Chapter 40 Solutions

40.1 $T = \dfrac{2.898 \times 10^{-3}\ \text{m} \cdot \text{K}}{560 \times 10^{-9}\ \text{m}} = \boxed{5.18 \times 10^{3}\ \text{K}}$

***40.2** (a) $\lambda_{\max} = \dfrac{2.898 \times 10^{-3}\ \text{m} \cdot \text{K}}{T} \sim \dfrac{2.898 \times 10^{-3}\ \text{m} \cdot \text{K}}{10^{4}\ \text{K}}\ \boxed{\sim 10^{-7}\ \text{m}}$ $\boxed{\text{ultraviolet}}$.

(b) $\lambda_{\max} \sim \dfrac{2.898 \times 10^{-3}\ \text{m} \cdot \text{K}}{10^{7}\ \text{K}}\ \boxed{\sim 10^{-10}\ \text{m}}$. $\boxed{\gamma - \text{ray}}$

40.3 (a) Using $\lambda_{\max} T = 2.898 \times 10^{-3}\ \text{m} \cdot \text{K}$

we get $\lambda_{\max} = \dfrac{2.898 \times 10^{-3}\ \text{m}}{2900\ \text{K}} = 9.99 \times 10^{-7}\ \text{m} = \boxed{999\ \text{nm}}$

(b) The $\boxed{\text{peak wavelength is in the infrared}}$ region of the electromagnetic spectrum, which is much wider than the visible region of the spectrum.

40.4 Planck's radiation law gives intensity-per-wavelength. Taking E to be the photon energy and n to be the number of photons emitted each second, we multiply by area and wavelength range to have energy-per-time leaving the hole:

$$\mathsf{P} = \frac{2\pi hc^{2}(\lambda_2 - \lambda_1)\pi(d/2)^{2}}{\left(\dfrac{\lambda_1 + \lambda_2}{2}\right)^{5}\left(e^{\frac{2hc}{(\lambda_1+\lambda_2)k_B T}} - 1\right)} = En = nhf \qquad \text{where} \qquad E = hf = \frac{2hc}{\lambda_1 + \lambda_2}$$

$$n = \frac{\mathsf{P}}{E} = \frac{8\pi^{2} c\, d^{2}(\lambda_2 - \lambda_1)}{(\lambda_1 + \lambda_2)^{4}\left(e^{2hc/(\lambda_1+\lambda_2)k_B T} - 1\right)} = \frac{8\pi^{2}\left(3.00 \times 10^{8}\ \text{m/s}\right)\left(5.00 \times 10^{-5}\ \text{m}\right)^{2}\left(1.00 \times 10^{-9}\ \text{m}\right)}{\left(1001 \times 10^{-9}\ \text{m}\right)^{4}\left(e^{\frac{2\left(6.626 \times 10^{-34}\ \text{J}\cdot\text{s}\right)\left(3.00 \times 10^{8}\ \text{m/s}\right)}{\left(1001 \times 10^{-9}\ \text{m}\right)\left(1.38 \times 10^{-23}\ \text{J/K}\right)\left(7.50 \times 10^{3}\ \text{K}\right)}} - 1\right)}$$

$$n = \frac{5.90 \times 10^{16}\ /\ \text{s}}{\left(e^{3.84} - 1\right)} = \boxed{1.30 \times 10^{15}\ /\ \text{s}}$$

***40.5** (a) $\mathcal{P} = eA\sigma T^4 = 1\left(20.0\times10^{-4}\text{ m}^2\right)\left(5.67\times10^{-8}\text{ W/m}^2\cdot\text{K}^4\right)(5000\text{ K})^4 = \boxed{7.09\times10^4\text{ W}}$

(b) $\lambda_{\max}T = \lambda_{\max}(5000\text{ K}) = 2.898\times10^{-3}\text{ m}\cdot\text{K} \Rightarrow \lambda_{\max} = \boxed{580\text{ nm}}$

(c) We compute: $\dfrac{hc}{k_BT} = \dfrac{\left(6.626\times10^{-34}\text{ J}\cdot\text{s}\right)\left(3.00\times10^8\text{ m/s}\right)}{\left(1.38\times10^{-23}\text{ J/K}\right)(5000\text{ K})} = 2.88\times10^{-6}\text{ m}$

The power per wavelength interval is $\mathcal{P}(\lambda) = AI(\lambda) = \dfrac{2\pi hc^2A}{\lambda^5\left[\exp\left(hc/\lambda k_BT\right)-1\right]}$, and

$$2\pi hc^2A = 2\pi\left(6.626\times10^{-34}\right)\left(3.00\times10^8\right)^2\left(20.0\times10^{-4}\right) = 7.50\times10^{-19}\ \frac{\text{J}\cdot\text{m}^4}{\text{s}}$$

$$\mathcal{P}(580\text{ nm}) = \frac{7.50\times10^{-19}\text{ J}\cdot\text{m}^4/\text{s}}{\left(580\times10^{-9}\text{ m}\right)^5\left[\exp\left(2.88\ \mu\text{m}/0.580\ \mu\text{m}\right)-1\right]} = \frac{1.15\times10^{13}\text{ J/m}\cdot\text{s}}{e^{4.973}-1} = \boxed{7.99\times10^{10}\text{ W/m}}$$

(d) - (i) The other values are computed similarly:

	λ	hc/k_BT	$e^{hc/\lambda k_BT}-1$	$2\pi hc^2A/\lambda^5$	$\mathcal{P}(\lambda)$, W/m
(d)	1.00 nm	2882.6	7.96×10^{1251}	7.50×10^{26}	9.42×10^{-1226}
(e)	5.00 nm	576.5	2.40×10^{250}	2.40×10^{23}	1.00×10^{-227}
(f)	400 nm	7.21	1347	7.32×10^{13}	5.44×10^{10}
(c)	580 nm	4.97	143.5	1.15×10^{13}	7.99×10^{10}
(g)	700 nm	4.12	60.4	4.46×10^{12}	7.38×10^{10}
(h)	1.00 mm	0.00288	0.00289	7.50×10^{-4}	0.260
(i)	10.0 cm	2.88×10^{-5}	2.88×10^{-5}	7.50×10^{-14}	2.60×10^{-9}

(j) We approximate the area under the $\mathcal{P}(\lambda)$ versus λ curve, between 400 nm and 700 nm, as two trapezoids:

$$\mathcal{P} \approx \frac{\left[(5.44+7.99)\times10^{10}\ \frac{\text{W}}{\text{m}}\right]\left[(580-400)\times10^{-9}\text{ m}\right]}{2} + \frac{\left[(7.99+7.38)\times10^{10}\ \frac{\text{W}}{\text{m}}\right]\left[(700-580)\times10^{-9}\text{ m}\right]}{2}$$

$\mathcal{P} = 2.13\times10^4\text{ W}$ so the power radiated as visible light is $\boxed{\text{approximately 20 kW}}$.

