Chapter 42 Solutions

42.1 (a) The point of closest approach is found when

$$E = K + U = 0 + \frac{k_e q_\alpha q_{\text{Au}}}{r} \quad \text{or} \quad r_{\min} = \frac{k_e(2e)(79e)}{E}$$

$$r_{\min} = \frac{\left(8.99\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2\right)158\left(1.60\times10^{-19}\ \text{C}\right)^2}{(4.00\ \text{MeV})\left(1.60\times10^{-13}\ \text{J/MeV}\right)} = \boxed{5.68\times10^{-14}\ \text{m}}$$

(b) The maximum force exerted on the alpha particle is

$$F_{\max} = \frac{k_e q_\alpha q_{\text{Au}}}{r_{\min}^2} = \frac{\left(8.99\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2\right)158\left(1.60\times10^{-19}\ \text{C}\right)^2}{\left(5.68\times10^{-14}\ \text{m}\right)^2} = \boxed{11.3\ \text{N}} \text{ away from the nucleus}$$

42.2 (a) The point of closest approach is found when

$$E = K + U = 0 + \frac{k_e q_\alpha q_{\text{T}}}{r} \quad \text{or} \quad r_{\min} = \frac{k_e(2e)(Ze)}{E} = \boxed{\frac{2Zk_e e^2}{E}}$$

(b) The maximum force exerted on the alpha particle is

$$F_{\max} = \frac{k_e q_\alpha q_{\text{T}}}{r_{\min}^2} = 2Zk_e e^2\left(\frac{E}{2Zk_e e^2}\right)^2 = \boxed{\frac{E^2}{2Zk_e e^2}} \text{ away from the target nucleus}$$

42.3 (a) The photon has energy 2.28 eV.

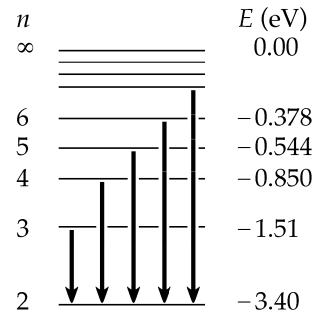
And $(13.6\ \text{eV})/2^2 = 3.40\ \text{eV}$ is required to ionize a hydrogen atom from state $n = 2$. So while the photon cannot ionize a hydrogen atom pre-excited to $n = 2$, it can ionize a hydrogen atom in the $n = \boxed{3}$ state, with energy

$$-\frac{13.6\ \text{eV}}{3^2} = -1.51\ \text{eV}$$

(b) The electron thus freed can have kinetic energy $K_e = 2.28\ \text{eV} - 1.51\ \text{eV} = 0.769\ \text{eV} = \frac{1}{2}m_e v^2$

$$v = \sqrt{\frac{2(0.769)(1.60\times10^{-19})\text{J}}{(9.11\times10^{-31})\text{kg}}} = \boxed{520\ \text{km/s}}$$

***42.4** (a) Longest wavelength implies lowest frequency and smallest energy: the electron falls from $n = 3$ to $n = 2$, losing energy



Balmer Series

$$-\frac{13.6\text{ eV}}{3^2} + \frac{13.6\text{ eV}}{2^2} = \boxed{1.89\text{ eV}}$$

The photon frequency is $f = \Delta E/h$ and its wavelength is

$$\lambda = \frac{c}{f} = \frac{ch}{\Delta E} = \frac{(3.00\times10^{8}\text{ m / s})(6.626\times10^{-34}\text{ J}\cdot\text{s})}{1.89\text{ eV}}\left(\frac{\text{eV}}{1.60\times10^{-19}\text{ J}}\right)$$

$$\lambda = \boxed{656\text{ nm}}$$

(b) The biggest energy loss is for an electron to fall from ionization, $n = \infty$, to the $n = 2$ state.

It loses energy $$-\frac{13.6\text{ eV}}{\infty} + \frac{13.6\text{ eV}}{2^2} = \boxed{3.40\text{ eV}}$$

to emit light of wavelength $$\lambda = \frac{hc}{\Delta E} = \frac{(3.00\times10^{8}\text{ m / s})(6.626\times10^{-34}\text{ J}\cdot\text{s})}{3.40\text{ eV}(1.60\times10^{-19}\text{ J / eV})} = \boxed{365\text{ nm}}$$

42.5 (a) For positronium, $\mu = \frac{m_e}{2}$, so $\lambda_{32} = (656\text{ nm})2 = 1312\text{ nm} = \boxed{1.31\ \mu\text{m}}$ (infrared region) .

(b) For He^+, $\mu \approx m_e$, $q_1 = e$, and $q_2 = 2e$, so $\lambda_{32} = (656/4)\text{ nm} = \boxed{164\text{ nm}}$ (ultraviolet region) .

Goal Solution

A general expression for the energy levels of one-electron atoms and ions is

$$E_n = -\left(\frac{\mu k_e^{\,2} q_1^{\,2} q_2^{\,2}}{2\text{h}^2 n^2}\right)$$

where k_e is the Coulomb constant, q_1 and q_2 are the charges of the two particles, and μ is the reduced mass, given by $\mu = m_1 m_2 / (m_1 + m_2)$. In Problem 4 we found that the wavelength for the $n = 3$ to $n = 2$ transition of the hydrogen atom is 656.3 nm (visible red light). What are the wavelengths for this same transition in (a) positronium, which consists of an electron and a positron, and (b) singly ionized helium? (**Note**: A positron is a positively charged electron.)

G: The reduced mass of positronium is **less** than hydrogen, so the photon energy will be **less** for positronium than for hydrogen. This means that the wavelength of the emitted photon will be **longer** than 656.3 nm. On the other hand, helium has about the same reduced mass but more charge than hydrogen, so its transition energy will be **larger**, corresponding to a wavelength **shorter** than 656.3 nm.

O: All the factors in the above equation are constant for this problem except for the reduced mass and the nuclear charge. Therefore, the wavelength corresponding to the energy difference for the transition can be found simply from the ratio of mass and charge variables.

A: For hydrogen, $\mu = \frac{m_p m_e}{m_p + m_e} \approx m_e$

The photon energy is $\Delta E = E_3 - E_2$

Its wavelength is $\lambda = 656.3 \text{ nm}$, where $\lambda = \frac{c}{f} = \frac{hc}{\Delta E}$

(a) For positronium, $\mu = \frac{m_e m_e}{m_e + m_e} = \frac{m_e}{2}$

so the energy of each level is one half as large as in hydrogen, which we could call "protonium." The photon energy is inversely proportional to its wavelength, so for positronium,

$\lambda_{32} = 2(656.3 \text{ nm}) = 1313 \text{ nm}$ (in the infrared region)

(b) For He^+, $\mu \approx m_e,\ q_1 = e$, and $q_2 = 2e$,

so the transition energy is $2^2 = 4$ times larger than hydrogen. Then,

$\lambda_{32} = \left(\frac{656}{4}\right) \text{nm} = 164 \text{ nm}$ (in the ultraviolet region)

L: As expected, the wavelengths for positronium and helium are respectively larger and smaller than for hydrogen. Other energy transitions should have wavelength shifts consistent with this pattern. It is important to remember that the reduced mass is not the total mass, but is generally close in magnitude to the smaller mass of the system (hence the name **reduced** mass).

***42.6** (a) For a particular transition from n_i to n_f,

$$\Delta E_{\text{H}} = -\frac{\mu_{\text{H}} k_e^2 e^4}{2\text{h}^2}\left(\frac{1}{n_f^2} - \frac{1}{n_i^2}\right) = \frac{hc}{\lambda_{\text{H}}}$$

and

$$\Delta E_{\text{D}} = -\frac{\mu_{\text{D}} k_e^2 e^4}{2\text{h}^2}\left(\frac{1}{n_f^2} - \frac{1}{n_i^2}\right) = \frac{hc}{\lambda_{\text{D}}}$$

where $\mu_{\text{H}} = \frac{m_e m_p}{m_e + m_p}$ and $\mu_{\text{D}} = \frac{m_e m_{\text{D}}}{m_e + m_{\text{D}}}$

By division, $\frac{\Delta E_{\text{H}}}{\Delta E_{\text{D}}} = \frac{\mu_{\text{H}}}{\mu_{\text{D}}} = \frac{\lambda_{\text{D}}}{\lambda_{\text{H}}}$ or $\lambda_{\text{D}} = \left(\frac{\mu_{\text{H}}}{\mu_{\text{D}}}\right)\lambda_{\text{H}}$

Then, $\boxed{\lambda_{\text{H}} - \lambda_{\text{D}} = \left(1 - \frac{\mu_{\text{H}}}{\mu_{\text{D}}}\right)\lambda_{\text{H}}}$

