Chapter 41 Solutions

41.1 (a) $$\lambda = \frac{h}{mv} = \frac{6.626\times10^{-34}\ \text{J}\cdot\text{s}}{(1.67\times10^{-27}\ \text{kg})(0.400\ \text{m/s})} = \boxed{9.92\times10^{-7}\ \text{m}}$$

(b) For destructive interference in a multiple-slit experiment,

$$d\sin\theta = \left(m+\frac{1}{2}\right)\lambda$$

with $m = 0$ for the first minimum. Then,

$$\theta = \sin^{-1}\left(\frac{\lambda}{2d}\right) = 0.0284°$$

$$\frac{y}{L} = \tan\theta \qquad \text{so} \qquad y = L\tan\theta = (10.0\ \text{m})(\tan\ 0.0284°) = \boxed{4.96\ \text{mm}}$$

(c) We cannot say the neutron passed through one slit. We can only say it passed through the slits.

41.2 Consider the first bright band away from the center: $d\sin\theta = m\lambda$

$$(6.00\times10^{-8}\ \text{m})\sin\left(\tan^{-1}\left[\frac{0.400}{200}\right]\right) = 1\lambda = 1.20\times10^{-10}\ \text{m}$$

$$\lambda = \frac{h}{m_e v} \qquad \text{so} \qquad m_e v = \frac{h}{\lambda} \qquad \text{and}$$

$$K = \tfrac{1}{2}m_e v^2 = \frac{m_e{}^2 v^2}{2m_e} = \frac{h^2}{2m_e\lambda^2} = e(\Delta V)$$

$$\Delta V = \frac{h^2}{2em_e\lambda^2} = \frac{(6.626\times10^{-34}\ \text{J}\cdot\text{s})^2}{2(1.60\times10^{-19}\ \text{C})(9.11\times10^{-31}\ \text{kg})(1.20\times10^{-10}\ \text{m})^2} = \boxed{105\ \text{V}}$$

41.3 (a) The wavelength of a non-relativistic particle of mass m is given by $\lambda = h/p = h/\sqrt{2mK}$ where the kinetic energy K is in joules. If the neutron kinetic energy K_n is given in electron volts, its kinetic energy in joules is $K = (1.60\times10^{-19}\ \text{J/eV})K_n$ and the equation for the wavelength becomes

$$\lambda = \frac{h}{\sqrt{2mK}} = \frac{6.626\times10^{-34}\ \text{J}\cdot\text{s}}{\sqrt{2(1.67\times10^{-27}\ \text{kg})(1.60\times10^{-19}\ \text{J/eV})K_n}} = \boxed{\frac{2.87\times10^{-11}}{\sqrt{K_n}}\ \text{m}}$$

where K_n is expressed in electron volts.

(b) If $K_n = 1.00\ \text{keV} = 1000\ \text{eV}$, then

41.4 $\lambda = \frac{h}{p} = \frac{h}{\sqrt{2mK}}$, so $K = \frac{h^2}{2m\lambda^2}$

If the particles are electrons and $\lambda \sim 0.1\text{ nm} = 10^{-10}\text{ m}$, the kinetic energy in electron volts is

$$K = \frac{\left(6.626\times10^{-34}\text{ J}\cdot\text{s}\right)^2}{2\left(9.11\times10^{-31}\text{ kg}\right)\left(10^{-10}\text{ m}\right)^2}\left(\frac{1\text{ eV}}{1.602\times10^{-19}\text{ J}}\right) = \boxed{\sim 10^2\text{ eV}}$$

41.5 $\lambda = \frac{h}{p}$ $\qquad p = \frac{h}{\lambda} = \frac{6.626\times10^{-34}\text{ J}\cdot\text{s}}{1.00\times10^{-11}\text{ m}} = 6.63\times10^{-23}\text{ kg}\cdot\text{m/s}$

(a) electrons: $K_e = \frac{p^2}{2m_e} = \frac{\left(6.63\times10^{-34}\text{ J}\cdot\text{s}\right)^2}{2\left(9.11\times10^{-31}\right)}\text{ J} = \boxed{15.1\text{ keV}}$

The relativistic answer is more precisely correct:

$$K_e = \left(p^2c^2 + m_e{}^2c^4\right)^{1/2} - m_ec^2 = 14/.9\text{ keV}$$

(b) photons: $E_\gamma = pc = \left(6.63\times10^{-23}\right)\left(3.00\times10^8\right) = \boxed{124\text{ keV}}$

41.6 The theoretical limit of the electron microscope is the wavelength of the electrons. If $K_e = 40.0\text{ keV}$, then $E = K_e + m_ec^2 = 551\text{ keV}$ and

$$p = \frac{1}{c}\sqrt{E^2 - m_e{}^2c^4} = \frac{\sqrt{(551\text{ keV})^2 - (511\text{ keV})^2}}{3.00\times10^8\text{ m/s}}\left(\frac{1.60\times10^{-16}\text{ J}}{1.00\text{ keV}}\right) = 1.10\times10^{-22}\text{ kg}\cdot\text{m/s}$$

The electron wavelength, and hence the theoretical limit of the microscope, is then

$$\lambda = \frac{h}{p} = \frac{6.626\times10^{-34}\text{ J}\cdot\text{s}}{1.10\times10^{-22}\text{ kg}\cdot\text{m/s}} = 6.03\times10^{-12}\text{ m} = \boxed{6.03\text{ pm}}$$

41.7 $E = K + m_ec^2 = 1.00\text{ MeV} + 0.511\text{ MeV} = 1.51\text{ MeV}$

$p^2c^2 = \sqrt{E^2 - m_e{}^2c^4} = \sqrt{(1.51\text{ MeV})^2 - (0.511\text{ MeV})^2}$ so $p = 1.42\text{ MeV}/c$

$$\lambda = \frac{h}{p} = \frac{hc}{1.42\text{ MeV}} = \frac{\left(6.626\times10^{-34}\text{ J}\cdot\text{s}\right)\left(3.00\times10^8\text{ m/s}\right)}{\left(1.42\times10^6\right)\left(1.60\times10^{-19}\text{ J}\right)} = 8.74\times10^{-13}\text{ m}$$

Suppose the array is like a flat diffraction grating with openings 0.250 nm apart: $d\sin\theta = m\lambda$

41.8 (a) $\Delta p \Delta x = m\Delta v \Delta x \geq \hbar / 2$ so

$$\Delta v \geq \frac{h}{4\pi m \Delta x} = \frac{2\pi \text{ J}\cdot\text{s}}{4\pi(2.00 \text{ kg})(1.00 \text{ m})} = \boxed{0.250 \text{ m/s}}$$

(b) The duck might move by $(0.25 \text{ m/s})(5 \text{ s}) = 1.25 \text{ m}$. With original position uncertainty of 1.00 m, we can think of Δx growing to $1.00 \text{ m} + 1.25 \text{ m} = \boxed{2.25 \text{ m}}$

41.9 For the electron,

$$\Delta p = m_e \Delta v = (9.11 \times 10^{-31} \text{ kg})(500 \text{ m/s})(1.00 \times 10^{-4}) = 4.56 \times 10^{-32} \text{ kg}\cdot\text{m/s}$$

$$\Delta x = \frac{h}{4\pi \Delta p} = \frac{6.626 \times 10^{-34} \text{ J}\cdot\text{s}}{4\pi(4.56 \times 10^{-32} \text{ kg}\cdot\text{m/s})} = \boxed{1.16 \text{ mm}}$$

For the bullet, $\Delta p = m\Delta v = (0.0200 \text{ kg})(500 \text{ m/s})(1.00 \times 10^{-4}) = 1.00 \times 10^{-3} \text{ kg}\cdot\text{m/s}$

$$\Delta x = \frac{h}{4\pi \Delta p} = \boxed{5.28 \times 10^{-32} \text{ m}}$$

Goal Solution

An electron ($m_e = 9.11 \times 10^{-31}$ kg) and a bullet $(m = 0.0200 \text{ kg})$ each have a speed of 500 m/s, accurate to within 0.0100%. Within what limits could we determine the position of the objects?

G: It seems reasonable that a tiny particle like an electron could be located within a more narrow region than a bigger object like a bullet, but we often find that the realm of the very small does not obey common sense.

O: Heisenberg's uncertainty principle can be used to find the uncertainty in position from the uncertainty in the momentum.

A: The uncertainty principle states: $\Delta x \Delta p_x \geq \hbar / 2$ where $\Delta p_x = m\Delta v$ and $h = \hbar / 2\pi$.

Both the electron and bullet have a velocity uncertainty,

$$\Delta v = (0.000100)(500 \text{ m/s}) = 0.0500 \text{ m/s}$$

For the electron, the minimum uncertainty in position is

$$\Delta x = \frac{h}{4\pi m \Delta v} = \frac{6.63 \times 10^{-34} \text{ J}\cdot\text{s}}{4\pi(9.11 \times 10^{-31} \text{ kg})(0.0500 \text{ m/s})} = 1.16 \text{ mm}$$

For the bullet,

$$\Delta x = \frac{h}{4\pi m \Delta v} = \frac{6.63 \times 10^{-34} \text{ J}\cdot\text{s}}{4\pi(0.0200 \text{ kg})(0.0500 \text{ m/s})} = 5.28 \times 10^{-32} \text{ m}$$

41.10 $\frac{\Delta y}{x} = \frac{\Delta p_y}{p_x}$ and $d\Delta p_y \ge h/4\pi$ Eliminate Δp_y and solve for x.

$$x = 4\pi p_x (\Delta y)\frac{d}{h} = 4\pi\left(1.00\times10^{-3}\text{ kg}\right)(100\text{ m/s})\left(1.00\times10^{-2}\text{ m}\right)\frac{\left(2.00\times10^{-3}\text{ m}\right)}{\left(6.626\times10^{-34}\text{ J}\cdot\text{s}\right)} =$$

$$\boxed{3.79\times10^{28}\text{ m}}$$

This is 190 times greater than the diameter of the Universe!