40.6 (a) $\mathsf{P} = eA\sigma T^4$, so

$$T = \left(\frac{\mathsf{P}}{eA\sigma}\right)^{1/4} = \left[\frac{3.77\times10^{26}\ \text{W}}{1\left[4\pi\left(6.96\times10^{8}\ \text{m}\right)^2\right]\left(5.67\times10^{-8}\ \frac{\text{W}}{\text{m}^2\cdot\text{K}^4}\right)}\right]^{1/4} = \boxed{5.75\times10^{3}\ \text{K}}$$

(b) $$\lambda_{\max} = \frac{2.898\times10^{-3}\ \text{m}\cdot\text{K}}{T} = \frac{2.898\times10^{-3}\ \text{m}\cdot\text{K}}{5.75\times10^{3}\ \text{K}} = 5.04\times10^{-7}\ \text{m} = \boxed{504\ \text{nm}}$$

40.7 (a) $$E = hf = \left(6.626\times10^{-34}\ \text{J}\cdot\text{s}\right)\left(620\times10^{12}\ \text{s}^{-1}\right)\left(\frac{1.00\ \text{eV}}{1.60\times10^{-19}\ \text{J}}\right) = \boxed{2.57\ \text{eV}}$$

(b) $$E = hf = \left(6.626\times10^{-34}\ \text{J}\cdot\text{s}\right)\left(3.10\times10^{9}\ \text{s}^{-1}\right)\left(\frac{1.00\ \text{eV}}{1.60\times10^{-19}\ \text{J}}\right) = \boxed{1.28\times10^{-5}\ \text{eV}}$$

(c) $$E = hf = \left(6.626\times10^{-34}\ \text{J}\cdot\text{s}\right)\left(46.0\times10^{6}\ \text{s}^{-1}\right)\left(\frac{1.00\ \text{eV}}{1.60\times10^{-19}\ \text{J}}\right) = \boxed{1.91\times10^{-7}\ \text{eV}}$$

(d) $$\lambda = \frac{c}{f} = \frac{3.00\times10^{8}\ \text{m/s}}{620\times10^{12}\ \text{Hz}} = 4.84\times10^{-7}\ \text{m} = \boxed{484\ \text{nm, visible light (blue)}}$$

$$\lambda = \frac{c}{f} = \frac{3.00\times10^{8}\ \text{m/s}}{3.10\times10^{9}\ \text{Hz}} = 9.68\times10^{-2}\ \text{m} = \boxed{9.68\ \text{cm, radio wave}}$$

$$\lambda = \frac{c}{f} = \frac{3.00\times10^{8}\ \text{m/s}}{46.0\times10^{6}\ \text{Hz}} = \boxed{6.52\ \text{m, radio wave}}$$

40.8 $$E = hf = \frac{hc}{\lambda} = \frac{\left(6.626\times10^{-34}\ \text{J}\cdot\text{s}\right)\left(3.00\times10^{8}\ \text{m/s}\right)}{589.3\times10^{-9}\ \text{m}} = 3.37\times10^{-19}\ \text{J/photon}$$

$$n = \frac{\mathsf{P}}{E} = \frac{10.0\ \text{J/s}}{3.37\times10^{-19}\ \text{J/photon}} = \boxed{2.96\times10^{19}\ \text{photons/s}}$$

40.9 Each photon has an energy $E = hf = (6.626\times10^{-34})(99.7\times10^{6}) = 6.61\times10^{-26}\ \text{J}$

This implies that there are $$\frac{150\times10^{3}\ \text{J/s}}{6.61\times10^{-26}\ \text{J/photons}} = \boxed{2.27\times10^{30}\ \text{photons/s}}$$

***40.10** Energy of a single 500-nm photon:

$$E_\gamma = hf = \frac{hc}{\lambda} = \frac{(6.626\times 10^{-34}\ \text{J}\cdot\text{s})(3.00\times 10^{8}\ \text{m/s})}{500\times 10^{-9}\ \text{m}} = 3.98\times 10^{-19}\ \text{J}$$

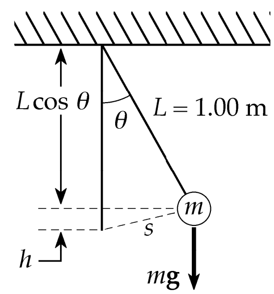
The energy entering the eye each second

$$E = \mathcal{P}t = (IA)t = (4.00\times 10^{-11}\ \text{W/m}^2)\frac{\pi}{4}(8.50\times 10^{-3}\ \text{m})^2(1.00\ \text{s}) = 2.27\times 10^{-15}\ \text{J}$$

The number of photons required to yield this energy

$$n = \frac{E}{E_\gamma} = \frac{2.27\times 10^{-15}\ \text{J}}{3.98\times 10^{-19}\ \text{J/photon}} = \boxed{5.71\times 10^{3}\ \text{photons}}$$

40.11 We take $\theta = 0.0300$ radians. Then the pendulum's total energy is



$$E = mgh = mg(L - L\cos\theta)$$

$$E = (1.00\ \text{kg})(9.80\ \text{m/s}^2)(1.00 - 0.9995) = 4.41\times 10^{-3}\ \text{J}$$

The frequency of oscillation is $f = \frac{\omega}{2\pi} = \frac{1}{2\pi}\sqrt{g/L} = 0.498\ \text{Hz}$

The energy is quantized, $E = nhf$

Therefore, $n = \frac{E}{hf} = \frac{4.41\times 10^{-3}\ \text{J}}{(6.626\times 10^{-34}\ \text{J}\cdot\text{s})(0.498\ \text{s}^{-1})} = \boxed{1.34\times 10^{31}}$

40.12 The radiation wavelength of $\lambda' = 500$ nm that is observed by observers on Earth is not the true wavelength, λ, emitted by the star because of the Doppler effect. The true wavelength is related to the observed wavelength using:

$$\frac{c}{\lambda'} = \frac{c}{\lambda}\sqrt{\frac{1-(v/c)}{1+(v/c)}}$$

$$\lambda = \lambda'\sqrt{\frac{1-(v/c)}{1+(v/c)}} = (500\ \text{nm})\sqrt{\frac{1-(0.280)}{1+(0.280)}} = 375\ \text{nm}$$

The temperature of the star is given by $\lambda_{\max}T = 2.898\times 10^{-3}\ \text{m}\cdot\text{K}$:

$$T = \frac{2.898\times 10^{-3}\ \text{m}\cdot\text{K}}{\lambda_{\max}} = \frac{2.898\times 10^{-3}\ \text{m}\cdot\text{K}}{375\times 10^{-9}} = \boxed{7.73\times 10^{3}\ \text{K}}$$

40.13 This follows from the fact that at low T or long λ, the exponential factor in the denominator of Planck's radiation law is large compared to 1, so the factor of 1 in the denominator can be neglected. In this approximation, one arrives at *Wien's radiation law*.

***40.14** Planck's radiation law is $I(\lambda,T)=\dfrac{2\pi hc^2}{\lambda^5\left(e^{hc/\lambda k_B T}-1\right)}$

Using the series expansion $e^x = 1+x+\dfrac{x^2}{2!}+\dfrac{x^3}{3!}+\ldots$

Planck's law reduces to $I(\lambda,T)=\dfrac{2\pi hc^2}{\lambda^5\left[\left(1+hc/\lambda k_B T+\ \ldots\right)-1\right]}\approx\dfrac{2\pi hc^2}{\lambda^5\left(hc/\lambda k_B T\right)}=\dfrac{2\pi ck_B T}{\lambda^4}$

which is the Rayleigh-Jeans law, for very long wavelengths.