(b) $$\frac{\mu_{\text{H}}}{\mu_{\text{D}}} = \left(\frac{m_e m_p}{m_e + m_p}\right)\left(\frac{m_e + m_{\text{D}}}{m_e m_{\text{D}}}\right) = \frac{(1.007\ 276 \text{ u})(0.000\ 549 \text{ u} + 2.013\ 553 \text{ u})}{(0.000\ 549 \text{ u} + 1.007\ 276 \text{ u})(2.013\ 553 \text{ u})} = 0.999728$$

$\lambda_{\text{H}} - \lambda_{\text{D}} = (1 - 0.999\ 728)(656.3 \text{ nm}) = \boxed{0.179 \text{ nm}}$

42.7 (a) In the 3*d* subshell, $n = 3$ and $\ell = 2$, we have

n	3	3	3	3	3	3	3	3	3	3
ℓ	2	2	2	2	2	2	2	2	2	2
m_ℓ	+2	+2	+1	+1	0	0	-1	-1	-2	-2
m_s	+1/2	-1/2	+1/2	-1/2	+1/2	-1/2	+1/2	-1/2	+1/2	-1/2

(A total of 10 states)

(b) In the 3*p* subshell, $n = 3$ and $\ell = 1$, we have

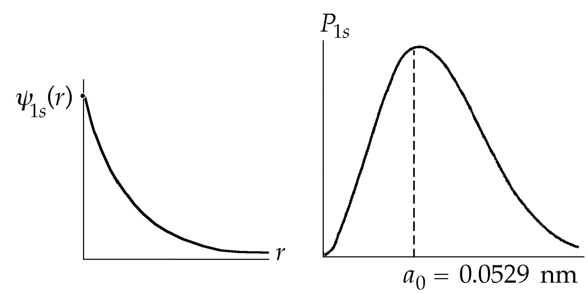
n	3	3	3	3	3	3
ℓ	1	1	1	1	1	1
m_ℓ	+1	+1	+0	+0	-1	-1
m_s	+1/2	-1/2	+1/2	-1/2	+1/2	-1/2

(A total of 6 states)

42.8

$$\psi_{1s}(r) = \frac{1}{\sqrt{\pi a_0^3}} e^{-r/a_0} \qquad \text{(Eq. 42.3)}$$

$$P_{1s}(r) = \frac{4r^2}{a_0^3} e^{-2r/a_0} \qquad \text{(Eq. 42.7)}$$



42.9 (a) $$\int |\psi|^2 dV = 4\pi \int_0^\infty |\psi|^2 r^2 \, dr = 4\pi \left(\frac{1}{\pi a_0^3}\right) \int_0^\infty r^2 e^{-2r/a_0} \, dr$$

Using integral tables, $$\int |\psi|^2 dV = -\frac{2}{a_0^2}\left[e^{-2r/a_0}\left(r^2 + a_0 r + \frac{a_0^2}{2}\right)\right]_{r=0}^{r=\infty} = \left(-\frac{2}{a_0^2}\right)\left(-\frac{a_0^2}{2}\right) = \boxed{1}$$

so the wave function as given is normalized.

(b) $$P_{a_0/2 \to 3a_0/2} = 4\pi \int_{a_0/2}^{3a_0/2} |\psi|^2 r^2 \, dr = 4\pi \left(\frac{1}{\pi a_0^3}\right) \int_{a_0/2}^{3a_0/2} r^2 e^{-2r/a_0} \, dr$$

Again, using integral tables,

$$P_{a_0/2 \to 3a_0/2} = -\frac{2}{a_0^2}\left[e^{-2r/a_0}\left(r^2 + a_0 r + \frac{a_0^2}{2}\right)\right]_{a_0/2}^{3a_0/2} = -\frac{2}{a_0^2}\left[e^{-3}\left(\frac{17 a_0^2}{4}\right) - e^{-1}\left(\frac{5 a_0^2}{4}\right)\right] = \boxed{0.497}$$

42.10 $\psi = \frac{1}{\sqrt{3}} \frac{1}{(2a_0)^{3/2}} \frac{r}{a_0} e^{-r/2a_0}$ so $P_r = 4\pi r^2 \left|\psi^2\right| = 4\pi r^2 \frac{r^2}{24a_0^5} e^{-r/a_0}$

Set $\frac{dP}{dr} = \frac{4\pi}{24a_0^5}\left[4r^3 e^{-r/a_0} + r^4\left(-\frac{1}{a_0}\right)e^{-r/a_0}\right] = 0$

Solving for r, this is a maximum at $\boxed{r = 4a_0}$

42.11 $\psi = \frac{1}{\sqrt{\pi a_0^3}} e^{-r/a_0}$ $\frac{2}{r}\frac{d\psi}{dr} = \frac{-2}{r\sqrt{\pi a_0^5}} e^{-r/a_0} = \frac{2}{ra_0}\psi$ $\frac{d^2\psi}{dr^2} = \frac{1}{\sqrt{\pi a_0^7}} e^{-r/a_0} = \frac{1}{a_0^2}\psi$

$$-\frac{\hbar^2}{2m_e}\left(\frac{1}{a_0^2} - \frac{2}{ra_0}\right)\psi - \frac{e^2}{4\pi\epsilon_0 r}\psi = E\psi$$

But $a_0 = \frac{\hbar^2 4\pi\epsilon_0}{m_e e^2}$, so $-\frac{e^2}{8\pi\epsilon_0 a_0} = E$ or $\boxed{E = -\frac{k_e e^2}{2a_0}}$

This is true, so the Schrödinger equation is satisfied.

42.12 The hydrogen ground-state radial probability density is

$$P(r) = 4\pi r^2 \left|\psi_{1s}\right|^2 = \frac{4r^2}{a_0^3}\exp\left(-\frac{2r}{a_0}\right)$$

The number of observations at $2a_0$ is, by proportion

$$N = 1000\frac{P(2a_0)}{P(a_0/2)} = 1000\frac{(2a_0)^2}{(a_0/2)^2}\frac{e^{-4a_0/a_0}}{e^{-a_0/a_0}} = 1000(16)e^{-3} = \boxed{797 \text{ times}}$$

42.13 (a) For the d state, $\ell = 2$, $L = \boxed{\sqrt{6}\hbar} = 2.58 \times 10^{-34}\ \text{J}\cdot\text{s}$

(b) For the f state, $\ell = 3$, $L = \sqrt{\ell(\ell+1)}\hbar = \boxed{\sqrt{12}\hbar} = 3.65 \times 10^{-34}\ \text{J}\cdot\text{s}$

***42.14** $L = \sqrt{\ell(\ell+1)}\hbar$ so $4.714\times 10^{-34} = \sqrt{\ell(\ell+1)}\dfrac{6.626\times 10^{-34}}{2\pi}$

$$\ell(\ell+1) = \frac{\left(4.714\times 10^{-34}\right)^2 (2\pi)^2}{\left(6.626\times 10^{-34}\right)^2} = 1.998\times 10^1 \approx 20 = 4(4+1) \quad \text{so} \quad \boxed{\ell = 4}$$

42.15 The 5th excited state has $n = 6$, energy $\dfrac{-13.6\text{ eV}}{36} = -0.378\text{ eV}$

The atom loses this much energy: $\dfrac{hc}{\lambda} = \dfrac{(6.626\times 10^{-34}\text{ J}\cdot\text{s})(3.00\times 10^8\text{ m/s})}{(1090\times 10^{-9}\text{ m})(1.60\times 10^{-19}\text{ J/eV})} = 1.14\text{ eV}$

to end up with energy $-0.378\text{ eV} - 1.14\text{ eV} = -1.52\text{ eV}$

which is the energy in state 3: $-\dfrac{13.6\text{ eV}}{3^3} = -1.51\text{ eV}$

While $n = 3$, ℓ can be as large as 2, giving angular momentum $\sqrt{\ell(\ell+1)}\hbar = \boxed{\sqrt{6}\,h}$

42.16 For a 3*d* state, $n = 3$ and $\ell = 2$. Therefore, $L = \sqrt{\ell(\ell+1)}\hbar = \sqrt{2(2+1)}\hbar = \boxed{\sqrt{6}\hbar} = 2.58\times 10^{-34}\text{ J}\cdot\text{s}$

m_ℓ can have the values $-2, -1, 0, 1,$ and 2, so $\boxed{L_z \text{ can have the values} -2\hbar, -\hbar, 0, \text{ and } 2\hbar}$

Using the relation $\cos\theta = L_z / L$, we find that the possible values of θ are equal to

$\boxed{145°, 114°, 90.0°, 65.9°, \text{and } 35.3°}$.