41.11 $\Delta p \Delta x \ge \frac{\hbar}{2}$ so $\Delta p = m_e \Delta v \ge \frac{h}{4\pi\,\Delta x}$

$$\Delta v \ge \frac{h}{4\pi\, m_e \Delta x} = \frac{6.626\times10^{-34}\text{ J}\cdot\text{s}}{4\pi\left(9.11\times10^{-31}\text{ kg}\right)\left(5.00\times10^{-11}\text{ m}\right)} = \boxed{1.16\times10^{6}\text{ m/s}}$$

41.12 With $\Delta x = 2\times10^{-15}$ m, the uncertainty principle requires $\Delta p_x \ge \frac{\hbar}{2\Delta x} = 2.6\times10^{-20}\text{ kg}\cdot\text{m/s}$

The average momentum of the particle bound in a stationary nucleus is zero. The uncertainty in momentum measures the root-mean-square momentum, so we take $p_{rms} = 3\times10^{-20}\text{ kg}\cdot\text{m/s}$.

For an electron, the non-relativistic approximation $p = m_e v$ would predict $v = 3\times10^{10}$ m/s, while v cannot be greater than c.

Thus, a better solution would be $E = \left[\left(m_e c^2\right)^2 + (pc)^2\right]^{1/2} = 56\text{ MeV} = \gamma m_e c^2$

$$\gamma \approx 110 = \frac{1}{\sqrt{1 - v^2/c^2}} \quad \text{so}$$

$$v \approx 0.99996c$$

For a proton, $v = p/m$ gives $v = 1.8\times10^{7}$ m / s, less than one-tenth the speed of light.

41.13 (a) At the top of the ladder, the woman holds a pellet inside a small region Δx_i. Thus, the uncertainty principle requires her to release it with typical horizontal momentum $\Delta p_x = m\Delta v_x = \hbar/2\Delta x_i$. It falls to the floor in time given by $H = 0 + \frac{1}{2}gt^2$ as $t = \sqrt{2H/g}$, so the total width of the impact points is

$$\Delta x_f = \Delta x_i + (\Delta v_x)t = \Delta x_i + \left(\frac{\hbar}{2m\Delta x_i}\right)\sqrt{\frac{2H}{g}} = \Delta x_i + \frac{A}{\Delta x_i}, \quad \text{where}$$

$$A = \frac{\hbar}{}\sqrt{\frac{2H}{}}$$

so $\Delta x_i = \sqrt{A}$, and the minimum width of the impact points is

$$\left(\Delta x_f\right)_{\min} = \left(\Delta x_i + \frac{A}{\Delta x_i}\right)\Bigg|_{\Delta x_i = \sqrt{A}} = 2\sqrt{A} = \boxed{\left(\frac{2\hbar}{m}\right)^{1/2} = \left(\frac{2H}{g}\right)^{1/4}}$$

(b) $$\left(\Delta x_f\right)_{\min} = \left[\frac{2\left(1.0546\times10^{-34}\ \text{J}\cdot\text{s}\right)}{5.00\times10^{-4}\ \text{kg}}\right]^{1/2}\left[\frac{2(2.00\ \text{m})}{9.80\ \text{m/s}^2}\right]^{1/4} = \boxed{5.19\times10^{-16}\ \text{m}}$$

41.14 Probability $P = \int_{-a}^{a} |\psi(x)|^2 = \int_{-a}^{a} \frac{a}{\pi(x^2 + a^2)} dx = \left(\frac{a}{\pi}\right)\left(\frac{1}{a}\right)\tan^{-1}\frac{x}{a}\Big|_{-a}^{a}$

$$P = \frac{1}{\pi}\left[\tan^{-1}1 - \tan^{-1}(-1)\right] = \frac{1}{\pi}\left[\frac{\pi}{4} - \left(-\frac{\pi}{4}\right)\right] = \boxed{1/2}$$

41.15 (a) $\psi(x) = A\sin\left(\frac{2\pi x}{\lambda}\right) = A\sin\left(5.00\times 10^{10} x\right)$

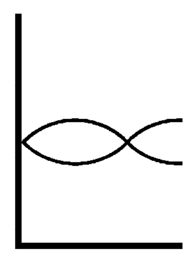
so $\frac{2\pi}{\lambda} = 5.00\times 10^{10}\ \text{m}^{-1}$ $\qquad \lambda = \frac{2\pi}{\left(5.00\times 10^{10}\right)} = \boxed{1.26\times 10^{-10}\ \text{m}}$

(b) $p = \frac{h}{\lambda} = \frac{6.626\times 10^{-34}\ \text{J}\cdot\text{s}}{1.26\times 10^{-10}\ \text{m}} = \boxed{5.27\times 10^{-24}\ \text{kg}\cdot\text{m/s}}$

(c) $m = 9.11\times 10^{-31}$ kg

$$K = \frac{p^2}{2m} = \frac{\left(5.27\times 10^{-24}\ \text{kg}\cdot\text{m/s}\right)^2}{\left(2\times 9.11\times 10^{-31}\ \text{kg}\right)} = 1.52\times 10^{-17}\ \text{J} = \frac{1.52\times 10^{-17}\ \text{J}}{1.602\times 10^{-19}\ \text{J/eV}} = \boxed{95.5\ \text{eV}}$$

41.16 For an electron to "fit" into an infinitely deep potential well, an integral number of half-wavelengths must equal the width of the well.



$\frac{n\lambda}{2} = 1.00\times 10^{-9}\ \text{m}$ so

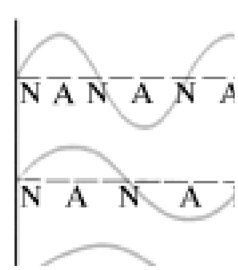
$$\lambda = \frac{2.00\times 10^{-9}}{n} = \frac{h}{p}$$

(a) Since $K = \frac{p^2}{2m_e} = \frac{\left(h^2/\lambda^2\right)}{2m_e} = \frac{h^2}{2m_e}\frac{n^2}{\left(2\times 10^{-9}\right)^2} = \left(0.377n^2\right)\ \text{eV}$

For $K \approx 6\ \text{eV}$, $\boxed{n = 4}$

(b) With $n = 4$, $\boxed{K = 6.03\ \text{eV}}$

41.17 (a) We can draw a diagram that parallels our treatment of standing mechanical waves. In each state, we measure the distance d from one node to another (N to N), and base our solution upon that:



Since $d_{N\text{ to N}} = \frac{\lambda}{2}$ and

Next,

$$K = \frac{p^2}{2m_e} = \frac{h^2}{8m_e d} = \frac{1}{d^2}\left[\frac{\left(6.626\times10^{-34}\ \text{J}\cdot\text{s}\right)^2}{8\left(9.11\times10^{-31}\ \text{kg}\right)}\right]$$

Evaluating, $K = \dfrac{6.02\times10^{-38}\ \text{J}\cdot\text{m}^2}{d^2}$

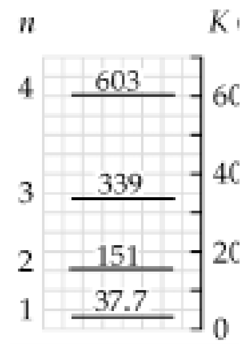
$$K = \frac{3.77\times10^{-19}\ \text{eV}\cdot\text{m}^2}{d^2}$$

In state 1, $d = 1.00\times10^{-10}\ \text{m}$
$K_1 = 37.7\ \text{eV}$

In state 2, $d = 5.00\times10^{-11}\ \text{m}$
$K_2 = 151\ \text{eV}$

In state 3, $d = 3.33\times10^{-11}\ \text{m}$
$K_3 = 339\ \text{eV}$

In state 4, $d = 2.50\times10^{-11}\ \text{m}$
$K_4 = 603\ \text{eV}$



(b) When the electron falls from state 2 to state 1, it puts out energy

$$E = 151\ \text{eV} - 37.7\ \text{eV} = 113\ \text{eV} = hf = \frac{hc}{\lambda}$$

into emitting a photon of wavelength

$$\lambda = \frac{hc}{E} = \frac{\left(6.626\times10^{-34}\ \text{J}\cdot\text{s}\right)\left(3.00\times10^{8}\ \text{m/s}\right)}{(113\ \text{eV})\left(1.60\times10^{-19}\ \text{J/eV}\right)} = 11.0\ \text{nm}$$

The wavelengths of the other spectral lines we find similarly:

Transition						
$E(\text{eV})$						
$\lambda(\text{nm})$						

41.18 $E_1 = 2.00 \text{ eV} = 3.20 \times 10^{-19} \text{ J}$

For the ground-state, $E_1 = \dfrac{h^2}{8m_e L^2}$

(a) $L = \dfrac{h}{\sqrt{8m_e E_1}} = 4.34 \times 10^{-10} \text{ m} = \boxed{0.434 \text{ nm}}$