40.15 (a) $\lambda_c = \dfrac{hc}{\phi} = \dfrac{(6.626\times10^{-34}\ \text{J}\cdot\text{s})(3.00\times10^{8}\ \text{m/s})}{(4.20\ \text{eV})(1.60\times10^{-19}\ \text{J/eV})} = \boxed{296\ \text{nm}}$

$f_c = \dfrac{c}{\lambda_c} = \dfrac{3.00\times10^{8}\ \text{m/s}}{296\times10^{-9}\ \text{m}} = \boxed{1.01\times10^{15}\ \text{Hz}}$

(b) $\dfrac{hc}{\lambda} = \phi + e(\Delta V_S)$: $\dfrac{(6.626\times10^{-34})(3.00\times10^{8})}{180\times10^{-9}} = (4.20\ \text{eV})(1.60\times10^{-19}\ \text{J/eV}) + (1.60\times10^{-19})(\Delta V_S)$

Therefore, $\boxed{\Delta V_S = 2.71\ \text{V}}$

40.16 $K_{\text{max}} = \frac{1}{2}mv_{\text{max}}^2 = \frac{1}{2}(9.11\times10^{-31})(4.60\times10^{5})^2 = 9.64\times10^{-20}\ \text{J} = 0.602\ \text{eV}$

(a) $\phi = E - K_{\text{max}} = \dfrac{1240\ \text{eV}\cdot\text{nm}}{625\ \text{nm}} - 0.602\ \text{eV} = \boxed{1.38\ \text{eV}}$

(b) $f_c = \dfrac{\phi}{h} = \dfrac{1.38\ \text{eV}}{6.626\times10^{-34}\ \text{J}\cdot\text{s}}\left(\dfrac{1.60\times10^{-19}\ \text{J}}{1\ \text{eV}}\right) = \boxed{3.34\times10^{14}\ \text{Hz}}$

40.17 (a) $\lambda_c = \dfrac{hc}{\phi}$

Li: $$\lambda_c = \frac{\left(6.626\times10^{-34}\text{ J}\cdot\text{s}\right)\left(3.00\times10^{8}\text{ m / s}\right)}{(2.30\text{ eV})\left(1.60\times10^{-19}\text{ J / eV}\right)} = 540\text{ nm}$$

Be: $$\lambda_c = \frac{\left(6.626\times10^{-34}\text{ J}\cdot\text{s}\right)\left(3.00\times10^{8}\text{ m / s}\right)}{(3.90\text{ eV})\left(1.60\times10^{-19}\text{ J / eV}\right)} = 318\text{ nm}$$

Hg: $$\lambda_c = \frac{\left(6.626\times10^{-34}\text{ J}\cdot\text{s}\right)\left(3.00\times10^{8}\text{ m / s}\right)}{(4.50\text{ eV})\left(1.60\times10^{-19}\text{ J / eV}\right)} = 276\text{ nm}$$

$\lambda < \lambda_c$ for photo current. $\boxed{\text{Thus, only lithium will exhibit the photoelectric effect.}}$

(b) For lithium, $$\frac{hc}{\lambda} = \phi + K_{\max}$$

$$\frac{\left(6.626\times10^{-34}\text{ J}\cdot\text{s}\right)\left(3.00\times10^{8}\text{ m / s}\right)}{400\times10^{-9}\text{ m}} = (2.30\text{ eV})\left(1.60\times10^{-19}\right) + K_{\max}$$

$$K_{\max} = 1.29\times10^{-19}\text{ J} = \boxed{0.808\text{ eV}}$$

40.18 From condition (i), $hf = e(\Delta V_{S1}) + \phi_1$ and $hf = e(\Delta V_{S2}) + \phi_2$

$$(\Delta V_{S1}) = (\Delta V_{S2}) + 1.48\text{ V}$$

Then $\phi_2 - \phi_1 = 1.48\text{ eV}$

From condition (ii), $h f_{c1} = \phi_1 = 0.600 h f_{c2} = 0.600\phi_2$

$$\phi_2 - 0.600\phi_2 = 1.48\text{ eV}$$

$$\boxed{\phi_2 = 3.70\text{ eV}} \quad \boxed{\phi_1 = 2.22\text{ eV}}$$

40.19 (a) $e(\Delta V_S) = \dfrac{hc}{\lambda} - \phi \quad \rightarrow \quad \phi = \dfrac{1240\text{ nm}\cdot\text{eV}}{546.1\text{ nm}} - 0.376\text{ eV} = \boxed{1.90\text{ eV}}$

(b) $e(\Delta V_S) = \dfrac{hc}{\lambda} - \phi = \dfrac{1240\text{ nm}\cdot\text{eV}}{587.5\text{ nm}} - 1.90\text{ eV} \quad \rightarrow \quad \Delta V_S = \boxed{0.216\text{ V}}$

Goal Solution

Two light sources are used in a photoelectric experiment to determine the work function for a particular metal surface. When green light from a mercury lamp (λ = 546.1 nm) is used, a retarding potential of 0.376 V reduces the photocurrent to zero. (a) Based on this measurement, what is the work function for this metal? (b) What stopping potential would be observed when using the yellow light from a helium discharge tube (λ = 587.5 nm)?

G: According to Table 40.1, the work function for most metals is on the order of a few eV, so this metal is probably similar. We can expect the stopping potential for the yellow light to be slightly lower than 0.376 V since the yellow light has a longer wavelength (lower frequency) and therefore less energy than the green light.

O: In this photoelectric experiment, the green light has sufficient energy hf to overcome the work function of the metal ϕ so that the ejected electrons have a maximum kinetic energy of 0.376 eV. With this information, we can use the photoelectric effect equation to find the work function, which can then be used to find the stopping potential for the less energetic yellow light.

A: (a) Einstein's photoelectric effect equation is $K_{\max} = hf - \phi$, and the energy required to raise an electron through a 1 V potential is 1 eV, so that $K_{\max} = eV_s = 0.376$ eV.

A photon from the mercury lamp has energy: $$hf = \frac{hc}{\lambda} = \frac{\left(4.14\times10^{-15}\text{ eV}\cdot\text{s}\right)\left(3.00\times10^{8}\text{ m/s}\right)}{546.1\times10^{-9}\text{ m}}$$

$$E = hf = 2.27\text{ eV}$$

Therefore, the work function for this metal is: $$\phi = hf - K_{\max} = 2.27\text{ eV} - (0.376\text{ eV}) = 1.90\text{ eV}$$

(b) For the yellow light, $\lambda = 587.5$ nm, and $$hf = \frac{hc}{\lambda} = \frac{\left(4.14\times10^{-15}\text{ eV}\cdot\text{s}\right)\left(3.00\times10^{8}\text{ m / s}\right)}{587.5\times10^{-9}\text{ m}}$$

$$E = 2.11\text{ eV}$$

Therefore, $K_{\max} = hf - \phi = 2.11\text{ eV} - 1.90\text{ eV} = 0.216\text{ eV}$, so $V_s = 0.216$ V

L: The work function for this metal is lower than we expected, and does not correspond with any of the values in Table 40.1. Further examination in the **CRC Handbook of Chemistry and Physics** reveals that all of the metal elements have work functions between 2 and 6 eV. However, a single metal's work function may vary by about 1 eV depending on impurities in the metal, so it is just barely possible that a metal might have a work function of 1.90 eV.

The stopping potential for the yellow light is indeed lower than for the green light as we expected. An interesting calculation is to find the wavelength for the lowest energy light that will eject electrons from this metal. That threshold wavelength for $K_{\max}$ = 0 is 658 nm, which is red light in the visible portion of the electromagnetic spectrum.)