42.17 (a) $n = 1$: For $n = 1$, $\ell = 0$, $m_\ell = 0$, $m_s = \pm\dfrac{1}{2}$, $\rightarrow$ 2 sets

n	ℓ	m_ℓ	m_s
1	0	0	–1/2
1	0	0	+1/2

$2n^2 = 1(1)^2 = \boxed{2}$

(b) For $n = 2$, we have

n	ℓ	m_ℓ	m_s
2	0	0	±1/2
2	1	–1	±1/2
2	1	0	±1/2
2	1	1	±1/2

yields 8 sets; $2n^2 = 2(2)^2 = \boxed{8}$

Note that the number is twice the number of m_l values. Also, for each l there are $(2l+1)$ different m_l values. Finally, l can take on values ranging from 0 to $n-1$. So the general expression is

$$s = \sum_{0}^{n-1} 2(2l+1)$$

The series is an arithmetic progression: 2 + 6 + 10 + 14, the sum of which is

$s = \frac{n}{2}[2a + (n-1)d]$ where $a = 2,\ d = 4$: $\qquad s = \frac{n}{2}[4 + (n-1)4] = 2n^2$

(c) $n = 3$: $2(1) + 2(3) + 2(5) = 2 + 6 + 10 = 18$ $\qquad 2n^2 = 2(3)^2 = \boxed{18}$

(d) $n = 4$: $2(1) + 2(3) + 2(5) + 2(7) = 32$ $\qquad 2n^2 = 2(4)^2 = \boxed{32}$

(e) $n = 5$: $32 + 2(9) = 32 + 18 = 50$ $\qquad 2n^2 = 2(5)^2 = \boxed{50}$

42.18 $\mu_B = \frac{e\hbar}{2m_e}$ $\qquad e = 1.60 \times 10^{-19}$ C $\qquad \hbar = 1.055 \times 10^{-34}$ J · s $\qquad m_e = 9.11 \times 10^{-31}$ kg

$$\mu_B = \boxed{9.27 \times 10^{-24} \text{ J/T} = 5.79 \times 10^{-5} \text{ eV/T}}$$

42.19 (a) Density of a proton: $\rho = \frac{m}{V} = \frac{1.67 \times 10^{-27} \text{ kg}}{(4/3)\pi(1.00 \times 10^{-15} \text{ m})^3} = \boxed{3.99 \times 10^{17} \text{ kg/m}^3}$

(b) Size of model electron: $r = \left(\frac{3m}{4\pi\rho}\right)^{1/3} = \left(\frac{3 \times 9.11 \times 10^{-31} \text{ kg} \cdot \text{m}^3}{4\pi \times 3.99 \times 10^{17} \text{ kg}}\right)^{1/3} = \boxed{8.17 \times 10^{-17} \text{ m}}$

(c) Moment of inertia: $I = \frac{2}{5}mr^2 = \frac{2}{5}(9.11 \times 10^{-31} \text{ kg})(8.17 \times 10^{-17} \text{ m})^2 = 2.43 \times 10^{-63} \text{ kg} \cdot \text{m}^2$

$$L_z = I\omega = \frac{\hbar}{2} = \frac{Iv}{r}$$

Therefore, $v = \frac{\hbar r}{2I} = \frac{(6.626 \times 10^{-34} \text{ J} \cdot \text{s})(8.17 \times 10^{-17} \text{ m})}{2\pi(2)(2.43 \times 10^{-63} \text{ kg} \cdot \text{m}^2)} = \boxed{1.77 \times 10^{12} \text{ m/s}}$

(d) This is $\boxed{5.91 \times 10^3 \text{ times larger}}$ than the speed of light.

42.20 (a) $L = mvr = m\frac{2\pi r}{T}r = \sqrt{\ell(\ell+1)}\hbar = \sqrt{(\ell^2+\ell)}\hbar \approx \ell\hbar$

$$(5.98\times10^{24}\text{ kg})\ \frac{2\pi(1.496\times10^{11}\text{ m})^2}{3.156\times10^{7}\text{ s}} = \ell\hbar \quad \text{so} \quad \frac{2.66\times10^{40}}{1.055\times10^{-34}\text{ J}\cdot\text{s}} = \ell = \boxed{2.52\times10^{74}}$$

(b) $|E| = |-U+K| = |-K| = \frac{1}{2}mv^2 = \frac{1}{2}\frac{mr^2}{mr^2}mv^2 = \frac{1}{2}\frac{L^2}{mr^2} = \frac{1}{2}\frac{\ell(\ell+1)\hbar^2}{mr^2} \approx \frac{1}{2}\frac{\ell^2\hbar^2}{mr^2}$

$$\frac{dE}{d\ell} = \frac{1}{2}\frac{2\ell\hbar^2}{mr^2}\frac{\ell}{\ell} = 2\frac{E}{\ell} \text{ so} \qquad dE = 2\frac{E}{\ell}d\ell = 2\frac{\frac{1}{2}(5.98\times10^{24}\text{ kg})\left(\frac{2\pi\times1.496\times10^{11}\text{ m}}{3.156\times10^{7}\text{ s}}\right)^2 1}{2.52\times10^{74}}$$

$$\Delta E = \frac{5.30\times10^{33}\text{ J}}{2.52\times10^{74}} = \boxed{2.10\times10^{-41}\text{ J}}$$

***42.21** $\mu_n = \frac{e\hbar}{2m_p}$ $\quad e = 1.60\times10^{-19}$ C $\quad \hbar = 1.055\times10^{-34}$ J · s $\quad m_p = 1.67\times10^{-27}$ kg

(a) $\mu_n = \boxed{5.05\times10^{-27}\text{ J/T}} = \boxed{31.6\text{ neV/T}}$

(b) $\frac{\mu_n}{\mu_B} = \frac{1}{1836} = \frac{m_e}{m_p}$

Apparently it is harder to "spin up" a nucleus than a electron, because of its greater mass.

42.22 In the N shell, $n = 4$. For $n = 4$, ℓ can take on values of 0, 1, 2, and 3. For each value of ℓ, m_ℓ can be $-\ell$ to ℓ in integral steps. Thus, the maximum value for m_ℓ is 3. Since $L_z = m_\ell\hbar$, the maximum value for L_z is $L_z = \boxed{3\hbar}$.

42.23 The 3*d* subshell has $\ell = 2$, and $n = 3$. Also, we have $s = 1$.
Therefore, we can have $\boxed{n = 3;\ \ell = 2;\ m_\ell = -2, -1, 0, 1, 2;\ s = 1;\text{ and } m_s = -1, 0, 1}$, leading to the following table:

n	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3
ℓ	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2
m_ℓ	–2	–2	–2	–1	–1	–1	0	0	0	1	1	1	2	2	2
s	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
m_s	–1	0	1	–1	0	1	–1	0	1	–1	0	1	–1	0	1

42.24 (a) $\boxed{1s^2 2s^2 2p^4}$

(b) For the 1s electrons, $n = 1$, $\ell = 0$, $m_\ell = 0$, $m_s = +1/2$ and $-1/2$
For the two 2s electrons, $n = 2$, $\ell = 0$, $m_\ell = 0$, $m_s = +1/2$ and $-1/2$
For the four 2p electrons, $n = 2$; $\ell = 1$; $m_\ell = -1, 0,$ or 1; and $m_s = +1/2$ or $-1/2$

42.25 The $\boxed{\text{4s subshell fills first}}$, for potassium and calcium, before the 3d subshell starts to fill for scandium through zinc. Thus, we would first suppose that $[\text{Ar}]3d^4 4s^2$ would have lower energy than $[\text{Ar}]3d^5 4s^1$. But the latter has more unpaired spins, six instead of four, and Hund's rule suggests that this could give the latter configuration lower energy. In fact it must, for $[\text{Ar}]3d^5 4s^1$ is the ground state for chromium.