(b) $\Delta E = E_2 - E_1 = 4\left(\dfrac{h^2}{8m_e L^2}\right) - \left(\dfrac{h^2}{8m_e L^2}\right) = \boxed{6.00 \text{ eV}}$

41.19 $\Delta E = \dfrac{hc}{\lambda} = \left(\dfrac{h^2}{8m_e L^2}\right)\left[2^2 - 1^2\right] = \dfrac{3h^2}{8m_e L^2}$

$L = \sqrt{\dfrac{3h\lambda}{8m_e c}} = 7.93 \times 10^{-10} \text{ m} = \boxed{0.793 \text{ nm}}$

41.20 $\Delta E = \dfrac{hc}{\lambda} = \left(\dfrac{h^2}{8m_e L^2}\right)\left[2^2 - 1^2\right] = \dfrac{3h^2}{8m_e L^2}$

so $L = \boxed{\sqrt{\dfrac{3h\lambda}{8m_e c}}}$

41.21 $E_n = \dfrac{n^2 h^2}{8mL^2}$

so $\Delta E = E_2 - E_1 = \dfrac{3h^2}{8mL^2} = \dfrac{3(hc)^2}{8mc^2 L^2}$

and $\Delta E = hf = \dfrac{hc}{\lambda}$

Hence, $\lambda = \dfrac{8mc^2 L^2}{3hc} = \dfrac{8\left(938 \times 10^6 \text{ eV}\right)\left(1.00 \times 10^{-5} \text{ nm}\right)^2}{3(1240 \text{ eV} \cdot \text{nm})}$

$\lambda = \boxed{2.02 \times 10^{-4} \text{ nm (gamma ray)}}$

$E = \dfrac{hc}{\lambda} = \boxed{6.15 \text{ MeV}}$

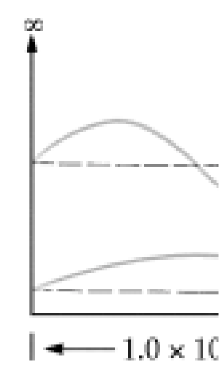


Figure for Goal Soluti

Goal Solution

The nuclear potential energy that binds protons and neutrons in a nucleus is often approximated by a square well. Imagine a proton confined in an infinitely high square well of width 10.0 fm, a typical nuclear diameter. Calculate the wavelength and energy associated with the photon emitted when the proton moves from the $n = 2$ state to the ground state. In what region of the electromagnetic spectrum does this wavelength belong?

G: Nuclear radiation from nucleon transitions is usually in the form of high energy gamma rays with short wavelengths.

O: The energy of the particle can be obtained from the wavelengths of the standing waves corresponding to each level. The transition between energy levels will result in the emission of a photon with this energy difference.

A: At level 1, the node-to-node distance of the standing wave is $1.00 \times 10^{-14}\text{ m}$, so the wavelength is twice this distance: $h / p = 2.00 \times 10^{-14}\text{ m}$. The proton's kinetic energy is

$$K = \tfrac{1}{2} m v^2 = \frac{p^2}{2m} = \frac{h^2}{2m\lambda^2} = \frac{\left(6.63 \times 10^{-34}\text{ J}\cdot\text{s}\right)^2}{2\left(1.67 \times 10^{-27}\text{ kg}\right)\left(2.00 \times 10^{-14}\text{ m}\right)^2} = \frac{3.29 \times 10^{-13}\text{ J}}{1.60 \times 10^{-19}\text{ J/eV}} = 2.06\text{ MeV}$$

In the first excited state, level 2, the node-to-node distance is two times smaller than in state 1. The momentum is two times larger and the energy is four times larger: $K = 8.23\text{ MeV}$.

The proton has mass, has charge, moves slowly compared to light in a standing-wave state, and stays inside the nucleus. When it falls from level 2 to level 1, its energy change is

$$2.06\text{ MeV} - 8.23\text{ MeV} = -6.17\text{ MeV}$$

Therefore, we know that a photon (a traveling wave with no mass and no charge) is emitted at the speed of light, and that it has an energy of $+6.17\text{ MeV}$.

Its frequency is
$$f = \frac{E}{h} = \frac{\left(6.17 \times 10^{6}\text{ eV}\right)\left(1.60 \times 10^{-19}\text{ J/eV}\right)}{6.63 \times 10^{-34}\text{ J}\cdot\text{s}} = 1.49 \times 10^{21}\text{ Hz}$$

and its wavelength is
$$\lambda = \frac{c}{f} = \frac{3.00 \times 10^{8}\text{ m/s}}{1.49 \times 10^{21}\text{ s}^{-1}} = 2.02 \times 10^{-13}\text{ m}$$

This is a gamma ray, according to Figure 34.17.

L: The radiated photons are energetic gamma rays as we expected for a nuclear transition. In the above calculations, we assumed that the proton was not relativistic ($v < 0.1c$), but we should check this assumption for the highest energy state we examined ($n = 2$):

$$v = \sqrt{\frac{2K}{m}} = \sqrt{\frac{2\left(8.23 \times 10^{6}\text{ eV}\right)\left(1.60 \times 10^{-19}\text{ J/eV}\right)}{1.67 \times 10^{-27}\text{ kg}}} = 3.97 \times 10^{7}\text{ m/s} = 0.133c$$

This appears to be a borderline case where we should probably use relativistic equations, but our classical treatment should give reasonable results, within $(0.133)^2 = 1\%$ accuracy.

41.22 $\lambda = 2D$ for the lowest energy state

$$K = \frac{p^2}{2m} = \frac{h^2}{2m\lambda^2} = \frac{h^2}{8mD} = \frac{(6.626\times10^{-34}\ \text{J}\cdot\text{s})^2}{8\left[4\left(1.66\times10^{-27}\ \text{kg}\right)\right]\left(1.00\times10^{-14}\ \text{m}\right)^2} = 8.27\times10^{-14}\ \text{J} = \boxed{0.517\ \text{MeV}}$$

$$p = \frac{h}{\lambda} = \frac{h}{2D} = \frac{6.626\times10^{-34}\ \text{J}\cdot\text{s}}{2\left(1.00\times10^{-14}\ \text{m}\right)} = \boxed{3.31\times10^{-20}\ \text{kg}\cdot\text{m/s}}$$

41.23 $E_n = \left(\dfrac{h^2}{8mL^2}\right)n^2$

$$E_1 = \frac{\left(6.626\times10^{-34}\ \text{J}\cdot\text{s}\right)^2}{8\left(1.67\times10^{-27}\ \text{kg}\right)\left(2.00\times10^{-14}\ \text{m}\right)^2} = 8.21\times10^{-14}\ \text{J}$$

$E_1 = \boxed{0.513\ \text{MeV}}$ $E_2 = 4E_1 = \boxed{2.05\ \text{MeV}}$ $E_3 = 9E_1 = \boxed{4.62\ \text{MeV}}$

41.24 (a) $$< x > = \int_0^L x\frac{2}{L}\sin^2\left(\frac{2\pi x}{L}\right)dx = \frac{2}{L}\int_0^L x\left(\frac{1}{2} - \frac{1}{2}\cos\frac{4\pi x}{L}\right)dx$$

$$< x > = \frac{1}{L}\frac{x^2}{2}\bigg|_0^L - \frac{1}{L}\frac{L^2}{16\pi^2}\left[\frac{4\pi x}{L}\sin\frac{4\pi x}{L} + \cos\frac{4\pi x}{L}\right]_0^L = \boxed{L/2}$$

(b) $$\text{Probability} = \int_{0.490L}^{0.510L}\frac{2}{L}\sin^2\left(\frac{2\pi x}{L}\right)dx = \left[\frac{1}{L}x - \frac{1}{L}\frac{L}{4\pi}\sin\frac{4\pi x}{L}\right]_{0.490L}^{0.510L}$$

$$\text{Probability} = 0.20 - \frac{1}{4\pi}\left(\sin 2.04\pi - \sin 1.96\pi\right) = \boxed{5.26\times10^{-5}}$$

(c) $$\text{Probability} = \left[\frac{x}{L} - \frac{1}{4\pi}\sin\frac{4\pi x}{L}\right]_{0.240L}^{0.260L} = \boxed{3.99\times10^{-2}}$$

(d) In the $n = 2$ graph in Figure 41.11 (b), it is more probable to find the particle either near

$x = \dfrac{L}{4}$ or $x = \dfrac{3L}{4}$ than at the center, where the probability density is zero.

$$\int_{x=0}^{L} A^2 \sin^2\left(\frac{n\pi x}{L}\right)dx = A^2\left(\frac{L}{2}\right) = 1 \qquad \text{or} \qquad \boxed{A = \sqrt{\frac{2}{L}}}$$

41.26 The desired probability is
$$P = \int_{x=0}^{x=L/4} |\psi|^2 dx = \frac{2}{L}\int_0^{L/4} \sin^2\left(\frac{2\pi x}{L}\right)dx$$

where
$$\sin^2\theta = \frac{1-\cos 2\theta}{2}$$

Thus,
$$P = \left(\frac{x}{L} - \frac{1}{4\pi}\sin\frac{4\pi x}{L}\right)\Bigg|_0^{L/4} = \left(\frac{1}{4} - 0 - 0 + 0\right) = \boxed{0.250}$$

41.27 In $0 \le x \le L$, the argument $2\pi x/L$ of the sine function ranges from 0 to 2π. The probability density $(2/L)\sin^2(2\pi x/L)$ reaches maxima at $\sin\theta = 1$ and $\sin\theta = -1$ at