40.20 From the photoelectric equation, we have: $e(\Delta V_{S1}) = E_{\gamma 1} - \phi$ and $e(\Delta V_{S2}) = E_{\gamma 2} - \phi$

Since $\Delta V_{S2} = 0.700(\Delta V_{S1})$, then $e(\Delta V_{S2}) = 0.700(E_{\gamma 1} - \phi) = E_{\gamma 2} - \phi$

or $(1 - 0.700)\phi = E_{\gamma 2} - 0.700 E_{\gamma 1}$

and the work function is: $\phi = \dfrac{E_{\gamma 2} - 0.700 E_{\gamma 1}}{0.300}$

The photon energies are: $E_{\gamma 1} = \dfrac{hc}{\lambda_1} = \dfrac{1240\text{ nm}\cdot\text{eV}}{410\text{ eV}} = 3.03\text{ eV}$

and $E_{\gamma 2} = \dfrac{hc}{\lambda_2} = \dfrac{1240\text{ nm}\cdot\text{eV}}{445\text{ eV}} = 2.79\text{ eV}$

Thus, the work function is $\phi = \dfrac{2.79\text{ eV} - 0.700(3.03\text{ eV})}{0.300} = 2.23\text{ eV}$

and we recognize this as characteristic of $\boxed{\text{potassium}}$.

***40.21** The energy needed is $E = 1.00\text{ eV} = 1.60 \times 10^{-19}\text{ J}$

The energy absorbed in time t is $E = Pt = (IA)t$

so $$t = \frac{E}{IA} = \frac{1.60 \times 10^{-19}\text{ J}}{(500\text{ J/s}\cdot\text{m}^2)[\pi(2.82 \times 10^{-15}\text{ m})^2]} = 1.28 \times 10^7\text{ s} = \boxed{148\text{ days}}$$

The gross failure of the classical theory of the photoelectric effect contrasts with the success of quantum mechanics.

***40.22** Ultraviolet photons will be absorbed to knock electrons out of the sphere with maximum kinetic energy $K_{\max} = hf - \phi$, or

$$K_{\max} = \frac{(6.626 \times 10^{-34}\text{ J}\cdot\text{s})(3.00 \times 10^8\text{ m/s})}{200 \times 10^{-9}\text{ m}}\left(\frac{1.00\text{ eV}}{1.60 \times 10^{-19}\text{ J}}\right) - 4.70\text{ eV} = 1.51\text{ eV}$$

The sphere is left with positive charge and so with positive potential relative to $V = 0$ at $r = \infty$. As its potential approaches 1.51 V, no further electrons will be able to escape, but will fall back onto the sphere. Its charge is then given by

$$V = \frac{k_e Q}{r} \quad\text{or}\quad Q = \frac{rV}{k_e} = \frac{(5.00 \times 10^{-2}\text{ m})(1.51\text{ N}\cdot\text{m/C})}{8.99 \times 10^9\text{ N}\cdot\text{m}^2/\text{C}^2} = \boxed{8.41 \times 10^{-12}\text{ C}}$$

40.23 (a) By having the photon source move toward the metal, the incident photons are Doppler shifted to higher frequencies, and hence, higher energy.

(b) If $v = 0.280c$, $f' = f\sqrt{\dfrac{1+v/c}{1-v/c}} = \left(7.00\times10^{14}\right)\sqrt{\dfrac{1.28}{0.720}} = 9.33\times10^{14}\ \text{Hz}$

Therefore, $\phi = \left(6.626\times10^{-34}\ \text{J}\cdot\text{s}\right)\left(9.33\times10^{14}\ \text{Hz}\right) = 6.18\times10^{-19}\ \text{J} = \boxed{3.87\ \text{eV}}$

(c) At $v = 0.900c$, $f = 3.05\times10^{15}\ \text{Hz}$

and $K_{\max} = hf - \phi = \left(6.626\times10^{-34}\ \text{J}\cdot\text{s}\right)\left(3.05\times10^{15}\ \text{Hz}\right)\left(\dfrac{1.00\ \text{eV}}{1.60\times10^{-19}\ \text{J}}\right) - 3.87\ \text{eV} = \boxed{8.78\ \text{eV}}$

***40.24** $$E = \frac{hc}{\lambda} = \frac{(6.626\times10^{-34}\ \text{J}\cdot\text{s})(3.00\times10^{8}\ \text{m/s})}{700\times10^{-9}\ \text{m}} = 2.84\times10^{-19}\ \text{J} = \boxed{1.78\ \text{eV}}$$

$$p = \frac{h}{\lambda} = \frac{6.626\times10^{-34}\ \text{J}\cdot\text{s}}{700\times10^{-9}\ \text{m}} = \boxed{9.47\times10^{-28}\ \text{kg}\cdot\text{m/s}}$$

40.25 (a) $\Delta\lambda = \dfrac{h}{m_e c}(1-\cos\theta) = \dfrac{6.626\times10^{-34}}{(9.11\times10^{-31})(3.00\times10^{8})}(1-\cos 37.0°) = \boxed{4.88\times10^{-13}\ \text{m}}$

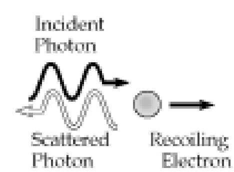
(b) $E_0 = hc/\lambda_0$: $\left(300\times10^{3}\ \text{eV}\right)\left(1.60\times10^{-19}\ \text{J/eV}\right) = \left(6.626\times10^{-34}\right)\left(3.00\times10^{8}\ \text{m/s}\right)/\lambda_0$

$\lambda_0 = 4.14\times10^{-12}\ \text{m}$ and $\lambda' = \lambda_0 + \Delta\lambda = 4.63\times10^{-12}\ \text{m}$

$$E' = \frac{hc}{\lambda'} = \frac{\left(6.626\times10^{-34}\ \text{J}\cdot\text{s}\right)\left(3.00\times10^{8}\ \text{m/s}\right)}{4.63\times10^{-12}\ \text{m}} = 4.30\times10^{14}\ \text{J} = \boxed{268\ \text{keV}}$$

(c) $K_e = E_0 - E' = 300\ \text{keV} - 268.5\ \text{keV} = \boxed{31.5\ \text{keV}}$

40.26 This is Compton scattering through 180°:



$$E_0 = \frac{hc}{\lambda_0} = \frac{(6.626\times10^{-34}\ \text{J}\cdot\text{s})(3.00\times10^{8}\ \text{m/s})}{(0.110\times10^{-9}\ \text{m})(1.60\times10^{-19}\ \text{J/eV})} = 11.3\ \text{keV}$$

$$\Delta\lambda = \frac{h}{m_e c}(1-\cos\theta) = (2.43\times10^{-12}\ \text{m})(1-\cos 180°) = 4.86\times10^{-12}\ \text{m}$$

$\lambda' = \lambda_0 + \Delta\lambda = 0.115\ \text{nm}$ so $E' = \dfrac{hc}{\lambda'} = 10.8\ \text{keV}$

Momentum conservation: $\dfrac{h}{\lambda_0}\,\mathbf{i} = \dfrac{h}{\lambda'}(-\mathbf{i}) + p_e(\mathbf{i})$ and $p_e = h\left(\dfrac{1}{\lambda_0} - \dfrac{1}{\lambda'}\right)$

$$p_e = \left(6.626\times10^{-34}\ \text{J}\cdot\text{s}\right)\left(\frac{\left(3.00\times10^{8}\ \text{m/s}\right)/c}{1.60\times10^{-19}\ \text{J/eV}}\right)\left(\frac{1}{0.110\times10^{-9}\ \text{m}} + \frac{1}{0.115\times10^{-9}\ \text{m}}\right) = \boxed{22.1\ \text{keV}/c}$$

Energy conservation: $11.3 \text{ keV} = 10.8 \text{ keV} + K_e$ so that $\boxed{K_e = 478 \text{ eV}}$