***42.26** (a) For electron one and also for electron two, $n = 3$ and $\ell = 1$. The possible states are listed here in columns giving the other quantum numbers:

electron one	m_ℓ	1	1	1	1	1	1	1	1	1	1	0	0	0	0	0
	m_s	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	$-\frac{1}{2}$	$-\frac{1}{2}$	$-\frac{1}{2}$	$-\frac{1}{2}$	$-\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$
electron two	m_ℓ	1	0	0	–1	–1	1	0	0	–1	–1	1	1	0	–1	–1
	m_s	$-\frac{1}{2}$	$\frac{1}{2}$	$-\frac{1}{2}$	$\frac{1}{2}$	$-\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	$-\frac{1}{2}$	$\frac{1}{2}$	$-\frac{1}{2}$	$\frac{1}{2}$	$-\frac{1}{2}$	$-\frac{1}{2}$	$\frac{1}{2}$	$-\frac{1}{2}$

electron one	m_ℓ	0	0	0	0	0	–1	–1	–1	–1	–1	–1	–1	–1	–1	–1
	m_s	$-\frac{1}{2}$	$-\frac{1}{2}$	$-\frac{1}{2}$	$-\frac{1}{2}$	$-\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	$-\frac{1}{2}$	$-\frac{1}{2}$	$-\frac{1}{2}$	$-\frac{1}{2}$	$-\frac{1}{2}$
electron two	m_ℓ	1	1	0	–1	–1	1	1	0	0	–1	1	1	0	0	–1
	m_s	$\frac{1}{2}$	$-\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	$-\frac{1}{2}$	$\frac{1}{2}$	$-\frac{1}{2}$	$\frac{1}{2}$	$-\frac{1}{2}$	$-\frac{1}{2}$	$\frac{1}{2}$	$-\frac{1}{2}$	$\frac{1}{2}$	$-\frac{1}{2}$	$\frac{1}{2}$

There are thirty allowed states, since electron one can have any of three possible values for m_ℓ for both spin up and spin down, amounting to six states, and the second electron can have any of the other five states.

(b) Were it not for the exclusion principle, there would be $\boxed{36}$ possible states, six for each electron independently.

42.27

Shell	K	L				M									N
n	1	2				3									4
ℓ	0	0	1			0	1			2					0
m_ℓ	0	0	1	0	–1	0	1	0	–1	2	1	0	–1	–2	0
m_s	↑↓	↑↓	↑↓	↑↓	↑↓	↑↓	↑↓	↑↓	↑↓	↑↓	↑↓	↑↓	↑↓	↑↓	↑↓
count	1 2	3 4			10	12			18	21				30	20
	He	Be			Ne	Mg			Ar					Zn	Ca

(a) $\boxed{\text{zinc or copper}}$

(b) $\boxed{1s^2 2s^2 2p^6 3s^2 3p^6 4s^2 3d^{10}}$ or $\boxed{1s^2 2s^2 2p^6 3s^2 3p^6 4s^1 3d^{10}}$

42.28 Listing subshells in the order of filling, we have for element 110,

$1s^2\,2s^2\,2p^6\,3s^2\,3p^6\,4s^2\,3d^{10}\,4p^6\,5s^2\,4d^{10}\,5p^6\,6s^2\,4f^{14}\,5d^{10}\,6p^6\,7s^2\,5f^{14}\,6d^8$

In order of increasing principal quantum number, this is

$$\boxed{1s^2\,2s^2\,2p^6\,3s^2\,3p^6\,3d^{10}\,4s^2\,4p^6\,4d^{10}\,4f^{14}\,5s^2\,5p^6\,5d^{10}\,5f^{14}\,6s^2\,6p^6\,6d^8\,7s^2}$$

42.29 (a)

$n + l$	1	2	3	4	5	6	7
subshell	$1s$	$2s$	$2p$, $3s$	$3p$, $4s$	$3d$, $4p$, $5s$	$4d$, $5p$, $6s$	$4f$, $5d$, $6p$, $7s$

(b) $Z = 15$:
Filled subshells: $1s$, $2s$, $2p$, $3s$ (12 electrons)
Valence subshell: 3 electrons in $3p$ subshell
Prediction: Valance = +3 or −5
Element is phosphorus Valence +3 or −5 (Prediction correct)

$Z = 47$:
Filled subshells: $1s$, $2s$, $2p$, $3s$, $3p$, $4s$, $3d$, $4p$, $5s$ (38 electrons)
Outer subshell: 9 electrons in $4d$ subshell
Prediction: Valence = −1
Element is silver, (Prediction fails) Valence is +1

$Z = 86$:
Filled subshells: $1s$, $2s$, $2p$, $3s$, $3p$, $4s$, $3d$, $4p$, $5s$, $4d$, $5p$, $6s$, $4f$, $5d$, $6p$ (86 electrons)
Prediction Outer subshell is full: inert gas
Element is radon, inert (Prediction correct)

42.30 Electronic configuration: Sodium to Argon

$[1s^2\,2s^2\,2p^6]\ +3s^1 \rightarrow \mathrm{Na}^{11}$
$+3s^2 \rightarrow \mathrm{Mg}^{12}$
$+3s^2\,3p^1 \rightarrow \mathrm{Al}^{13}$
$+3s^2\,3p^2 \rightarrow \mathrm{Si}^{14}$
$+3s^2\,3p^3 \rightarrow \mathrm{P}^{15}$
$+3s^2\,3p^4 \rightarrow \mathrm{S}^{16}$
$+3s^2\,3p^5 \rightarrow \mathrm{Cl}^{17}$
$+3s^2\,3p^6 \rightarrow \mathrm{Ar}^{18}$
$[1s^2\,2s^2\,2p^6\,3s^2\,3p^6]4s^1 \rightarrow \mathrm{K}^{19}$

***42.31** $\boxed{n = 3,\ \ell = 0,\ m_\ell = 0}$

ψ_{300} corresponds to $E_{300} = -\dfrac{Z^2 E_0}{n^2} = -\dfrac{2^2(13.6)}{(3)^2} = \boxed{-6.05\text{ eV}}$

$\boxed{n = 3,\ \ell = 1,\ m_\ell = -1, 0, 1}$

ψ_{31-1}, ψ_{310}, ψ_{311} have the same energy since n is the same.

For $\boxed{n = 3,\ \ell = 2,\ m_\ell = -2, -1, 0, 1, 2}$

ψ_{32-2}, ψ_{32-1}, ψ_{320}, ψ_{321}, ψ_{322} have the same energy since n is the same.

All states are degenerate.

42.32 $E = \dfrac{hc}{\lambda} = e(\Delta V) \Rightarrow \dfrac{(6.626\times 10^{-34}\text{ J}\cdot\text{s})(3.00\times 10^{8}\text{ m/s})}{10.0\times 10^{-9}\text{ m}} = (1.60\times 10^{-19})(\Delta V)$

$\Delta V = \boxed{124\text{ V}}$

***42.33** $E_{\text{photon max}} = \dfrac{hc}{\lambda_{\text{min}}} = e(\Delta V) = 40.0\text{ keV}$

$$\lambda_{\text{min}} = \frac{hc}{E_{\text{max}}} = \frac{(6.626\times 10^{-34}\text{ J}\cdot\text{s})(3.00\times 10^{8}\text{ m/s})}{40.0\times 10^{3}\text{ eV}}\left(\frac{1.00\text{ eV}}{1.60\times 10^{-19}\text{ J}}\right) = \boxed{0.031\,0\text{ nm}}$$

42.34 Some electrons can give all their kinetic energy $K_e = e(\Delta V)$ to the creation of a single photon of x-radiation, with

$hf = \dfrac{hc}{\lambda} = e(\Delta V)$

$$\lambda = \frac{hc}{e(\Delta V)} = \frac{(6.6261\times 10^{-34}\text{ J}\cdot\text{s})(2.9979\times 10^{8}\text{ m/s})}{(1.6022\times 10^{-19}\text{ C})(\Delta V)} = \boxed{\frac{1240\text{ nm}\cdot\text{V}}{\Delta V}}$$

42.35 Following Example 42.7, $E_\gamma = \dfrac{3}{4}(42-1)^2(13.6\text{ eV}) = 1.71\times 10^{4}\text{ eV} = 2.74\times 10^{-15}\text{ J}$

$f = 4.14\times 10^{18}\text{ Hz}$ and $\lambda = \boxed{0.072\,5\text{ nm}}$

42.36 The K_β x-rays are emitted when there is a vacancy in the (n = 1) K shell and an electron from the (n = 3) M shell falls down to fill it. Then this electron is shielded by nine electrons originally and by one in its final state.

$$\frac{hc}{\lambda} = -\frac{13.6(Z-9)^2}{3^2}\text{ eV} + \frac{13.6(Z-1)^2}{1^2}\text{ eV}$$

$$\frac{(6.626\times10^{-34}\text{ J}\cdot\text{s})(3.00\times10^{8}\text{ m/s})}{(0.152\times10^{-9}\text{ m})(1.60\times10^{-19}\text{ J/eV})} = 13.6\text{ eV}\left(-\frac{Z^2}{9}+\frac{18Z}{9}-\frac{81}{9}+Z^2-2Z+1\right)$$

$$8.17\times10^{3}\text{ eV} = 13.6\text{ eV}\left(\frac{8Z^2}{9}-8\right) \quad \text{so} \quad 601 = \frac{8Z^2}{9}-8 \quad \text{and} \quad Z = 26\ \boxed{\text{Iron}}$$