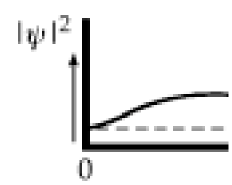
$$\frac{2\pi x}{L} = \frac{\pi}{2} \qquad \text{and} \qquad \frac{2\pi x}{L} = \frac{3\pi}{2}$$

$\therefore$ The most probable positions of the particle are at $\boxed{x = \frac{L}{4} \quad \text{and} \quad x = \frac{3L}{4}}$

41.28 (a) The probability is
$$P = \int_0^{L/3} |\psi|^2 dx = \int_0^{L/3} \frac{2}{L}\sin^2\left(\frac{\pi x}{L}\right)dx = \frac{2}{L}\int_0^{L/3}\left(\frac{1}{2} - \frac{1}{2}\cos\frac{2\pi x}{L}\right)dx$$

$$P = \left(\frac{x}{L} - \frac{1}{2\pi}\sin\frac{2\pi x}{L}\right)\Bigg|_0^{L/3} = \left(\frac{1}{3} - \frac{1}{2\pi}\sin\frac{2\pi}{3}\right) = \left(\frac{1}{3} - \frac{\sqrt{3}}{4\pi}\right) = \boxed{0.196}$$

(b) The probability density is symmetric about $x = L/2$. Thus, the probability of finding the particle between $x = 2L/3$ and $x = L$ is the same 0.196. Therefore, the probability of finding it in the range $L/3 \le x \le 2L/3$ is $P = 1.00 - 2(0.196) = 0.609$.



(c) Classically, the electron moves back and forth with constant speed between the walls, and the probability of finding the electron is the same for all points between the walls. Thus, the classical probability of finding the electron in any range equal to one-third of the available space is $P_{\text{classical}} = \boxed{1/3}$.

41.29 The ground state energy of a particle (mass m) in a 1-dimensional box of width L is $E_1 = \dfrac{h^2}{8mL^2}$.

(a) For a proton $\left(m = 1.67 \times 10^{-27}\ \text{kg}\right)$ in a 0.200-nm wide box:

$$E_1 = \frac{\left(6.626 \times 10^{-34}\ \text{J}\cdot\text{s}\right)^2}{8\left(1.67 \times 10^{-27}\ \text{kg}\right)\left(2.00 \times 10^{-10}\ \text{m}\right)^2} = 8.22 \times 10^{-22}\ \text{J} = \boxed{5.13 \times 10^{-3}\ \text{eV}}$$

(b) For an electron $\left(m = 9.11 \times 10^{-31}\ \text{kg}\right)$ in the same size box:

$$E_1 = \frac{\left(6.626 \times 10^{-34}\ \text{J}\cdot\text{s}\right)^2}{8\left(9.11 \times 10^{-31}\ \text{kg}\right)\left(2.00 \times 10^{-10}\ \text{m}\right)^2} = 1.51 \times 10^{-18}\ \text{J} = \boxed{9.41\ \text{eV}}$$

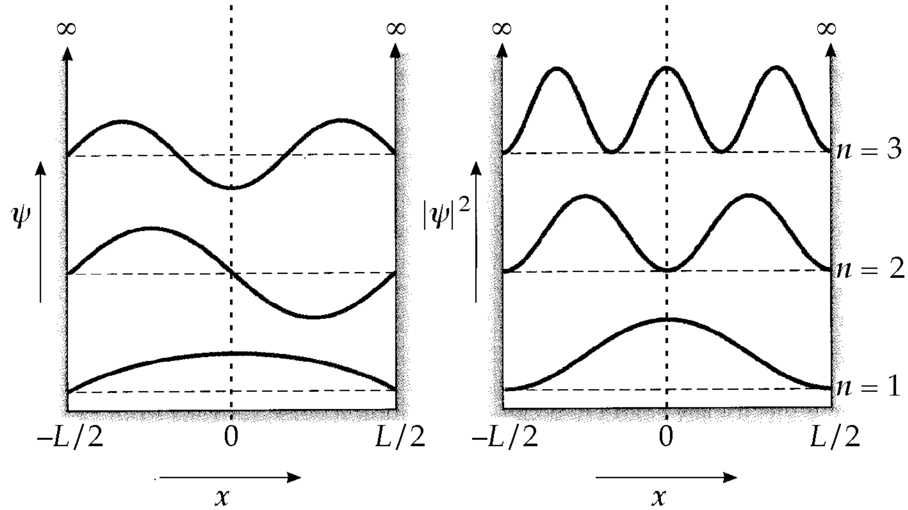
(c) The electron has a much higher energy because it is much less massive.

41.30 (a)

$$\psi_1(x) = \sqrt{\frac{2}{L}}\cos\left(\frac{\pi x}{L}\right) \qquad P_1(x) = \left|\psi_1(x)\right|^2 = \frac{2}{L}\cos^2\left(\frac{\pi x}{L}\right)$$

$$\psi_2(x) = \sqrt{\frac{2}{L}}\sin\left(\frac{2\pi x}{L}\right) \qquad P_2(x) = \left|\psi_2(x)\right|^2 = \frac{2}{L}\sin^2\left(\frac{2\pi x}{L}\right)$$

$$\psi_3(x) = \sqrt{\frac{2}{L}}\cos\left(\frac{3\pi x}{L}\right) \qquad P_3(x) = \left|\psi_3(x)\right|^2 = \frac{2}{L}\cos\left(\frac{3\pi x}{L}\right)$$



41.31 We have $\psi = Ae^{i(kx-\omega t)}$ and $\dfrac{\partial^2\psi}{\partial x^2} = -k^2\psi$

Schrödinger's equation: $$\frac{\partial^2\psi}{\partial x^2} = -k^2\psi = \frac{2m}{h^2}(E-U)\psi$$

Since $k^2 = \dfrac{(2\pi)^2}{\lambda^2} = \dfrac{(2\pi p)^2}{h^2} = \dfrac{p^2}{h^2}$ and $(E-U) = p^2/2m$

41.32 $\psi(x) = A\cos kx + B\sin kx$ $\qquad \dfrac{\partial\psi}{\partial x} = -kA\sin kx + kB\cos kx$

$$\frac{\partial^2\psi}{\partial x^2} = -k^2A\cos kx - k^2B\sin kx \qquad -\frac{2m}{\hbar^2}(E-U)\psi = -\frac{2mE}{\hbar^2}(A\cos kx + B\sin kx)$$

Therefore the Schrödinger equation is satisfied if

$$\frac{\partial^2\psi}{\partial x^2} = \left(-\frac{2m}{\hbar^2}\right)(E-U)\psi \qquad \text{or}$$

$$-k^2(A\cos kx + B\sin kx) = \left(-\frac{2mE}{\hbar^2}\right)(A\cos kx + B\sin kx)$$

This is true as an identity (functional equality) for all x if $\boxed{E = \dfrac{\hbar^2k^2}{2m}}$

41.33 Problem 45 in Ch. 16 helps students to understand how to draw conclusions from an identity.

(a) $\psi(x) = A\left(1 - \dfrac{x^2}{L^2}\right)$ $\qquad \dfrac{d\psi}{dx} = -\dfrac{2Ax}{L^2}$ $\qquad \dfrac{\partial^2\psi}{\partial x^2} = -\dfrac{2A}{L^2}$

Schrödinger's equation $\qquad \dfrac{\partial^2\psi}{\partial x^2} = -\dfrac{2m}{\hbar^2}(E-U)\psi$

becomes

$$-\frac{2A}{L^2} = \frac{2m}{\hbar^2}EA\left(1-\frac{x^2}{L^2}\right) + \frac{2m}{\hbar^2}\frac{\left(-\hbar^2x^2\right)A\left(1-\dfrac{x^2}{L^2}\right)}{mL^2\left(L^2-x^2\right)}$$

$$-\frac{1}{L^2} = -\frac{mE}{\hbar^2} + \frac{mEx^2}{\hbar^2L^2} - \frac{x^2}{L^4}$$

This will be true for all x if both $\qquad \dfrac{1}{L^2} = \dfrac{mE}{\hbar^2}$ $\qquad$ and $\qquad \dfrac{mE}{\hbar^2L^2} - \dfrac{1}{L^4} = 0$

Both of these conditions are satisfied for a particle of energy $\boxed{E = \dfrac{\hbar^2}{L^2m}}$.