Check: $E^2 = p^2c^2 + m_e^2c^4$ or $(m_ec^2 + K_e)^2 = (pc)^2 + (m_ec^2)^2$

$(511 \text{ keV} + 0.478 \text{ keV})^2 = (22.1 \text{ keV})^2 + (511 \text{ keV})^2$

$2.62 \times 10^{11} = 2.62 \times 10^{11}$

40.27 $K_e = E_0 - E'$

With $K_e = E'$, $E' = E_0 - E'$: $E' = \dfrac{E_0}{2}$

$$\lambda' = \frac{hc}{E'} = \frac{hc}{\frac{1}{2}E_0} = 2\frac{hc}{E_0} = 2\lambda_0 \qquad \lambda' = \lambda_0 + \lambda_C(1 - \cos\theta)$$

$$2\lambda_0 = \lambda_0 + \lambda_C(1 - \cos\theta) \qquad 1 - \cos\theta = \frac{\lambda_0}{\lambda_C} = \frac{0.00160}{0.00243} \to \theta = \boxed{70.0^\circ}$$

40.28 We may write down four equations, not independent, in the three unknowns λ_0, λ', and v using the conservation laws:

$$\frac{hc}{\lambda_0} = \frac{hc}{\lambda'} + \gamma m_ec^2 - m_ec^2 \qquad \text{(Energy conservation)}$$

$$\frac{h}{\lambda_0} = \gamma m_e v \cos 20.0^\circ \qquad \text{(momentum in } x\text{-direction)}$$

$$0 = \frac{h}{\lambda'} - \gamma m_e v \sin 20.0^\circ \qquad \text{(momentum in } y\text{-direction)}$$

and Compton's equation $\lambda' - \lambda_0 = \dfrac{h}{m_ec}(1 - \cos 90.0^\circ)$.

It is easiest to ignore the energy equation and, using the two momentum equations, write

$$\frac{h/\lambda_0}{h/\lambda'} = \frac{\gamma m_e v \cos 20.0^\circ}{\gamma m_e v \sin 20.0^\circ} \quad \text{or} \quad \lambda_0 = \lambda' \tan 20.0^\circ$$

Then, the Compton equation becomes $\lambda' - \lambda' \tan 20.0^\circ = 0.00243 \text{ nm}$,

$$\text{or} \quad \lambda' = \frac{0.00243 \text{ nm}}{1 - \tan 20.0^\circ} = 0.00382 \text{ nm} = \boxed{3.82 \text{ pm}}$$

40.29 (a) Conservation of momentum in the x direction gives: $p_\gamma = p'_\gamma \cos\theta + p_e \cos\phi$

or since $\theta = \phi$, $\dfrac{h}{\lambda_0} = \left(p_e + \dfrac{h}{\lambda'}\right)\cos\theta$ [1]

Conservation of momentum in the y direction gives: $0 = p'_\gamma \sin\theta - p_e \sin\theta$,

which (neglecting the trivial solution $\theta = 0$) gives: $p_e = p'_\gamma = \dfrac{h}{\lambda'}$ [2]

Substituting [2] into [1] gives: $\dfrac{h}{\lambda_0} = \dfrac{2h}{\lambda'}\cos\theta$, or $\lambda' = 2\lambda_0 \cos\theta$ [3]

Then the Compton equation is $\lambda' - \lambda_0 = \dfrac{h}{m_e c}(1 - \cos\theta)$

giving $2\lambda_0 \cos\theta - \lambda_0 = \dfrac{h}{m_e c}(1 - \cos\theta)$

or $2\cos\theta - 1 = \dfrac{hc}{\lambda_0}\dfrac{1}{m_e c^2}(1 - \cos\theta)$

Since $E_\gamma = \dfrac{hc}{\lambda_0}$, this may be written as: $2\cos\theta - 1 = \left(\dfrac{E_\gamma}{m_e c^2}\right)(1 - \cos\theta)$

which reduces to: $\left(2 + \dfrac{E_\gamma}{m_e c^2}\right)\cos\theta = 1 + \dfrac{E_\gamma}{m_e c^2}$

or $\cos\theta = \dfrac{m_e c^2 + E_\gamma}{2m_e c^2 + E_\gamma} = \dfrac{0.511\text{ MeV} + 0.880\text{ MeV}}{1.02\text{ MeV} + 0.880\text{ MeV}} = 0.732$ so that $\boxed{\theta = \phi = 43.0°}$

(b) Using Equation (3): $E'_\gamma = \dfrac{hc}{\lambda'} = \dfrac{hc}{\lambda_0(2\cos\theta)} = \dfrac{E_\gamma}{2\cos\theta} = \dfrac{0.880\text{ MeV}}{2\cos 43.0°} = 0.602\text{ MeV} = \boxed{602\text{ keV}}$

Then, $p'_\gamma = \dfrac{E'_\gamma}{c} = 0.602\ \text{MeV}/c = \boxed{3.21\times 10^{-22}\ \text{kg}\cdot\text{m/s}}$

(c) From Equation (2), $p_e = p'_\gamma = 0.602\ \text{MeV}/c = \boxed{3.21\times 10^{-22}\ \text{kg}\cdot\text{m/s}}$

From energy conservation: $K_e = E_\gamma - E'_\gamma = 0.880\text{ MeV} - 0.602\text{ MeV} = 0.278\text{ MeV} = \boxed{278\text{ keV}}$

40.30 The energy of the incident photon is $E_0 = p_\gamma c = hc/\lambda_0$.

(a) Conserving momentum in the x direction gives

$$p_\gamma = p_e \cos\phi + p'_\gamma \cos\theta, \text{ or since } \phi = \theta, \quad \frac{E_0}{c} = \left(p_e + p'_\gamma\right)\cos\theta \qquad [1]$$

Conserving momentum in the y direction (with $\phi = \theta$) yields

$$0 = p'_\gamma \sin\theta - p_e \sin\theta, \quad \text{or} \quad p_e = p'_\gamma = \frac{h}{\lambda'} \qquad [2]$$

Substituting Equation [2] into Equation [1] gives

$$\frac{E_0}{c} = \left(\frac{h}{\lambda'} + \frac{h}{\lambda'}\right)\cos\theta, \quad \text{or} \quad \lambda' = \frac{2hc}{E_0}\cos\theta \qquad [3]$$

By the Compton equation, $\lambda' - \lambda_0 = \frac{h}{m_e c}(1 - \cos\theta)$, $\qquad \frac{2hc}{E_0}\cos\theta - \frac{2hc}{E_0} = \frac{h}{m_e c}(1 - \cos\theta)$

which reduces to $\qquad \left(2m_e c^2 + E_0\right)\cos\theta = m_e c^2 + E_0$

Thus, $\qquad \boxed{\phi = \theta = \cos^{-1}\left(\frac{m_e c^2 + E_0}{2m_e c^2 + E_0}\right)}$

(b) From Equation [3], $\qquad \lambda' = \frac{2hc}{E_0}\cos\theta = \frac{2hc}{E_0}\left(\frac{m_e c^2 + E_0}{2m_e c^2 + E_0}\right)$

Therefore, $\qquad E'_\gamma = \frac{hc}{\lambda'} = \frac{hc}{\left(2hc/E_0\right)\left(m_e c^2 + E_0\right)/\left(2m_e c^2 + E_0\right)} = \boxed{\frac{E_0}{2}\left(\frac{2m_e c^2 + E_0}{m_e c^2 + E_0}\right)}$,

and $\qquad p'_\gamma = \frac{E'_\gamma}{c} = \boxed{\frac{E_0}{2c}\left(\frac{2m_e c^2 + E_0}{m_e c^2 + E_0}\right)}$

