mo}}$.

44.69 $K = \frac{1}{2}mv^2$,

so $$v = \sqrt{\frac{2K}{m}} = \sqrt{\frac{2(0.0400\ \mathrm{eV})(1.60 \times 10^{-19}\ \mathrm{J/eV})}{1.67 \times 10^{-27}\ \mathrm{kg}}} = 2.77 \times 10^3\ \mathrm{m\ s}$$

The time for the trip is $t = \frac{x}{v} = \frac{1.00 \times 10^4\ \mathrm{m}}{2.77 \times 10^3\ \mathrm{m/s}} = 3.61\ \mathrm{s}$

The number of neutrons finishing the trip is given by $N = N_0 e^{-\lambda t}$.

The fraction decaying is $1 - \frac{N}{N_0} = 1 - e^{-(\ln 2)t/T_{1/2}} = 1 - e^{-(\ln 2)(3.61\ \mathrm{s}/624\ \mathrm{s})} = 0.004\ 00 = \boxed{0.400\%}$

44.70 (a) At threshold, the particles have no kinetic energy relative to each other. That is, they move like two particles which have suffered a perfectly inelastic collision. Therefore, in order to calculate the reaction threshold energy, we can use the results of a perfectly inelastic collision. Initially, the projectile M_a moves with velocity v_a while the target M_X is at rest. We have from momentum conservation: $M_a v_a = (M_a + M_X) v_c$

The initial energy is: $E_i = \frac{1}{2} M_a v_a^2$

The final kinetic energy is: $E_f = \frac{1}{2}(M_a + M_X) v_c^2 = \frac{1}{2}(M_a + M_X)\left[\frac{M_a v_a}{M_a + M_X}\right]^2 = \left[\frac{M_a}{(M_a + M_X)}\right] E_i$

From this, we see that E_f is always less than E_i and the *loss* in energy, $E_i - E_f$, is given by

$$E_i - E_f = \left[1 - \frac{M_a}{M_a + M_X}\right] E_i = \left[\frac{M_X}{M_a + M_X}\right] E_i$$

In this problem, the energy loss is the disintegration energy $-Q$ and the initial energy is the threshold energy E_{th}. Therefore,

$$-Q = \left[\frac{M_X}{M_a + M_X}\right] E_{th} \quad \text{or} \quad E_{th} = -Q\left[\frac{M_X + M_a}{M_X}\right] = \boxed{-Q\left[1 + \frac{M_a}{M_X}\right]}$$

(b) First, calculate the $Q-$value for the reaction: $Q = \left[M_{^{14}\text{N}} + M_{^{4}\text{He}} - M_{^{17}\text{O}} - M_{^{1}\text{H}}\right] c^2$

$$Q = [14.003\,074 + 4.002\,602 - 16.999\,132 - 1.007\,825]\,\text{u}\,(931.5\ \text{MeV/u}) = -1.19\ \text{MeV}$$

Then, $E_{th} = -Q\left[\frac{M_X + M_a}{M_X}\right] = -(-1.19\ \text{MeV})\left[1 + \frac{4.002\,602}{14.003\,074}\right] = \boxed{1.53\ \text{MeV}}$

44.71 $R = R_0 \exp(-\lambda t)$

$\ln R = \ln R_0 - \lambda t$ (the equation of a straight line)

$|\text{slope}| = \lambda$

The logarithmic plot shown in Figure P44.71 is fitted by

$\ln R = 8.44 - 0.262t$.

If t is measured in minutes, then the decay constant λ is 0.262 per minute. The half-life is

$$T_{1/2} = \frac{\ln 2}{\lambda} = \frac{\ln 2}{0.262\ /\ \text{min}} = \boxed{2.64\ \text{min}}$$

The reported half-life of ^{137}Ba is 2.55 min. The difference reflects experimental uncertainties.