(b) For normalization,

$$1 = \int_{-L}^{L} A^2\left(1-\frac{x^2}{L^2}\right)^2 dx = A^2\int_{-L}^{L}\left(1-\frac{2x^2}{L^2}+\frac{x^4}{L^4}\right)dx$$

$$1 = A^2\left[x - \frac{2x^3}{3L^2} + \frac{x^5}{5L^4}\right]_{-L}^{L} = A^2\left[L - \frac{2}{3}L + \frac{L}{5} + L - \frac{2}{3}L + \frac{L}{5}\right] = A^2\frac{16L}{15} \qquad \boxed{A = \sqrt{\frac{15}{16L}}}$$

(c)

$$P = \frac{47}{81} = \boxed{0.580}$$

41.34 (a) Setting the total energy E equal to zero and rearranging the Schrödinger equation to isolate the potential energy function gives

$$U(x)=\left(\frac{\hbar^2}{2m}\right)\frac{1}{\psi}\frac{d^2\psi}{dx^2}$$

If $\psi(x)=Axe^{-x^2/L^2}$ Then $\dfrac{d^2\psi}{dx^2}=\left(4Ax^3-6AxL^2\right)\dfrac{e^{-x^2/L^2}}{L^4}$

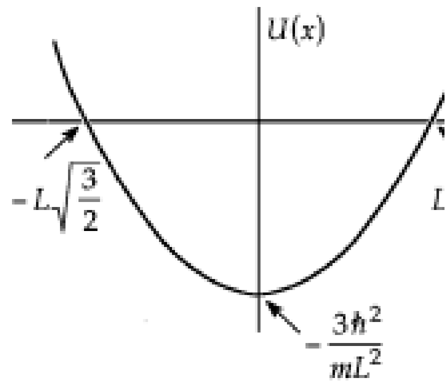
or

$$\frac{d^2\psi}{dx^2}=\frac{\left(4x^2-6L^2\right)}{L^4}\psi(x)$$

and

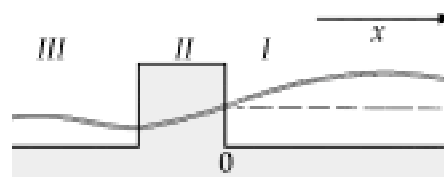
$$U(x)=\frac{\hbar^2}{2mL^2}\left(\frac{4x^2}{L^2}-6\right)$$

See figure to the right.



41.35 (a) See figure to the right.

(b) The wavelength of the transmitted wave traveling to the left is the same as the original wavelength, which equals $2L$.



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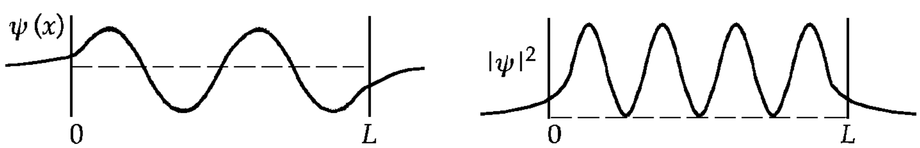
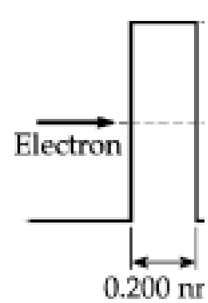


41.37 $T=e^{-2CL}$ (Use Equation 41.17)



$$2CL=\frac{2\sqrt{2\left(9.11\times10^{-31}\right)\left(8.00\times10^{-19}\right)}}{1.055\times10^{-34}}\left(2.00\times10^{-10}\right)=4.58$$

F
i

or GoalSolution

Goal Solution

An electron with kinetic energy $E = 5.00\ \text{eV}$ is incident on a barrier with thickness $L = 0.100\ \text{nm}$ and height $U = 10.0\ \text{eV}$ (Fig. P41.37). What is the probability that the electron (a) will tunnel through the barrier and (b) will be reflected?

G: Since the barrier energy is higher than the kinetic energy of the electron, transmission is not likely, but should be possible since the barrier is not infinitely high or thick.

O: The probability of transmission is found from the transmission coefficient equation 41.18.

A: The transmission coefficient is

$$C = \frac{\sqrt{2m(U-E)}}{\hbar} = \frac{\sqrt{2\left(9.11\times10^{-31}\ \text{kg}\right)(10.0\ \text{eV} - 5.00\ \text{eV})\left(1.60\times10^{-19}\ \text{J/eV}\right)}}{6.63\times10^{-34}\ \text{J}\cdot\text{s}/2\pi} = 1.14\times10^{10}\ \text{m}^{-1}$$

(a) The probability of transmission is

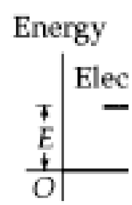
$$T = e^{-2CL} = e^{-2\left(1.14\times10^{10}\ \text{m}^{-1}\right)\left(2.00\times10^{-10}\ \text{m}\right)} = e^{-4.58} = 0.0103$$

(b) If the electron does not tunnel, it is reflected, with probability $1 - 0.0103 = 0.990$

L: Our expectation was correct: there is only a 1% chance that the electron will penetrate the barrier. This tunneling probability would be greater if the barrier were thinner, shorter, or if the kinetic energy of the electron were greater.

41.38

$$C = \frac{\sqrt{2\left(9.11\times10^{-31}\right)(5.00-4.50)\left(1.60\times10^{-19}\right)}\ \text{kg}\cdot\text{m/s}}{1.055\times10^{-34}\ \text{J}\cdot\text{s}}$$



$$T = e^{-2CL} = \exp\left[-2\left(3.62\times10^{9}\ \text{m}^{-1}\right)\left(950\times10^{-12}\ \text{m}\right)\right] = \exp(-6.88)$$

$$T = \boxed{1.03\times10^{-3}}$$

41.39 From problem 38, $C = 3.62\times10^{9}\ \text{m}^{-1}$

$$10^{-6} = \exp\left[-2\left(3.62\times10^{9}\ \text{m}^{-1}\right)L\right]$$

Taking logarithms, $-13.816 = -2\left(3.62\times10^{9}\ \text{m}^{-1}\right)L$

New $L = 1.91\ \text{nm}$

41.40 With the wave function proportional to e^{-CL}, the transmission coefficient and the tunneling current are proportional to $|\psi|^2$, to e^{-CL}.

Then, $$\frac{I(0.500\text{ nm})}{I(0.515\text{ nm})} = \frac{e^{-2(10.0\text{ /nm})(0.500\text{ nm})}}{e^{-2(10.0\text{ /nm})(0.515\text{ nm})}} = e^{20.0(0.015)} = \boxed{1.35}$$

41.41 With transmission coefficient e^{-CL}, the fractional change in transmission is

$$\frac{e^{-2(10.0\text{ /nm})L} - e^{-2(10.0\text{ /nm})(L+0.00200\text{ nm})}}{e^{-2(10.0\text{ /nm})L}} = 1 - e^{29.0(0.00200)} = 0.0392 = \boxed{3.92\%}$$

41.42 $\psi = Be^{-(m\omega/2h)x^2}$ so $\dfrac{d\psi}{dx} = -\left(\dfrac{m\omega}{h}\right)x\psi$ and $\dfrac{d^2\psi}{dx^2} = \left(\dfrac{m\omega}{h}\right)^2 x^2\psi + \left(-\dfrac{m\omega}{h}\right)\psi$

Substituting into Equation 41.19 gives

$$\left(\frac{m\omega}{h}\right)^2 x^2\psi + \left(-\frac{m\omega}{h}\right)\psi = \left(\frac{2mE}{h^2}\right)\psi + \left(\frac{m\omega}{h}\right)^2 x^2\psi$$

which is satisfied provided that $E = \dfrac{h\omega}{2}$.

41.43 Problem 45 in Chapter 16 helps students to understand how to draw conclusions from an identity.

$\psi = Axe^{-bx^2}$ so $\dfrac{d\psi}{dx} = Ae^{-bx^2} - 2bx^2Ae^{-bx^2}$

and

$$\frac{d^2\psi}{dx^2} = -2bxAe^{-bx^2} - 4bxAe^{-bx^2} + 4b^2x^3Ae^{-bx^2} = -6b\psi + 4b^2x^2\psi$$

Substituting into Equation 41.19, $$-6b\psi + 4b^2x^2\psi = -\left(\frac{2mE}{h}\right)\psi + \left(\frac{m\omega}{h}\right)^2 x^2\psi$$

For this to be true as an identity, it must be true for all values of x.

So we must have both $-6b = -\dfrac{2mE}{h^2}$ and $4b^2 = \left(\dfrac{m\omega}{h}\right)^2$

(a) Therefore $\boxed{b = \dfrac{m\omega}{2h}}$

(b) and $E = \dfrac{3bh^2}{m} = \boxed{\dfrac{3}{2}h\omega}$

(c) The wave function is that of the $\boxed{\text{first excited state}}$.

41.44 The longest wavelength corresponds to minimum photon energy, which must be equal to the spacing between energy levels of the oscillator:

$$\frac{hc}{\lambda} = \hbar\omega = \hbar\sqrt{\frac{k}{m}} \qquad \text{so}$$

$$\lambda = 2\pi c\sqrt{\frac{m}{k}} = 2\pi\left(3.00\times10^{8}\ \text{m/s}\right)\left(\frac{9.11\times10^{-31}\ \text{kg}}{8.99\ \text{N/m}}\right)^{1/2} = \boxed{600\ \text{nm}}$$

41.45 (a) With $\psi = Be^{-(m\omega/2\hbar)x^2}$, the normalization condition $\int_{\text{all}} |\psi|^2 dx = 1$

becomes
$$1 = \int_{-\infty}^{\infty} B^2 e^{-2(m\omega/2\hbar)x^2} dx = 2B^2\int_0^{\infty} e^{-2(m\omega/2\hbar)x^2} dx = 2B^2\,\frac{1}{2}\sqrt{\frac{\pi}{m\omega/\hbar}}$$

where Table B.6 in Appendix B was used to evaluate the integral.