(c) From conservation of energy, $\qquad K_e = E_0 - E'_\gamma = E_0 - \frac{E_0}{2}\left(\frac{2m_e c^2 + E_0}{m_e c^2 + E_0}\right)$

or $\qquad K_e = \frac{E_0}{2}\left(\frac{2m_e c^2 + 2E_0 - 2m_e c^2 - E_0}{m_e c^2 + E_0}\right) = \boxed{\frac{E_0^2}{2\left(m_e c^2 + E_0\right)}}$

Finally, from Equation (2), $\qquad p_e = p'_\gamma = \boxed{\frac{E_0}{2c}\left(\frac{2m_e c^2 + E_0}{m_e c^2 + E_0}\right)}$

40.31 (a) Thanks to Compton we have four equations in the unknowns ϕ, v, and λ':

$$\frac{hc}{\lambda_0} = \frac{hc}{\lambda'} + \gamma m_e c^2 - m_e c^2 \qquad \text{(energy conservation)} \qquad [1]$$

$$\frac{h}{\lambda_0} = \frac{h}{\lambda'}\cos 2\phi + \gamma m_e v \cos\phi \qquad \text{(momentum in } x \text{ direction)} \qquad [2]$$

$$0 = \frac{h}{\lambda'}\sin 2\phi - \gamma m_e v \sin\phi \qquad \text{(momentum in } y \text{ direction)} \qquad [3]$$

$$\lambda' - \lambda_0 = \frac{h}{m_e c}(1 - \cos 2\phi) \qquad \text{(Compton equation)} \qquad [4]$$

Using $\sin 2\phi = 2\sin\phi\cos\phi$ in Equation [3] gives $\gamma m_e v = \frac{2h}{\lambda'}\cos\phi$.

Substituting this into Equation [2] and using $\cos 2\phi = 2\cos^2\phi - 1$ yields

$$\frac{h}{\lambda_0} = \frac{h}{\lambda'}(2\cos^2\phi - 1) + \frac{2h}{\lambda'}\cos^2\phi = \frac{h}{\lambda'}(4\cos^2\phi - 1),$$

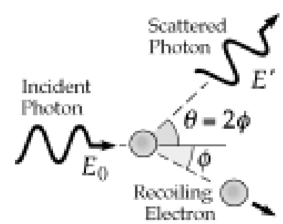
or $\lambda' = 4\lambda_0\cos^2\phi - \lambda_0$ [5]

Substituting the last result into the Compton equation gives

$$4\lambda_0\cos^2\phi - 2\lambda_0 = \frac{h}{m_e c}\left[1 - \left(2\cos^2\phi - 1\right)\right] = 2\frac{hc}{m_e c^2}\left(1 - \cos^2\phi\right).$$

With the substitution $\lambda_0 = hc/E_0$, this reduces to

$$\cos^2\phi = \frac{m_e c^2 + E_0}{2m_e c^2 + E_0} = \frac{1+x}{2+x} \quad \text{where} \quad x \equiv \frac{E_0}{m_e c^2}.$$



For $x = \frac{0.700 \text{ MeV}}{0.511 \text{ MeV}} = 1.37$, this gives $\phi = \cos^{-1}\sqrt{\frac{1+x}{2+x}} = \boxed{33.0°}$

(b) From Equation [5], $\lambda' = \lambda_0\left(4\cos^2\phi - 1\right) = \lambda_0\left[4\left(\frac{1+x}{2+x}\right) - 1\right] = \lambda_0\left(\frac{2+3x}{2+x}\right)$.

Then, Equation [1] becomes

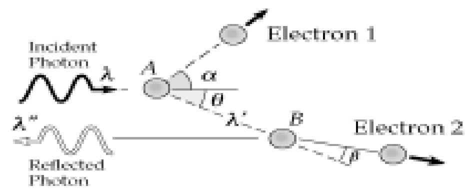
$$\frac{hc}{\lambda_0} = \frac{hc}{\lambda_0}\left(\frac{2+x}{2+3x}\right) + \gamma m_e c^2 - m_e c^2 \quad \text{or} \quad \frac{E_0}{m_e c^2} - \frac{E_0}{m_e c^2}\left(\frac{2+x}{2+3x}\right) + 1 = \gamma.$$

Thus, $\gamma = 1 + x - x\left(\frac{2+x}{2+3x}\right)$, and with $x = 1.37$ we get $\gamma = 1.614$.

Therefore, $\frac{v}{c} = \sqrt{1 - \gamma^{-2}} = \sqrt{1 - 0.384} = 0.785$ or $v = \boxed{0.785c}$.

40.32

$$\lambda' - \lambda = \frac{h}{m_e c}(1 - \cos\theta)$$



$$\lambda'' - \lambda' = \frac{h}{m_e c}\left[1 - \cos(\pi - \theta)\right]$$

$$\lambda'' - \lambda = \frac{h}{m_e c} - \frac{h}{m_e c}\cos(\pi - \theta) + \frac{h}{m_e c} - \frac{h}{m_e c}\cos\theta$$

Now $\cos(\pi - \theta) = -\cos\theta$, so $\lambda'' - \lambda = 2\frac{h}{m_e c} = \boxed{0.00486 \text{ nm}}$

40.33 (a) $K = \frac{1}{2} m_e v^2 = \frac{1}{2}\left(9.11\times10^{-31} \text{ kg}\right)\left(1.40\times10^6 \text{ m/s}\right)^2 = 8.93\times10^{-19} \text{ J} = 5.58 \text{ eV}$

$$E_0 = \frac{hc}{\lambda_0} = \frac{1240 \text{ eV}\cdot\text{nm}}{0.800 \text{ nm}} = 1550 \text{ eV}$$

$E' = E_0 - K$, and $\lambda' = \frac{hc}{E'} = \frac{1240 \text{ eV}\cdot\text{nm}}{1550 \text{ eV} - 5.58 \text{ eV}} = 0.803 \text{ nm}$

$$\Delta\lambda = \lambda' - \lambda_0 = 0.00288 \text{ nm} = \boxed{2.88 \text{ pm}}$$

(b) $\Delta\lambda = \lambda_C\left(1 - \cos\theta\right) \Rightarrow \cos\theta = 1 - \frac{\Delta\lambda}{\lambda_C} = 1 - \frac{0.00288 \text{ nm}}{0.00243 \text{ nm}} = -0.189$, so $\boxed{\theta = 101°}$

***40.34** Maximum energy loss appears as maximum increase in wavelength, which occurs for scattering angle 180°. Then $\Delta\lambda = \left(1 - \cos 180°\right)\left(h/mc\right) = 2h/mc$ where m is the mass of the target particle. The fractional energy loss is

$$\frac{E_0 - E'}{E_0} = \frac{hc/\lambda_0 - hc/\lambda'}{hc/\lambda_0} = \frac{\lambda' - \lambda_0}{\lambda'} = \frac{\Delta\lambda}{\lambda_0 + \Delta\lambda} = \frac{2h/mc}{\lambda_0 + 2h/mc}$$

Further, $\lambda_0 = hc/E_0$, so $\frac{E_0 - E'}{E_0} = \frac{2h/mc}{hc/E_0 + 2h/mc} = \frac{2E_0}{mc^2 + 2E_0}$.