42.37 (a) Suppose the electron in the M shell is shielded from the nucleus by two K plus seven L electrons. Then its energy is

$$-\frac{13.6\text{ eV}(83-9)^2}{3^2} = -8.27\text{ keV}$$

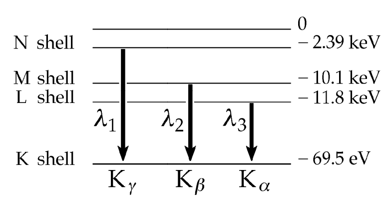
Suppose, after it has fallen into the vacancy in the L shell, it is shielded by just two K-shell electrons. Then its energy is

$$\frac{-13.6\text{ eV}(83-2)^2}{2^2} = -22.3\text{ keV}$$

Thus the electron's energy loss is the photon energy: $(22.3-8.27)\text{ keV} = \boxed{14.0\text{ keV}}$

(b) $\Delta E = \frac{hc}{\lambda}$ so $\lambda = \frac{6.626\times10^{-34}\text{ J}\cdot\text{s}\,(3.00\times10^{8}\text{ m/s})}{14.0\times10^{3}\times1.60\times10^{-19}\text{ J}} = \boxed{8.85\times10^{-11}\text{ m}}$

***42.38** $E = \frac{hc}{\lambda} = \frac{1240\text{ eV}\cdot\text{nm}}{\lambda} = \frac{1.240\text{ keV}\cdot\text{nm}}{\lambda}$



for λ_1 = 0.0185 nm, E = 67.11 keV

λ_2 = 0.0209 nm, E = 59.4 keV

λ_3 = 0.0215 nm, E = 57.7 keV

The ionization energy for K shell = 69.5 keV, so, the ionization energies for the other shells are: $\boxed{\text{L shell} = 11.8\text{ keV}}$: $\boxed{\text{M shell} = 10.1\text{ keV}}$: $\boxed{\text{N shell} = 2.39\text{ keV}}$

***42.39** (a) The outermost electron in sodium has a 3*s* state for its ground state. The longest wavelength means minimum photon energy and smallest step on the energy level diagram. Since $n = 3$, n' must be 4. With $\ell = 0$, ℓ' must be $\boxed{1}$, since ℓ must change by 1 in a photon absorption process.

(b) $$\frac{1}{330 \times 10^{-9}\text{ m}} = \left(1.097 \times 10^{7}\ \frac{1}{\text{m}}\right)\left[\frac{1}{(3 - 1.35)^2} - \frac{1}{(4 - \delta_1)^2}\right]$$

$$0.276 = \frac{1}{(1.65)^2} - \frac{1}{(4 - \delta_1)^2} = 0.367 - \frac{1}{(4 - \delta_1)^2} \quad \text{so} \quad (4 - \delta_1)^2 = 10.98 \quad \text{and} \quad \boxed{\delta_1 = 0.686}$$

42.40 $$\lambda = \frac{c}{f} = \frac{hc}{hf} = \frac{(6.626 \times 10^{-34}\text{ J}\cdot\text{s})(3.00 \times 10^{8}\text{ m / s})}{(2.10\text{ eV})(1.60 \times 10^{19}\text{ J / eV})} = \boxed{590\text{ nm}}$$

***42.41** We require $A = u_f B = \frac{16\pi^2 \hbar}{\lambda^3} B$ or $$u_f = \frac{16\pi^2 \hbar}{\lambda^3} = \frac{16\pi^2\left(1.055 \times 10^{-34}\text{ J}\cdot\text{s}\right)}{\left(645 \times 10^{-9}\text{ m}\right)^3} = \boxed{6.21 \times 10^{-14}\ \frac{\text{J}\cdot\text{s}}{\text{m}^3}}$$

42.42 $$f = \frac{E}{h} = \boxed{2.82 \times 10^{13}\text{ s}^{-1}}$$

$$\lambda = \frac{c}{f} = \boxed{10.6\ \mu\text{m}}, \quad \boxed{\text{infrared}}$$

42.43 $E = \mathcal{P}t = (1.00 \times 10^6\text{ W})(1.00 \times 10^{-8}\text{ s}) = 0.0100\text{ J}$

$$E_\gamma = hf = \frac{hc}{\lambda} = \frac{(6.626 \times 10^{-34})(3.00 \times 10^{8})}{694.3 \times 10^{-9}}\text{ J} = 2.86 \times 10^{-19}\text{ J}$$

$$N = \frac{E}{E_\gamma} = \frac{0.0100}{2.86 \times 10^{-19}} = \boxed{3.49 \times 10^{16}\text{ photons}}$$

Goal Solution

A ruby laser delivers a 10.0-ns pulse of 1.00 MW average power. If the photons have a wavelength of 694.3 nm, how many are contained in the pulse?

G: Lasers generally produce concentrated beams that are bright (except for IR or UV lasers that produce invisible beams). Since our eyes can detect light levels as low as a few photons, there are probably at least 1000 photons in each pulse.

O: From the pulse width and average power, we can find the energy delivered by each pulse. The number of photons can then be found by dividing the pulse energy by the energy of each photon, which is determined from the photon wavelength.

A: The energy in each pulse is $E = \mathcal{P}t = (1.00 \times 10^{6}\text{ W})(1.00 \times 10^{-8}\text{ s}) = 1.00 \times 10^{-2}\text{ J}$

The energy of each photon is $E_\gamma = hf = \dfrac{hc}{\lambda} = \dfrac{(6.626 \times 10^{-34})(3.00 \times 10^{8})}{694.3 \times 10^{-9}}\text{ J} = 2.86 \times 10^{-19}\text{ J}$

So $N = \dfrac{E}{E_\gamma} = \dfrac{1.00 \times 10^{-2}\text{ J}}{2.86 \times 10^{-19}\text{ J / photon}} = 3.49 \times 10^{16}\text{ photons}$

L: With 10^{16} photons/pulse, this laser beam should produce a bright red spot when the light reflects from a surface, even though the time between pulses is generally much longer than the width of each pulse. For comparison, this laser produces more photons in a single ten-nanosecond pulse than a typical 5 mW helium-neon laser produces over a full second (about 1.6×10^{16} photons/second).

***42.44** In $G = e^{\sigma(n_u - n_\ell)L}$ we require $1.05 = e^{(1.00 \times 10^{-18}\text{ m}^2)(n_u - n_\ell)(0.500\text{ m})}$

Thus, $\ln(1.05) = (5.00 \times 10^{-19}\text{ m}^3)(n_u - n_\ell)$ so $n_u - n_\ell = \dfrac{\ln(1.05)}{5.00 \times 10^{-19}\text{ m}^3} = \boxed{9.76 \times 10^{16}\text{ m}^{-3}}$

42.45 (a) $\dfrac{N_3}{N_2} = \dfrac{N_g e^{-E_3/(k_B \cdot 300\text{ K})}}{N_g e^{-E_2/(k_B \cdot 300\text{ K})}} = e^{-(E_3 - E_2)/(k_B \cdot 300\text{ K})} = e^{-hc/\lambda(k_B \cdot 300\text{ K})}$

where λ is the wavelength of light radiated in the $3 \to 2$ transition:

$$\frac{N_3}{N_2} = e^{-(6.626 \times 10^{-34}\text{ J}\cdot\text{s})(3.00 \times 10^{8}\text{ m/s})/(632.8 \times 10^{-9}\text{ m})(1.38 \times 10^{-23}\text{ J/K})(300\text{ K})} = e^{-75.8} = \boxed{1.22 \times 10^{-33}}$$

(b) $$\frac{N_3}{N_2} = e^{hc/\lambda k_B T} = e^{-(6.626 \times 10^{-34}\text{ J}\cdot\text{s})(3.00 \times 10^{8}\text{ m/s})/(694.3 \times 10^{-9}\text{ m})(1.38 \times 10^{-23}\text{ J/K})(4.00\text{ K})} = e^{-5187}$$

To avoid overflowing your calculator, note that $10 = e^{\ln 10}$. Take

$$\frac{N_3}{N_2} = e^{\ln 10 \times (-5187/\ln 10)} = \boxed{10^{-2253}}$$

***42.46** $N_u/N_\ell = e^{-(E_u - E_\ell)/k_B T}$ where the subscript u refers to an upper energy state and the subscript ℓ to a lower energy state.