Thus, $1 = B^2\sqrt{\dfrac{\pi\hbar}{m\omega}}$ and $\boxed{B = \left(\dfrac{m\omega}{\pi\hbar}\right)^{1/4}}$

(b) For small δ, the probability of finding the particle in the range $-\delta/2 \le x \le \delta/2$ is

$$\int_{-\delta/2}^{\delta/2} |\psi|^2 dx = \delta|\psi(0)|^2 = \delta B^2 e^{-0} = \boxed{\delta\left(\frac{m\omega}{\pi\hbar}\right)^{1/2}}$$

41.46 (a) With $\langle x\rangle = 0$ and $\langle p_x\rangle = 0$, the average value of x^2 is $(\Delta x)^2$ and the average value of $p_x^{\,2}$ is $(\Delta p_x)^2$. Then $\Delta x \ge \hbar/2\Delta p_x$ requires

$$E \ge \frac{p_x^{\,2}}{2m} + \frac{k}{2}\frac{\hbar^2}{4p_x^{\,2}} = \boxed{\frac{p_x^{\,2}}{2m} + \frac{k\hbar^2}{8p_x^{\,2}}}$$

(b) To minimize this as a function of $p_x^{\,2}$, we require
$$\frac{dE}{dp_x^{\,2}} = 0 = \frac{1}{2m} + \frac{k\hbar^2}{8}(-1)\frac{1}{p_x^{\,4}}$$

Then
$$\frac{k\hbar^2}{8p_x^{\,4}} = \frac{1}{2m}$$

$$p_x^{\,2} = \left(\frac{2mk\hbar^2}{8}\right)^{1/2} = \frac{\hbar\sqrt{mk}}{2}$$

and
$$E \ge \frac{\hbar\sqrt{mk}}{2(2m)} + \frac{k\hbar^2 3}{8\hbar\sqrt{mk}} = \frac{\hbar}{4}\sqrt{\frac{k}{m}} + \frac{\hbar}{4}\sqrt{\frac{k}{m}}.$$

41.47 Suppose the marble has mass 20 g. Suppose the wall of the box is 12 cm high and 2 mm thick. While it is inside the wall,

$$U = mgy = (0.02 \text{ kg})(9.8 \text{ m/s}^2)(0.12 \text{ m}) = 0.0235 \text{ J}$$

and $$E = K = \frac{1}{2}mv^2 = \frac{1}{2}(0.02 \text{ kg})(0.8 \text{ m/s})^2 = 0.0064 \text{ J}$$

Then $$C = \frac{\sqrt{2m(U-E)}}{\hbar} = \frac{\sqrt{2(0.02 \text{ kg})(0.0171 \text{ J})}}{1.055\times10^{-34} \text{ J}\cdot\text{s}} = 2.5\times10^{32} \text{ m}^{-1}$$

and the transmission coefficient is

$$e^{-2CL} = e^{-2(2.5\times10^{32})(2\times10^{-3})} = e^{-10\times10^{29}} = e^{-2.30(4.3\times10^{29})} = 10^{-4.3\times10^{29}} = \boxed{\sim 10^{-10^{30}}}$$

41.48 (a) $\lambda = 2L = \boxed{2.00\times10^{-10} \text{ m}}$

(b) $p = \dfrac{h}{\lambda} = \dfrac{6.626\times10^{-34} \text{ J}\cdot\text{s}}{2.00\times10^{-10} \text{ m}} = \boxed{3.31\times10^{-24} \text{ kg}\cdot\text{m/s}}$

(c) $E = \dfrac{p^2}{2m} = \boxed{0.172 \text{ eV}}$

41.49 (a) See the first figure to the right.

(b) See the second figure to the right.

(c) ψ is continuous and $\psi \to 0$ as $x \to \pm\infty$

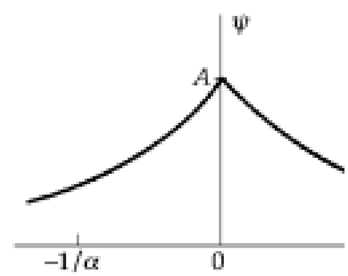
(d) Since ψ is symmetric,

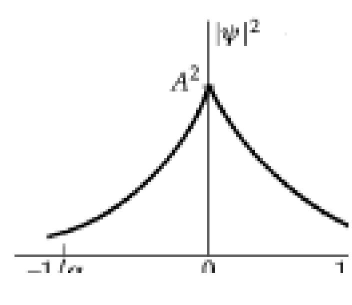
$$\int_{-\infty}^{\infty}|\psi|^2 dx = 2\int_0^{\infty}|\psi|^2 dx = 1$$

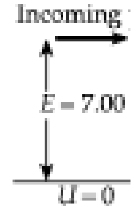
or

$$2A^2\int_0^{\infty} e^{-2\alpha x}dx = \left(\frac{2A^2}{-2\alpha}\right)\left(e^{-\infty} - e^0\right) = 1$$

This gives $A = \sqrt{\alpha}$







(a) Use Schrödinger's equation

$$\frac{\partial^2\psi}{\partial x^2} = -\frac{2m}{\hbar^2}(E-U)\psi$$

with solutions $\psi_1 = Ae^{ik_1x} + Be^{-ik_1x}$ [region *I*]

$$\psi_2 = Ce^{ik_2x}$$ [region *II*]

Where $k_1 = \frac{\sqrt{2mE}}{\hbar}$

and

$$k_2 = \frac{\sqrt{2m(E-U)}}{\hbar}$$

Then, matching functions and derivatives at $x = 0$: $(\psi_1)_0 = (\psi_2)_0 \quad \Rightarrow \quad A + B = C$

and

$$\left(\frac{d\psi_1}{dx}\right)_0 = \left(\frac{d\psi_2}{dx}\right)_0 \quad \Rightarrow \quad k_1(A-B) = k_2C$$

Then $B = \frac{1-k_2/k_1}{1+k_2/k_1}A$

$$C = \frac{2}{1+k_2/k_1}A$$

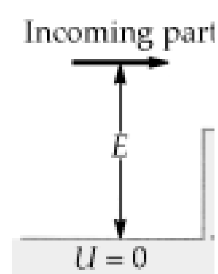
Incident wave Ae^{ikx} reflects Be^{-ikx}, with probability $R = \frac{B^2}{A^2} = \frac{(1-k_2/k_1)^2}{(1+k_2/k_1)^2} = \boxed{\frac{(k_1-k_2)^2}{(k_1+k_2)^2}}$

(b) With $E = 7.00$ eV and $U = 5.00$ eV, $\frac{k_2}{k_1} = \sqrt{\frac{E-U}{E}} = \sqrt{\frac{2.00}{7.00}} = 0.535$

The reflection probability is $R = \frac{(1-0.535)^2}{(1+0.535)^2} = \boxed{0.0920}$

The probability of transmission is $T = 1 - R = \boxed{0.908}$

41.51 $R = \frac{(k_1-k_2)^2}{(k_1+k_2)^2} = \frac{(1-k_2/k_1)^2}{(1+k_2/k_1)^2}$



$$\frac{\hbar^2k^2}{2m} = E - U \text{ for constant } U$$

$$\frac{h^2k_1^{\,2}}{2m} = E \text{ since } U = 0 \qquad (1)$$

Dividing (2) by (1), $\frac{k_2^{\ 2}}{k_1^{\ 2}} = 1 - \frac{U}{E} = 1 - \frac{1}{2} = \frac{1}{2}$ so

$$\frac{k_2}{k_1} = \frac{1}{\sqrt{2}}$$

and therefore,

$$R = \frac{\left(1 - 1/\sqrt{2}\right)^2}{\left(1 + 1/\sqrt{2}\right)^2} = \frac{\left(\sqrt{2} - 1\right)^2}{\left(\sqrt{2} + 1\right)^2} = \boxed{0.0294}$$

41.52 (a) The wave functions and probability densities are the same as those shown in the two lower curves in Figure 41.11 of the textbook.

(b)

$$P_1 = \int_{0.150\text{ nm}}^{0.350\text{ nm}} |\psi_1|^2 dx = \left(\frac{2}{1.00\text{ nm}}\right)\int_{0.150\text{ nm}}^{0.350\text{ nm}} \sin^2\left(\frac{\pi x}{1.00\text{ nm}}\right)dx = \frac{2.00}{\text{nm}}\left[\frac{x}{2} - \frac{1.00\text{ nm}}{4\pi}\sin\left(\frac{2\pi x}{1.00\text{ nm}}\right)\right]_{0.150\text{ nm}}^{0.350\text{ nm}}$$

In the above result we used $\int \sin^2 ax dx = (x/2) - (1/4a)\sin(2ax)$

$$P_1 = \frac{1.00}{\text{nm}}\left(x - \frac{1.00\text{ nm}}{2\pi}\sin\left(\frac{2\pi x}{1.00\text{ nm}}\right)\right)_{0.150\text{ nm}}^{0.350\text{ nm}}$$

$$P_1 = \frac{1.00}{\text{nm}}\left\{0.350\text{ nm} - 0.150\text{ nm} - \frac{1.00\text{ nm}}{2\pi}\left[\sin(0.700\pi) - \sin(0.300\pi)\right]\right\} = \boxed{0.200}$$

(c) $$P_2 = \frac{2}{1.00}\int_{0.150}^{0.350}\sin^2\left(\frac{2\pi x}{1.00}\right)dx = 2.00\left[\frac{x}{2} - \frac{1.00}{8\pi}\sin\left(\frac{4\pi x}{1.00}\right)\right]_{0.150}^{0.350}$$

$$P_2 = 1.00\left[x - \frac{1.00}{4\pi}\sin\left(\frac{4\pi x}{1.00}\right)\right]_{0.150}^{0.350} = 1.00\left\{(0.350 - 0.150) - \frac{1.00}{4\pi}\left[\sin(1.40\pi) - \sin(0.600\pi)\right]\right\} =$$
$\boxed{0.351}$