(a) For scattering from a free electron, $mc^2 = 0.511$ MeV, so

$$\frac{E_0 - E'}{E_0} = \frac{2(0.511 \text{ MeV})}{0.511 \text{ MeV} + 2(0.511 \text{ MeV})} = \boxed{0.667}$$

(b) For scattering from a free proton, $mc^2 = 938$ MeV, and

$$\frac{E_0 - E'}{E_0} = \frac{2(0.511 \text{ MeV})}{938 \text{ MeV} + 2(0.511 \text{ MeV})} = \boxed{0.00109}$$

40.35 Start with Balmer's equation, $\frac{1}{\lambda} = R_H\left(\frac{1}{2^2} - \frac{1}{n^2}\right)$, or $\lambda = \frac{(4n^2 / R_H)}{(n^2 - 4)}$.

Substituting $R_H = 1.0973732 \times 10^7 \text{ m}^{-1}$, we obtain

$$\lambda = \frac{(3.645 \times 10^{-7}\text{ m})n^2}{n^2 - 4} = \frac{364.5n^2}{n^2 - 4}\text{ nm}, \text{ where } n = 3, 4, 5, \ldots$$

40.36 (a) Using $\frac{1}{\lambda} = R_H\left(\frac{1}{n_f^2} - \frac{1}{n_i^2}\right)$, for $n_f = 2$, and $n_i \geq 3$, we get:

$$\lambda = \frac{4n^2}{R_H\left(n^2 - 4\right)} = \frac{4n^2}{\left(2.00 \times 10^7\text{ m}^{-1}\right)\left(n^2 - 4\right)} = \frac{(200.0)n^2}{n^2 - 4}\text{ nm}$$

This says that $200\text{ nm} \leq \lambda \leq 360\text{ nm}$, which is $\boxed{\text{ultraviolet}}$.

(b) Using $n \geq 3$, $\lambda = \frac{4n^2}{R_H\left(n^2 - 4\right)} = \frac{4n^2}{\left(0.500 \times 10^7\text{ m}^{-1}\right)\left(n^2 - 4\right)} = \frac{(800.0)n^2}{n^2 - 4}\text{ nm}$

This says that $800\text{ nm} \leq \lambda \leq 1440\text{ nm}$, which is in the $\boxed{\text{infrared}}$.

40.37 (a) Lyman series: $\frac{1}{\lambda} = R\left(1 - \frac{1}{n^2}\right)$ $n = 2, 3, 4, \ldots$

$\frac{1}{\lambda} = \frac{1}{94.96 \times 10^{-9}} = (1.097 \times 10^7)\left(1 - \frac{1}{n^2}\right)$ $\boxed{n = 5}$

(b) Paschen series: $\frac{1}{\lambda} = R\left(\frac{1}{3^2} - \frac{1}{n^2}\right)$ $n = 4, 5, 6, \ldots$

The shortest wavelength for this series corresponds to $n = \infty$ for ionization

$\frac{1}{\lambda} = 1.097 \times 10^7\left(\frac{1}{9} - \frac{1}{n^2}\right)$ For $n = \infty$, this gives $\lambda = 820$ nm

This is larger than 94.96 nm, so this wave length $\boxed{\text{cannot be associated with the Paschen series}}$

Brackett series: $\frac{1}{\lambda} = R\left(\frac{1}{4^2} - \frac{1}{n^2}\right)$ $n = 5, 6, 7, \ldots$

$\frac{1}{\lambda} = 1.097 \times 10^7\left(\frac{1}{16} - \frac{1}{n^2}\right)$ $n = \infty$ for ionization $\lambda_{min} = 1458$ nm

Once again this wavelength $\boxed{\text{cannot be associated with the Brackett series}}$

40.38 (a) $\lambda_{\min} = \dfrac{hc}{E_{\max}}$

Lyman ($n_f = 1$): $\lambda_{\min} = \dfrac{hc}{|E_1|} = \dfrac{1240\text{ eV}\cdot\text{nm}}{13.6\text{ eV}} = \boxed{91.2\text{ nm}}$ (Ultraviolet)

Balmer ($n_f = 2$): $\lambda_{\min} = \dfrac{hc}{|E_2|} = \dfrac{1240\text{ eV}\cdot\text{nm}}{\left(\frac{1}{4}\right)13.6\text{ eV}} = \boxed{365\text{ nm}}$ (UV)

Paschen ($n_f = 3$): $\lambda_{\min} = \ldots = 3^2(91.2\text{ nm}) = \boxed{821\text{ nm}}$ (Infrared)

Bracket ($n_f = 4$): $\lambda_{\min} = \ldots = 4^2(91.2\text{ nm}) = \boxed{1460\text{ nm}}$ (IR)

(b) $E_{\max} = \dfrac{hc}{\lambda_{\min}}$

Lyman: $E_{\max} = \boxed{13.6\text{ eV}}$ $\left(=|E_1|\right)$

Balmer: $E_{\max} = \boxed{3.40\text{ eV}}$ $\left(=|E_2|\right)$

Paschen: $E_{\max} = \boxed{1.51\text{ eV}}$ $\left(=|E_3|\right)$

Brackett: $E_{\max} = \boxed{0.850\text{ eV}}$ $\left(=|E_4|\right)$

40.39 Liquid O_2 $\lambda_{abs} = 1269\text{ nm}$

$E = \dfrac{hc}{\lambda} = \dfrac{1.2398\times 10^{-6}}{1.269\times 10^{-6}} = 0.977\text{ eV}$ for each molecule.

For two molecules, $\lambda = \dfrac{hc}{2E} = \boxed{634\text{ nm, red}}$

By absorbing the red photons, the liquid O_2 appears to be blue.

***40.40** (a) $v_1 = \sqrt{\dfrac{k_e e^2}{m_e r_1}}$ where $r_1 = (1)^2 a_0 = 0.00529\text{ nm} = 5.29\times 10^{-11}\text{ m}$

$$v_1 = \sqrt{\frac{\left(8.99\times 10^9\text{ N}\cdot\text{m}^2/\text{C}^2\right)\left(1.60\times 10^{-19}\text{ C}\right)^2}{\left(9.11\times 10^{-31}\text{ kg}\right)\left(5.29\times 10^{-11}\text{ m}\right)}} = \boxed{2.19\times 10^6\text{ m/s}}$$

(b) $K_1 = \frac{1}{2}m_e v_1^2 = \frac{1}{2}\left(9.11\times 10^{-31}\text{ kg}\right)\left(2.19\times 10^6\text{ m/s}\right)^2 = 2.18\times 10^{-18}\text{ J} = \boxed{13.6\text{ eV}}$

(c) $U_1 = -\dfrac{k_e e^2}{r_1} = -\dfrac{\left(8.99\times 10^9\text{ N}\cdot\text{m}^2/\text{C}^2\right)\left(1.60\times 10^{-19}\text{ C}\right)^2}{5.29\times 10^{-11}\text{ m}} = -4.35\times 10^{-18}\text{ J} = \boxed{-27.2\text{ eV}}$

40.41 (a) $r_2^2 = (0.0529 \text{ nm})(2)^2 = \boxed{0.212 \text{ nm}}$

(b) $m_e v_2 = \sqrt{\dfrac{m_e k_e e^2}{r_2}} = \sqrt{\dfrac{(9.11\times 10^{-31} \text{ kg})(8.99\times 10^9 \text{ N}\cdot\text{m}^2/\text{C}^2)(1.60\times 10^{-19} \text{ C})^2}{0.212\times 10^{-9} \text{ m}}} = \boxed{9.95\times 10^{-25} \text{ kg}\cdot\text{m/s}}$