(a) Since $E_u - E_\ell = E_{\text{photon}} = hc/\lambda$, $N_u/N_\ell = e^{-hc/\lambda k_B T}$

Thus, we require $1.02 = e^{-hc/\lambda k_B T}$ or $\ln(1.02) = -\dfrac{(6.626\times10^{-34}\text{ J}\cdot\text{s})(3.00\times10^{8}\text{ m/s})}{(694.3\times10^{-9}\text{ m})(1.38\times10^{-23}\text{ J/K})T}$

$$T = -\frac{2.07\times10^{4}\text{ K}}{\ln(1.02)} = \boxed{-1.05\times10^{6}\text{ K}}$$

A negative-temperature state is not achieved by cooling the system below 0 K, but by heating it above $T=\infty$, for as $T\to\infty$ the populations of upper and lower states approach equality.

(b) Because $E_u - E_\ell > 0$, and in any real equilibrium state $T>0$, $e^{-(E_u-E_\ell)/k_B T} < 1$ and $N_u < N_\ell$.

Thus, a population inversion cannot happen in thermal equilibrium.

42.47 (a) $I = \dfrac{(3.00\times10^{-3}\text{ J})}{(1.00\times10^{-9}\text{ s})\pi(15.0\times10^{-6}\text{ m})^2} = \boxed{4.24\times10^{15}\text{ W/m}^2}$

(b) $(3.00\times10^{-3}\text{ J})\dfrac{(0.600\times10^{-9}\text{ m})^2}{(30.0\times10^{-6}\text{ m})^2} = \boxed{1.20\times10^{-12}\text{ J}} = 7.50\text{ MeV}$

***42.48** (a) The energy difference between these two states is equal to the energy that is absorbed.

Thus, $E = E_2 - E_1 = \dfrac{(-13.6\text{ eV})}{4} - \dfrac{(-13.6\text{ eV})}{1} = 10.2\text{ eV} = \boxed{1.63\times10^{-18}\text{ J}}$

(b) We have $E = \dfrac{3}{2}k_B T$, or $T = \dfrac{2}{3k_B}E = \dfrac{2(1.63\times10^{-18}\text{ J})}{3(1.38\times10^{-23}\text{ J/K})} = \boxed{7.88\times10^{4}\text{ K}}$

42.49 $r_{\text{av}} = \int_0^\infty rP(r)dr = \int_0^\infty\left(\dfrac{4r^3}{a_0^3}\right)(e^{-2r/a_0})dr$

Make a change of variables with $\dfrac{2r}{a_0} = x$ and $dr = \dfrac{a_0}{2}dx$.

Then $r_{\text{av}} = \dfrac{a_0}{4}\int_0^\infty x^3 e^{-x}dx = \dfrac{a_0}{4}\left[-x^3e^{-x} + 3\left(-x^2e^{-x} + 2e^{-x}(-x-1)\right)\right]\Big|_0^\infty = \boxed{\dfrac{3}{2}a_0}$

***42.50** $\left\langle \frac{1}{r} \right\rangle = \int_0^\infty \frac{4r^2}{a_0^3} e^{-2r/a_0} \frac{1}{r} dr = \frac{4}{a_0^3} \int_0^\infty r e^{-(2/a_0)r} dr = \frac{4}{a_0^3} \frac{1}{(2/a_0)^2} = \boxed{\frac{1}{a_0}}$

We compare this to $\frac{1}{\langle r \rangle} = \frac{1}{3a_0/2} = \frac{2}{3a_0}$, and find that the average reciprocal value is $\boxed{\text{NOT}}$ the reciprocal of the average value.

42.51 The wave equation for the $2s$ state is given by Eq. 42.7: $\psi_{2s}(r) = \frac{1}{4\sqrt{2\pi}} \left(\frac{1}{a_0} \right)^{3/2} \left[2 - \frac{r}{a_0} \right] e^{-r/2a_0}$

(a) Taking $r = a_0 = 0.529 \times 10^{-10}$ m, we find

$$\psi_{2s}(a_0) = \frac{1}{4\sqrt{2\pi}} \left(\frac{1}{0.529 \times 10^{-10} \text{ m}} \right)^{3/2} [2-1] e^{-1/2} = \boxed{1.57 \times 10^{14} \text{ m}^{-3/2}}$$

(b) $\left| \psi_{2s}(a_0) \right|^2 = \left(1.57 \times 10^{14} \text{ m}^{-3/2} \right)^2 = \boxed{2.47 \times 10^{28} \text{ m}^{-3}}$

(c) Using Equation 42.5 and the results to (b) gives $P_{2s}(a_0) = 4\pi a_0^2 \left| \psi_{2s}(a_0) \right|^2 = \boxed{8.69 \times 10^8 \text{ m}^{-1}}$

***42.52** We define the reduced mass to be μ, and the ground state energy to be E_1:

$$\mu = \frac{m_1 m_2}{m_1 + m_2} = \frac{207 m_e m_p}{207 m_e + m_p} = \frac{207(9.11 \times 10^{-31} \text{ kg})(1.67 \times 10^{-27} \text{ kg})}{207(9.11 \times 10^{-31} \text{ kg}) + (1.67 \times 10^{-27} \text{ kg})} = 1.69 \times 10^{-28} \text{ kg}$$

$$E_1 = -\frac{\mu k_e^2 q_1^2 q_2^2}{2\hbar^2 (1)^2} = -\frac{(1.69 \times 10^{-28} \text{ kg})(8.99 \times 10^9 \text{ N} \cdot \text{m}^2/\text{C}^2)^2 (1.60 \times 10^{-19} \text{ C})^3 e}{2(1.055 \times 10^{-34} \text{ J} \cdot \text{s})^2} = -2.52 \times 10^3 \text{ eV}$$

To ionize the muonium "atom" one must supply energy $\boxed{+2.52 \text{ keV}}$.

42.53 (a) $(3.00 \times 10^8 \text{ m/s})(14.0 \times 10^{-12} \text{ s}) = \boxed{4.20 \text{ mm}}$

(b) $E = \frac{hc}{\lambda} = 2.86 \times 10^{-19}$ J $\qquad N = \frac{3.00 \text{ J}}{2.86 \times 10^{-19} \text{ J}} = \boxed{1.05 \times 10^{19} \text{ photons}}$

(c) $V = (4.20 \text{ mm})\pi(3.00 \text{ mm})^2 = 119 \text{ mm}^3$ $\qquad n = \frac{1.05 \times 10^{19}}{119} = \boxed{8.82 \times 10^{16} \text{ mm}^{-3}}$

42.54 (a) The length of the pulse is $\Delta L = \boxed{ct}$

(b) The energy of each photon is $E_\gamma = \dfrac{hc}{\lambda}$ so $N = \dfrac{E}{E_\gamma} = \boxed{\dfrac{E\lambda}{hc}}$

(c) $V = \Delta L \pi \dfrac{d^2}{4}$ $\quad n = \dfrac{N}{V} = \boxed{\left(\dfrac{4}{ct\pi d^2}\right)\left(\dfrac{E\lambda}{hc}\right)}$

42.55 We use $\psi_{2s}(r) = \dfrac{1}{4}(2\pi a_0^3)^{-1/2}\left(2 - \dfrac{r}{a_0}\right)e^{-r/2a_0}$

By Equation 42.5, $P(r) = 4\pi r^2\psi^2 = \dfrac{1}{8}\left(\dfrac{r^2}{a_0^3}\right)\left(2 - \dfrac{r}{a_0}\right)^2 e^{-r/a_0}$

(a)
$$\frac{dP(r)}{dr} = \frac{1}{8}\left[\frac{2r}{a_0^3}\left(2 - \frac{r}{a_0}\right)^2 - \frac{2r^2}{a_0^3}\left(\frac{1}{a_0}\right)\left(2 - \frac{r}{a_0}\right) - \frac{r^2}{a_0^3}\left(2 - \frac{r}{a_0}\right)^2\left(\frac{1}{a_0}\right)\right]e^{-r/a_0} = 0$$

or
$$\frac{1}{8}\left(\frac{r}{a_0^3}\right)\left(2 - \frac{r}{a_0}\right)\left[2\left(2 - \frac{r}{a_0}\right) - \frac{2r}{a_0} - \frac{r}{a_0}\left(2 - \frac{r}{a_0}\right)\right]e^{-r/a_0} = 0$$

Therefore we require the roots of $\dfrac{dP}{dr} = 0$ at $r = 0$, $r = 2a_0$, and $r = \infty$ to be minima with $P(r) = 0$.

$[\,.\,.\,.\,.\,.\,] = 4 - (6r/a_0) + (r/a_0)^2 = 0$ with solutions $r = (3 \pm \sqrt{5})a_0$.