(d) Using $E_n = \dfrac{n^2h^2}{8mL^2}$, we find that $E_1 = \boxed{0.377\text{ eV}}$ and $E_2 = \boxed{1.51\text{ eV}}$

41.53 (a) $mgy_i = \frac{1}{2}mv_f^2$ $\quad v_f = \sqrt{2gy_i} = \sqrt{2(9.80\text{ m/s}^2)(50.0\text{ m})} = 31.3\text{ m/s}$

$$\lambda = \frac{h}{mv} = \frac{6.626\times10^{-34}\text{ J}\cdot\text{s}}{(75.0\text{ kg})(31.3\text{ m/s})} = \boxed{2.82\times10^{-37}\text{ m}} \text{ (not observable)}$$

(b) $\Delta E\Delta t \geq \hbar/2$ so $\quad \Delta E \geq \dfrac{6.626\times10^{-34}\text{ J}\cdot\text{s}}{4\pi(5.00\times10^{-3}\text{ s})} = \boxed{1.06\times10^{-32}\text{ J}}$

(c) $$\frac{\Delta E}{E} = \frac{1.06\times10^{-32}\text{ J}}{(75.0\text{ kg})(9.80\text{ m/s}^2)(50.0\text{ m})} = \boxed{2.87\times10^{-35}\text{ \%}}$$

41.54 From the uncertainty principle $\Delta E \Delta t - \hbar / 2$ or $\Delta\left(mc^2\right)\Delta t = \hbar / 2$. Therefore,

$$\frac{\Delta m}{m} = \frac{h}{4\pi c^2 (\Delta t) m} = \frac{h}{4\pi (\Delta t) E_R} = \frac{6.626 \times 10^{-34}\ \text{J}\cdot\text{s}}{4\pi\left(8.70 \times 10^{-17}\ \text{s}\right)(135\ \text{MeV})}\left(\frac{1\ \text{MeV}}{1.60 \times 10^{-13}\ \text{J}}\right) =$$

$$\boxed{2.81 \times 10^{-8}}$$

41.55 (a) $f=\dfrac{E}{h}=\dfrac{180\text{ eV}}{6.626\times10^{-34}\text{ J}\cdot\text{s}}\left(\dfrac{1.60\times10^{-19}\text{ J}}{1.00\text{ eV}}\right)=\boxed{4.34\times10^{14}\text{ Hz}}$

(b) $\lambda=\dfrac{c}{f}=\dfrac{3.00\times10^{8}\text{ m/s}}{4.34\times10^{14}\text{ Hz}}=6.91\times10^{-7}\text{ m}=\boxed{691\text{ nm}}$

(c) $\Delta E\Delta t\geq\dfrac{\hbar}{2}$ so

$$\Delta E\geq\frac{\hbar}{2\Delta t}=\frac{h}{4\pi\Delta t}=\frac{6.626\times10^{-34}\text{ J}\cdot\text{s}}{4\pi\left(2.00\times10^{-8}\text{ s}\right)}=2.64\times10^{-29}\text{ J}=\boxed{1.65\times10^{-10}\text{ eV}}$$

41.56 (a) $f=\boxed{\dfrac{E}{h}}$

(b) $\lambda=\dfrac{c}{f}=\boxed{\dfrac{hc}{E}}$

(c) $\Delta E\Delta t\geq\dfrac{\hbar}{2}$ so $\Delta E\geq\dfrac{\hbar}{2\Delta t}=\boxed{\dfrac{h}{4\pi T}}$

41.57 $\left\langle x^2\right\rangle=\int_{-\infty}^{\infty}x^2|\psi|^2dx$

For a one-dimensional box of width L, $\psi_n=\sqrt{\dfrac{2}{L}}\sin\left(\dfrac{n\pi x}{L}\right)$

Thus, $\left\langle x^2\right\rangle=\dfrac{2}{L}\int_0^L x^2\sin^2\left(\dfrac{n\pi x}{L}\right)dx=\boxed{\dfrac{L^2}{3}-\dfrac{L^2}{2n^2\pi^2}}$ (from integral tables)

41.58 (a) $\int_{-\infty}^{\infty}|\psi|^2dx=1$ becomes

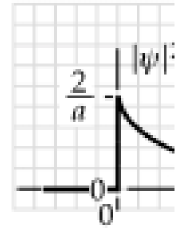
$$A^2\int_{-L/4}^{L/4}\cos^2\left(\frac{2\pi x}{L}\right)dx=A^2\left(\frac{L}{2\pi}\right)\left[\frac{\pi x}{L}+\frac{1}{4}\sin\left(\frac{4\pi x}{L}\right)\right]_{-L/4}^{L/4}=A^2\left(\frac{L}{2\pi}\right)\left(\frac{\pi}{2}\right)=1$$

or $A^2=\dfrac{4}{L}$ and $\boxed{A=\dfrac{2}{\sqrt{L}}}$

(b) The probability of finding the particle between 0 and $L/8$ is

$$\int_0^{L/8}|\psi|^2dx=A^2\int_0^{L/8}\cos^2\left(\frac{2\pi x}{L}\right)dx=\frac{1}{4}+\frac{1}{2\pi}=\boxed{0.409}$$

41.59 For a particle with wave function $\psi(x) = \sqrt{\frac{2}{a}}e^{-x/a}$ for $x > 0$ and 0 for $x < 0$



(a) $\left|\psi(x)\right|^2 = 0, \; x < 0$ and $\left|\psi^2(x)\right| = \frac{2}{a}e^{-2x/a}, \; x > 0$

(b) $\text{Prob}(x < 0) = \int_{-\infty}^{0}\left|\psi(x)\right|^2 dx = \int_{-\infty}^{0}(0)\,dx = \boxed{0}$

(c) Normalization

$$\int_{-\infty}^{\infty}\left|\psi(x)\right|^2 dx = \int_{-\infty}^{0}\left|\psi\right|^2 dx + \int_{0}^{\infty}\left|\psi\right|^2 dx = 1$$

$$\int_{-\infty}^{0} 0\,dx + \int_{0}^{\infty}(2/a)e^{-2x/a}dx = 0 - e^{-2x/a}\Big|_{0}^{\infty} = -\left(e^{-\infty} - 1\right) = 1$$

$$\text{Prob}(0 < x < a) = \int_{0}^{a}\left|\psi\right|^2 dx = \int_{0}^{a}(2/a)\,e^{-2x/a}dx = e^{-2x/a}\Big|_{0}^{a} = 1 - e^{-2} =$$
$\boxed{0.865}$

41.60 (a) $\lambda = \frac{h}{p} = \frac{hc}{\sqrt{E^2 - m_e{}^2c^4}} = \frac{hc}{\sqrt{\left(m_ec^2 + K\right)^2 - \left(m_ec^2\right)^2}}$

$$\lambda = \frac{\left(6.626\times10^{-34}\ \text{J}\cdot\text{s}\right)\left(3.00\times10^{8}\ \text{m/s}\right)}{\sqrt{(576\ \text{keV})^2 - (511\ \text{keV})^2}}\left(\frac{1\ \text{keV}}{1.60\times10^{-16}\ \text{J}}\right) = \boxed{4.68\times10^{-12}\ \text{m}}$$

(b) $50.0\lambda = \boxed{2.34\times10^{-10}\ \text{m}}$

41.61 (a) $\Delta x\Delta p \ge \hbar/2$ so if $\Delta x = r$, $\Delta p \ge \boxed{\hbar/2r}$

(b) Choosing $\Delta p = \frac{\hbar}{r}$, $K = \frac{p^2}{2m_e} = \frac{(\Delta p)^2}{2m_e} = \boxed{\frac{\hbar^2}{2m_er^2}}$

$U = -\frac{k_ee^2}{r}$, so $E = K + U = \boxed{\frac{\hbar^2}{2m_er^2} - \frac{k_ee^2}{r}}$

(c) To minimize E,

dE $\hbar$ $k_e e^2$ $\hbar^2$

Then, $E=\frac{\hbar^2}{2m_e}\left(\frac{m_ek_ee^2}{\hbar^2}\right)^2-k_ee^2\left(\frac{m_ek_ee^2}{\hbar^2}\right)=-\left(\frac{m_ek_e{}^2e^4}{2\hbar^2}\right)=\boxed{-13.6\text{ eV}}$

41.62 (a) The requirement that $\frac{n\lambda}{2}=L$ so $p=\frac{h}{\lambda}=\frac{nh}{2L}$ is still valid.

$$E=\sqrt{(pc)^2+\left(mc^2\right)^2}\quad\Rightarrow\quad E_n=\sqrt{\left(\frac{nhc}{2L}\right)^2+\left(mc^2\right)^2}$$

$$K_n=E_n-mc^2=\boxed{\sqrt{\left(\frac{nhc}{2L}\right)^2+\left(mc^2\right)^2}-mc^2}$$

(b) Taking $L=1.00\times10^{-12}$ m, $m=9.11\times10^{-31}$ kg, and $n=1$, we find $K_1=\boxed{4.69\times10^{-14}\text{ J}}$

Nonrelativistic, $E_1=\frac{h^2}{8mL^2}=\frac{\left(6.626\times10^{-34}\text{ J}\cdot\text{s}\right)^2}{8\left(9.11\times10^{-31}\text{ kg}\right)\left(1.00\times10^{-12}\text{ m}\right)^2}=6.02\times10^{-14}\text{ J}$

Comparing this to K_1, we see that this value is too large by $\boxed{28.6\%}$.

41.63 (a) $U=\frac{e^2}{4\pi\epsilon_0d}\left[-1+\frac{1}{2}-\frac{1}{3}+\left(-1+\frac{1}{2}\right)+(-1)\right]=\frac{(-7/3)e^2}{4\pi\epsilon_0d}=\boxed{-\frac{7k_ee^2}{3d}}$

(b) From Equation 41.9, $K=2E_1=\frac{2h^2}{8m_e\left(9d^2\right)}=\boxed{\frac{h^2}{36m_ed^2}}$

(c) $E=U+K$ and $\frac{dE}{dd}=0$ for a minimum: $\frac{7k_ee^2}{3d^2}-\frac{h^2}{18m_ed^3}=0$

$$d=\frac{3h^2}{(7)\left(18k_ee^2m_e\right)}=\frac{h^2}{42m_ek_ee^2}=\frac{\left(6.626\times10^{-34}\right)^2}{(42)\left(9.11\times10^{-31}\right)\left(8.99\times10^9\right)\left(1.602\times10^{-19}\text{ C}\right)^2}=$$

$\boxed{0.0499\text{ nm}}$

(d) Since the lithium spacing is a, where $Na^3=V$, and the density is Nm/V, where m is the mass of one atom, we get:

$$a=\left(\frac{Vm}{Nm}\right)^{1/3}=\left(\frac{m}{\text{density}}\right)^{1/3}=\left(\frac{1.66\times10^{-27}\text{ kg}\times7}{530\text{ kg}}\right)^{1/3}\text{ m}=2.80\times10^{-10}\text{ m}=\boxed{0.280\text{ nm}}$$

(5.62 times larger than c).

41.64 (a) $\psi = Bxe^{-(m\omega/2\hbar)x^2}$

$$\frac{d\psi}{dx} = Be^{-(m\omega/2\hbar)x^2} + Bx\left(\frac{-m\omega}{2\hbar}\right)2xe^{-(m\omega/2\hbar)x^2} = Be^{-(m\omega/2\hbar)x^2} - B\left(\frac{m\omega}{\hbar}\right)x^2e^{-(m\omega/2\hbar)x^2}$$

$$\frac{d^2\psi}{dx^2} = Bx\left(\frac{-m\omega}{\hbar}\right)xe^{-(m\omega/2\hbar)x^2} - B\left(\frac{m\omega}{\hbar}\right)2xe^{-(m\omega/2\hbar)x^2} - B\left(\frac{m\omega}{\hbar}\right)x^2\left(\frac{-m\omega}{\hbar}\right)xe^{-(m\omega/2\hbar)x^2}$$

$$\frac{d^2\psi}{dx^2} = -3B\left(\frac{m\omega}{\hbar}\right)xe^{-(m\omega/2\hbar)x^2} + B\left(\frac{m\omega}{\hbar}\right)x^3e^{-(m\omega/2\hbar)x^2}$$

Substituting into the Schrödinger Equation (41.19), we have

$$-3B\left(\frac{m\omega}{\hbar}\right)xe^{-(m\omega/2\hbar)x^2} + B\left(\frac{m\omega}{\hbar}\right)x^3e^{-(m\omega/2\hbar)x^2} = -\frac{2mE}{\hbar^2}Bxe^{-(m\omega/2\hbar)x^2} + \left(\frac{m\omega}{\hbar}\right)^2x^2Bxe^{-(m\omega/2\hbar)x^2}$$

This is true if $-3\omega = -\frac{2E}{\hbar}$; it is true if $\boxed{E = \frac{3}{2}\hbar\omega}$

(b) We never find the particle at $\boxed{x = 0}$ because $\psi = 0$ there.

(c) ψ is maximized if $\frac{d\psi}{dx} = 0 = 1 - x^2\left(\frac{m\omega}{\hbar}\right)$, which is true at $\boxed{x = \pm\sqrt{\frac{\hbar}{m\omega}}}$

(d) We require $\int_{-\infty}^{\infty}|\psi|^2dx = 1$:

$$1 = \int_{-\infty}^{\infty}B^2x^2e^{-(m\omega/\hbar)x^2}dx = 2B^2\int_0^{\infty}x^2e^{-(m\omega/\hbar)x^2}dx = 2B^2\frac{1}{4}\sqrt{\frac{\pi}{(m\omega/\hbar)^3}} = \frac{B^2}{2}\frac{\pi^{1/2}\hbar^{3/2}}{(m\omega)^{3/2}}$$

Then $B = \frac{2^{1/2}}{\pi^{1/4}}\left(\frac{m\omega}{\hbar}\right)^{3/4} = \boxed{\left(\frac{4m^3\omega^3}{\pi\hbar^3}\right)^{1/4}}$

(e) At $x = 2\sqrt{\hbar/m\omega}$, the potential energy is $\frac{1}{2}m\omega^2x^2 = \frac{1}{2}m\omega^2(4\hbar/m\omega) = 2\hbar\omega$. This is larger than the total energy $3\hbar\omega/2$, so there is $\boxed{\text{zero}}$ classical probability of finding the particle here.

(f) Probability $= |\psi|^2dx = \left(Bxe^{-(m\omega/2\hbar)x^2}\right)^2\delta = \delta B^2x^2e^{-(m\omega/\hbar)x^2}$

Probability $= \delta\frac{2}{\pi^{1/2}}\left(\frac{m\omega}{\hbar}\right)^{3/2}\left(\frac{4\hbar}{m\omega}\right)e^{-(m\omega/\hbar)4(\hbar/m\omega)} = \boxed{8\delta\left(\frac{m\omega}{\hbar\pi}\right)^{1/2}e^{-4}}$

41.65 (a) $\int_0^L |\psi|^2 dx = 1$: $A^2\int_0^L\left[\sin^2\left(\frac{\pi x}{L}\right)+16\sin^2\left(\frac{2\pi x}{L}\right)+8\sin\left(\frac{\pi x}{L}\right)\sin\left(\frac{2\pi x}{L}\right)\right]dx = 1$

$$A^2\left[\left(\frac{L}{2}\right)+16\left(\frac{L}{2}\right)+8\int_0^L \sin\left(\frac{\pi x}{L}\right)\sin\left(\frac{2\pi x}{L}\right)dx\right]=1$$

$$A^2\left[\frac{17L}{2}+16\int_0^L \sin^2\left(\frac{\pi x}{L}\right)\cos\left(\frac{\pi x}{L}\right)dx\right]=A^2\left[\frac{17L}{2}+\frac{16L}{3\pi}\sin^3\left(\frac{\pi x}{L}\right)\Bigg|_{x=0}^{x=L}\right]=1$$

$A^2=\frac{2}{17L}$, so the normalization constant is $\boxed{A=\sqrt{2/17L}}$

(b) $\int_{-a}^{a}|\psi|^2 dx = 1$:

$$\int_{-a}^{a}\left[|A|^2\cos^2\left(\frac{\pi x}{2a}\right)+|B|^2\sin^2\left(\frac{\pi x}{a}\right)+2|A||B|\cos\left(\frac{\pi x}{2a}\right)\sin\left(\frac{\pi x}{a}\right)\right]dx=1$$

The first two terms are $|A|^2 a$ and $|B|^2 a$. The third term is:

$$2|A||B|\int_{-a}^{a}\cos\left(\frac{\pi x}{2a}\right)\left[2\sin\left(\frac{\pi x}{2a}\right)\cos\left(\frac{\pi x}{2a}\right)\right]dx=4|A||B|\int_{-a}^{a}\cos^2\left(\frac{\pi x}{2a}\right)\sin\left(\frac{\pi x}{2a}\right)dx=\frac{8a|A||B|}{3\pi}\cos^3\left(\frac{\pi x}{2a}\right)\Bigg|_{-a}^{a}=0$$

so that $a\left(|A|^2+|B|^2\right)=1$, giving $\boxed{|A|^2+|B|^2=1/a}$.

41.66 With one slit open $P_1=|\psi_1|^2$ or $P_2=|\psi_2|^2$

With both slits open, $P=|\psi_1+\psi_2|^2$

At a maximum, the wave functions are in phase $P_{\max}=\left(|\psi_1|+|\psi_2|\right)^2$

At a minimum, the wave functions are out of phase $P_{\min}=\left(|\psi_1|-|\psi_2|\right)^2$

Now $\frac{P_1}{P_2}=\frac{|\psi_1|^2}{|\psi_2|^2}=25.0$, so $\frac{|\psi_1|}{|\psi_2|}=5.00$

‘41.67 (a) The light is unpolarized. It contains both horizontal and vertical field oscillations.

(b) The interference pattern appears, but with diminished overall intensity.

(c) The results are the same in each case.

(d) The interference pattern appears and disappears as the polarizer turns, with alternately increasing and decreasing contrast between the bright and dark fringes. The intensity on the screen is precisely zero at the center of a dark fringe four times in each revolution, when the filter axis has turned by 45°, 135°, 225°, and 315° from the vertical.

(e) Looking at the overall light energy arriving at the screen, we see a low-contrast interference pattern. After we sort out the individual photon runs into those for trial 1, those for trial 2, and those for trial 3, we have the original results replicated: The runs for trials 1 and 2 form the two blue graphs in Figure 41.3, and the runs for trial 3 build up the red graph.