We substitute the last two roots into $P(r)$ to determine the most probable value:

When $r = (3 - \sqrt{5})a_0 = 0.7639a_0$, then $P(r) = 0.0519/a_0$
When $r = (3 + \sqrt{5})a_0 = 5.236a_0$, then $P(r) = 0.191/a_0$

Therefore, the most probable value of r is $(3 + \sqrt{5})a_0 = \boxed{5.236a_0}$

(b) $\displaystyle\int_0^\infty P(r)dr = \int_0^\infty \frac{1}{8}\left(\frac{r^2}{a_0{}^3}\right)\left(2 - \frac{r}{a_0}\right)^2 e^{-r/a_0}\,dr$ Let $u = \dfrac{r}{a_0}$, $dr = a_0\,du$,

$$\int_0^\infty P(r)dr = \int_0^\infty \frac{1}{8}u^2(4 - 4u + u^2)e^{-u}\,dr = \int_0^\infty \frac{1}{8}(u^4 - 4u^3 + 4u^2)e^{-u}\,du = -\frac{1}{8}(u^4 + 4u^2 + 8u + 8)e^{-u}\Big|_0^\infty = 1$$

$\boxed{\text{This is as desired}}$.

***42.56** $\Delta z = \dfrac{at^2}{2} = \dfrac{1}{2}\left(\dfrac{F_z}{m_0}\right)t^2 = \dfrac{\mu_z(dB_z/dz)}{2m_0}\left(\dfrac{\Delta x}{v}\right)^2$ and $\mu_z = \dfrac{e\hbar}{2m_e}$

$$\frac{dB_z}{dz} = \frac{2m_0(\Delta z)v^2 2m_e}{\Delta x^2 e\hbar} = \frac{2(108)(1.66\times10^{-27}\text{ kg})(10^4\text{ m}^2/\text{s}^2)2(9.11\times10^{-31}\text{ kg})(10^{-3}\text{ m})}{(1.00\text{ m}^2)(1.60\times10^{-19}\text{ C})(1.05\times10^{-34}\text{ J}\cdot\text{s})} = \boxed{0.389\text{ T/m}}$$

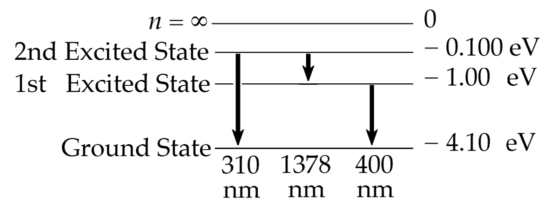
42.57 With one vacancy in the K shell, excess energy $\Delta E \approx -(Z-1)^2(13.6 \text{ eV})\left(\frac{1}{2^2}-\frac{1}{1^2}\right) = 5.40 \text{ keV}$

We suppose the outermost 4*s* electron is shielded by 20 electrons inside its orbit:

$$E_{\text{ionization}} \approx \frac{2^2(13.6 \text{ eV})}{4^2} = 3.40 \text{ eV}$$

Note the experimental ionization energy is 6.76 eV. $K = \Delta E - E_{\text{ionization}} \approx \boxed{5.39 \text{ keV}}$

***42.58** $E = \dfrac{hc}{\lambda} = \dfrac{1240 \text{ eV}\cdot\text{nm}}{\lambda} = \Delta E$



$\lambda_1 = 310 \text{ nm}$, so $\Delta E_1 = 4.00 \text{ eV}$

$\lambda_2 = 400 \text{ nm}$, $\Delta E_2 = 3.10 \text{ eV}$

$\lambda_3 = 1378 \text{ nm}$, $\Delta E_3 = 0.900 \text{ eV}$

and the ionization energy = 4.10 eV

The energy level diagram having the fewest levels and consistent with these energies is shown at the right.

42.59 (a) One molecule's share of volume

Al: $V = \dfrac{\text{mass per molecule}}{\text{density}} = \left(\dfrac{27.0 \text{ g/mol}}{6.02\times10^{23} \text{ molecules/mole}}\right)\left(\dfrac{1.00\times10^{-6} \text{ m}^3}{2.70 \text{ g}}\right) = 1.66\times10^{-29} \text{ m}^3$

$\sqrt[3]{V} = \boxed{2.55\times10^{-10} \text{ m} \sim 10^{-1} \text{ nm}}$

U: $V = \left(\dfrac{238 \text{ g}}{6.02\times10^{23} \text{ molecules}}\right)\left(\dfrac{1.00\times10^{-6} \text{ m}^3}{18.9 \text{ g}}\right) = 2.09\times10^{-29} \text{ m}^3$

$\sqrt[3]{V} = \boxed{2.76\times10^{-10} \text{ m} \sim 10^{-1} \text{ nm}}$

(b) The outermost electron in any atom sees the nuclear charge screened by all the electrons below it. If we can visualize a single outermost electron, it moves in the electric field of net charge, $+Ze-(Z-1)e=+e$, the charge of a single proton, as felt by the electron in hydrogen. So the Bohr radius sets the scale for the outside diameter of every atom. An innermost electron, on the other hand, sees the nuclear charge unscreened, and the scale size of its (K-shell) orbit is a_0/Z.

42.60 (a) No orbital magnetic moment to consider: higher energy for $\boxed{\begin{matrix}N\\S\end{matrix}}\ \boxed{\begin{matrix}N\\S\end{matrix}}$ parallel magnetic moments, for $\boxed{\text{antiparallel spins}}$ of the electron and proton.

(b) $E = \dfrac{hc}{\lambda} = 9.42 \times 10^{-25}\ \text{J} = \boxed{5.89\ \mu\text{eV}}$

(c) $\Delta E \Delta t \approx \dfrac{h}{2}$ so $\Delta E \approx \dfrac{1.055 \times 10^{-34}\ \text{J}\cdot\text{s}}{2\left(10^7\ \text{yr}\right)\left(3.16 \times 10^7\ \text{s/yr}\right)}\left(\dfrac{1.00\ \text{eV}}{1.60 \times 10^{-19}\ \text{J}}\right) = \boxed{1.04 \times 10^{-30}\ \text{eV}}$

42.61 $P = \int_{2.50 a_0}^{\infty} \dfrac{4r^2}{a_0^{\ 3}} e^{-2r/a_0}\, dr = \frac{1}{2}\int_{5.00}^{\infty} z^2 e^{-z}\, dz$ where $z \equiv \dfrac{2r}{a_0}$

$$P = -\frac{1}{2}(z^2 + 2z + 2)e^{-z}\Big|_{5.00}^{\infty} = -\frac{1}{2}[0] + \frac{1}{2}(25.0 + 10.0 + 2.00)e^{-5} = \left(\frac{37}{2}\right)(0.00674) = \boxed{0.125}$$

Goal Solution

For hydrogen in the 1*s* state, what is the probability of finding the electron farther than $2.50 a_o$ from the nucleus?

G: From the graph shown in Figure 42.8, it appears that the probability of finding the electron beyond $2.5\ a_0$ is about 20%.

O: The precise probability can be found by integrating the 1*s* radial probability distribution function from $r = 2.50 a_o$ to ∞.

A: The general radial probability distribution function is $P(r) = 4\pi r^2 |\psi|^2$

With $\psi_{1s} = \left(\pi a_0^{\ 3}\right)^{-1/2} e^{-r/a_0}$ it is $P(r) = 4r^2 a_0^{\ -3} e^{-2r/a_0}$

The required probability is then $P = \int_{2.50 a_0}^{\infty} P(r) dr = \int_{2.50 a_0}^{\infty} \dfrac{4r^2}{a_0^{\ 3}} e^{-2r/a_0} dr$

Let $z = 2r/a_0$ and $dz = 2dr/a_0$: $P = \frac{1}{2}\int_{5.00}^{\infty} z^2 e^{-z} dz$

Performing this integration by parts, $P = -\frac{1}{2}\left(z^2 + 2z + 2\right)e^{-z}\Big]_{5.00}^{\infty}$

$$P = -\tfrac{1}{2}(0) + \tfrac{1}{2}(25.0 + 10.0 + 2.00)e^{-5.00} = \left(\tfrac{37}{2}\right)(0.00674) = 0.125$$

L: The probability of 12.5% is less than the 20% we estimated, but close enough to be a reasonable result. In comparing the 1*s* probability density function with the others in Figure 42.8, it appears that the ground state is the most narrow, indicating that a 1*s* electron will probably be found in the narrow range of 0 to 4 Bohr radii, and most likely at $r = a_0$.

42.62 The probability, P, of finding the electron within the Bohr radius is

$$P = \int_{r=0}^{a_0} P_{1s}(r)\,dr = \int_{r=0}^{a_0}\left(\frac{4r^2}{a_0{}^3}\right)e^{-2r/a_0}\,dr$$

Defining $z \equiv 2r/a_0$, this becomes

$$P = -\frac{1}{2}(z^2+2z+2)e^{-z}\Big|_0^2 = -\frac{1}{2}\left[(4+4+2)e^{-2}-(0+0+2)e^0\right] = \frac{1}{2}\left(2-\frac{10}{e^2}\right) = \boxed{0.323}$$

The electron is likely to be within the Bohr radius about one-third of the time. The Bohr model indicates *none* of the time.

42.63 (a) For a classical atom, the centripetal acceleration is

$$a = \frac{v^2}{r} = \frac{1}{4\pi e_0}\frac{e^2}{r^2 m_e}$$

$$E = -\frac{e^2}{4\pi e_0 r} + \frac{m_e v^2}{2} = -\frac{e^2}{8\pi e_0 r} \quad \text{so} \quad \frac{dE}{dt} = \frac{e^2}{8\pi e_0 r^2}\frac{dr}{dt} = \frac{-1}{6\pi e_0}\frac{e^2 a^2}{c^3} = \frac{-e^2}{6\pi e_0 c^3}\left(\frac{e^2}{4\pi e_0 r^2 m_e}\right)^2$$

Therefore,

$$\frac{dr}{dt} = -\frac{e^4}{12\pi^2 e_0{}^2 r^2 m_e{}^2 c^3}$$

(b) $$-\int_{r=2.00\times10^{-10}\text{ m}}^{r=0} 12\pi^2 e_0{}^2 r^2 m_e{}^2 c^3\,dr = e^4\int_{t=0}^{T} dt$$

$$\frac{12\pi^2 e_0{}^2 m_e{}^2 c^3}{e^4}\frac{r^3}{3}\Big|_0^{2.00\times10^{-10}} = T = \boxed{8.46\times10^{-10}\text{ s}}$$

Since atoms last a lot longer than 0.8 ns, the classical laws (fortunately!) do not hold for systems of atomic size.

42.64 (a) $+3e - 0.85e - 0.85e = \boxed{1.30e}$

(b) The valence electron is in an $n = 2$ state, with energy

$$\frac{-13.6\text{ eV } Z_{\text{eff}}^2}{n^2} = \frac{-13.6\text{ eV}(1.30)^2}{2^2} = -5.75\text{ eV}$$

To ionize the atom you must put in $\boxed{+5.75\text{ eV}}$

This differs from the experimental value by 6%, so we could say the effective value of Z is accurate within 3%.

42.65 $\Delta E = 2\mu_B B = hf$ so $2\left(9.27\times10^{-24}\text{ J / T}\right)\left(0.350\text{ T}\right) = \left(6.626\times10^{-34}\text{ J}\cdot\text{s}\right)f$

and $f = \boxed{9.79\times10^{9}\text{ Hz}}$

42.66 The photon energy is $E_4 - E_3 = 20.66 - 18.70\text{ eV} = 1.96\text{ eV} = \frac{hc}{\lambda}$

$$\lambda = \frac{(6.626\times10^{-34}\text{ J}\cdot\text{s})(3.00\times10^{8}\text{ m / s})}{1.96\times1.60\times10^{-19}\text{ J}} = \boxed{633\text{ nm}}$$

42.67 (a) $\frac{1}{\alpha} = \frac{\hbar c}{k_e e^2} = \frac{\left(6.626\times10^{-34}\right)\left(3.00\times10^{8}\right)}{2\pi\left(8.99\times10^{9}\right)\left(1.60\times10^{-19}\right)^2} = \boxed{137}$

(b) $\frac{\lambda_C}{r_e} = \frac{h}{mc}\frac{mc^2}{k_e e^2} = \frac{hc}{ke^2} = \boxed{\frac{2\pi}{\alpha}}$

(c) $\frac{a_0}{\lambda_C} = \frac{\hbar^2}{mk_e e^2}\frac{mc}{h} = \frac{1}{2\pi}\frac{\hbar c}{k_e e^2} = \frac{137}{2\pi} = \boxed{\frac{1}{2\pi\alpha}}$

(d) $\frac{1/R_H}{a_0} = \frac{1}{R_H a_0} = \frac{4\pi c\hbar^3}{mk_e^2 e^4}\frac{mk_e e^2}{\hbar^2} = 4\pi\frac{\hbar c}{k_e e^2} = \boxed{\frac{4\pi}{\alpha}}$

42.68 $\psi = \frac{1}{4}(2\pi)^{-1/2}\left(\frac{1}{a_0}\right)^{3/2}\left(2-\frac{r}{a_0}\right)e^{-r/2a_0} = A\left(2-\frac{r}{a_0}\right)e^{-r/2a_0}$ $\qquad \frac{\partial^2\psi}{\partial r^2} = \left(\frac{Ae^{-r/2a_0}}{a_0^2}\right)\left(\frac{3}{2}-\frac{r}{4a_0}\right)$

Substituting into Schrödinger's equation and dividing by ψ,

$$\frac{1}{a_0^2}\left(\frac{1}{2}-\frac{r}{4a_0}\right) = -\frac{2m}{\hbar^2}[E-U]\left(2-\frac{r}{a_0}\right)$$

Now $$E-U = \frac{\hbar^2}{2m\,a_0^2}\left(\frac{1}{4}\right) - \frac{\left(ke^2/4a_0\right)\left(m/\hbar^2\right)}{\left(m/\hbar^2\right)} = -\frac{1}{4}\left(\frac{\hbar^2}{2m\,a_0^2}\right)$$

and $$\left(\frac{1}{a_0^2}\right)\left(\frac{1}{2}-\frac{r}{4a_0}\right) = \frac{1}{4a_0^2}\left(2-\frac{r}{a_0}\right)$$ $\therefore \psi$ is a solution.

***42.69** The beam intensity is reduced by absorption of photons into atoms in the lower state. The number of transitions per time and per area is $-BN_l I(x) n\,dx/c$. The beam intensity is increased by stimulating emission in atoms in the upper state, with transition rate $+BN_u I(x) n\,dx/c$. The net rate of change in photon numbers per area is then $-B(N_l - N_u) I(x) n\,dx/c$.

Each photon has energy hf, so the net change in intensity is

$$dI(x) = -hfB(N_l - N_u) I(x) n\,dx/c = -hfB\Delta N\, I(x) n\,dx/c$$

Then, $\dfrac{dI(x)}{I(x)} = -\dfrac{hfB\Delta N\, n}{c}dx$ so $\displaystyle\int_{I_0}^{I(L)} \frac{dI(x)}{I(x)} = \int_{x=0}^{L}\left(-\frac{hfB\Delta N\, n}{c}\right)dx$

$$\ln[I(L)] - \ln[I_0] = \ln\left[\frac{I(L)}{I_0}\right] = -\frac{hfB\Delta N\, n}{c}(L-0)$$

$$I(L) = I_0\, e^{-hfB\Delta N\, nL/c} = I_0\, e^{-\alpha L}$$

This result is also expressed in problem 42.44 as $\dfrac{I(L)}{I_0} = G = e^{-\sigma(n_l - n_u)L} = e^{+\sigma(n_u - n_l)L}$

***42.70** (a) Suppose the atoms move in the $+x$ direction. The absorption of a photon by an atom is a completely inelastic collision, described by

$mv_i\,\mathbf{i} + \dfrac{h}{\lambda}(-\mathbf{i}) = mv_f\,\mathbf{i}$ so $v_f - v_i = -\dfrac{h}{m\lambda}$

This happens promptly every time an atom has fallen back into the ground state, so it happens every $10^{-8}\ \text{s} = \Delta t$. Then,

$$a = \frac{v_f - v_i}{\Delta t} = -\frac{h}{m\lambda\,\Delta t} \sim -\frac{6.626\times10^{-34}\ \text{J}\cdot\text{s}}{(10^{-25}\ \text{kg})(500\times10^{-9}\ \text{m})(10^{-8}\ \text{s})} \quad \boxed{-10^{6}\ \text{m/s}^2}$$

(b) With constant average acceleration, $v_f^2 = v_i^2 + 2a(\Delta x)$

$0 \sim (10^3\ \text{m/s})^2 + 2(-10^6\ \text{m/s}^2)\Delta x$ so $\Delta x \sim \dfrac{(10^3\ \text{m/s})^2}{10^6\ \text{m/s}^2}$ $\boxed{\sim 1\ \text{m}}$