(c) $L_2 = m_e v_2 r_2 = (9.95\times 10^{-25} \text{ kg}\cdot\text{m/s})(0.212\times 10^{-9} \text{ m}) = \boxed{2.11\times 10^{-34} \text{ kg}\cdot\text{m}^2/\text{s}}$

(d) $K_2 = \frac{1}{2} m_e v_2^2 = \dfrac{(m_e v_2)^2}{2m_e} = \dfrac{(9.95\times 10^{-25} \text{ kg}\cdot\text{m/s})^2}{2(9.11\times 10^{-31} \text{ kg})} = 5.43\times 10^{-19} \text{ J} = \boxed{3.40 \text{ eV}}$

(e) $U_2 = -\dfrac{k_e e^2}{r_2} = -\dfrac{(8.99\times 10^9 \text{ N}\cdot\text{m}^2/\text{C}^2)(1.60\times 10^{-19} \text{ C})^2}{0.212\times 10^{-9} \text{ m}} = -1.09\times 10^{-18} \text{ J} = \boxed{-6.80 \text{ eV}}$

(f) $E_2 = K_2 + U_2 = 3.40 \text{ eV} - 6.80 \text{ eV} = \boxed{-3.40 \text{ eV}}$

40.42 $\Delta E = (13.6 \text{ eV})\left(\dfrac{1}{n_i^2} - \dfrac{1}{n_f^2}\right)$

Where for $\Delta E > 0$ we have absorption and for $\Delta E < 0$ we have emission.

(A) for $n_i = 2$ and $n_f = 5$ $\Delta E = 2.86$ eV (absorption)

(B) for $n_i = 5$ and $n_f = 3$ $\Delta E = -0.967$ eV (emission)

(C) for $n_i = 7$ and $n_f = 4$ $\Delta E = -0.572$ eV (emission)

(D) for $n_i = 4$ and $n_f = 7$ $\Delta E = 0.572$ eV (absorption)

(a) $E = \dfrac{hc}{\lambda}$ so the shortest wavelength is emitted in transition $\boxed{\text{B}}$.

(b) The atom gains most energy in transition $\boxed{\text{A}}$.

(c) The atom loses energy in transitions $\boxed{\text{B and C}}$.

40.43 (b) $\dfrac{1}{\lambda} = R\left(\dfrac{1}{n_f^2} - \dfrac{1}{n_i^2}\right) = (1.097\times 10^7 \text{ m}^{-1})\left(\dfrac{1}{2^2} - \dfrac{1}{6^2}\right)$ so $\boxed{\lambda = 410 \text{ nm}}$

(a) $E = \dfrac{hc}{\lambda} = \dfrac{(6.626\times 10^{-34} \text{ J}\cdot\text{s})(3.00\times 10^8 \text{ m / s})}{410\times 10^{-9} \text{ m}} = 4.85\times 10^{-19} \text{ J} = \boxed{3.03 \text{ eV}}$

(c) $f = \dfrac{c}{\lambda} = \dfrac{3.00\times 10^8}{410\times 10^{-9}} = \boxed{7.32\times 10^{14} \text{ Hz}}$

***40.44** We use $E_n = \dfrac{-13.6 \text{ eV}}{n^2}$

To ionize the atom when the electron is in the n^{th} level, it is necessary to add an amount of energy given by

$$E = -E_n = \frac{13.6 \text{ eV}}{n^2}$$

(a) Thus, in the ground state where $n = 1$, we have $\boxed{E = 13.6 \text{ eV}}$

(b) In the $n = 3$ level, $E = \dfrac{13.6 \text{ eV}}{9} = \boxed{1.51 \text{ eV}}$

***40.45** Starting with $\frac{1}{2} m_e v^2 = \dfrac{k_e e^2}{2r}$, we have $v^2 = \dfrac{k_e e^2}{m_e r}$

and using $r_n = \dfrac{n^2 \hbar^2}{m_e k_e e^2}$

gives $v_n^{\,2} = \dfrac{k_e e^2}{m_e \dfrac{n^2 \hbar^2}{m_e k_e e^2}}$ or $v_n = \dfrac{k_e e^2}{n\hbar}$

***40.46** (a) The velocity of the moon in its orbit is $v = \dfrac{2\pi r}{T} = \dfrac{2\pi(3.84 \times 10^8 \text{ m})}{2.36 \times 10^6 \text{ s}} = 1.02 \times 10^3 \text{ m/s}$

So, $L = mvr = (7.36 \times 10^{22} \text{ kg})(1.02 \times 10^3 \text{ m/s})(3.84 \times 10^8 \text{ m}) = \boxed{2.89 \times 10^{34} \text{ kg} \cdot \text{m}^2/\text{s}}$

(b) We have $L = n\hbar$

or $n = \dfrac{L}{\hbar} = \dfrac{2.89 \times 10^{34} \text{ kg} \cdot \text{m}^2/\text{s}}{1.055 \times 10^{-34} \text{ J} \cdot \text{s}} = \boxed{2.74 \times 10^{68}}$

(c) We have $n\hbar = L = mvr = m(GM_e/r)^{1/2} r$,

so $r = \dfrac{\hbar^2}{m^2 GM_e} n^2 = Rn^2$ and $\dfrac{\Delta r}{r} = \dfrac{(n+1)^2 R - n^2 R}{n^2 R} = \dfrac{2n+1}{n^2}$

which is approximately equal to $\dfrac{2}{n} = \boxed{7.30 \times 10^{-69}}$

40.47 The batch of excited atoms must make these six transitions to get back to state one: $2 \to 1$, and also $3 \to 2$ and $3 \to 1$, and also $4 \to 3$ and $4 \to 2$ and $4 \to 1$. Thus, the incoming light must have just enough energy to produce the $1 \to 4$ transition. It must be the third line of the Lyman series in the absorption spectrum of hydrogen. The absorbing atom changes from energy

$$E_i = -\frac{13.6\text{ eV}}{1^2} = -13.6\text{ eV} \quad \text{to} \quad E_f = -\frac{13.6\text{ eV}}{4^2} = -0.850\text{ eV},$$

so the incoming photons have wavelength

$$\lambda = \frac{hc}{E_f - E_i} = \frac{\left(6.626\times10^{-34}\text{ J}\cdot\text{s}\right)\left(3.00\times10^{8}\text{ m/s}\right)}{-0.850\text{ eV} - \left(-13.6\text{ eV}\right)}\left(\frac{1.00\text{ eV}}{1.60\times10^{-19}\text{ J}}\right) = 9.75\times10^{-8}\text{ m} = \boxed{97.5\text{ nm}}$$

40.48 Each atom gives up its kinetic energy in emitting a photon, so

$$\frac{1}{2}mv^2 = \frac{hc}{\lambda} = \frac{(6.626\times10^{-34}\text{ J}\cdot\text{s})(3.00\times10^{8}\text{ m/s})}{(1.216\times10^{-7}\text{ m})} = 1.63\times10^{-18}\text{ J}$$

$$v = \boxed{4.42\times10^{4}\text{ m/s}}$$

40.49 (a) The energy levels of a hydrogen-like ion whose charge number is Z are given by

$$E_n = (-13.6\text{ eV})\frac{Z^2}{n^2}$$

Thus for He lium $(Z = 2)$, the energy levels are

$$\boxed{E_n = -\frac{54.4\text{ eV}}{n^2} \quad n = 1, 2, 3, \ldots}$$

(b) For He^{+}, $Z = 2$, so we see that the ionization energy (the energy required to take the electron from the $n = 1$ to the $n = \infty$ state is

$$E = E_\infty - E_1 = 0 - \frac{(-13.6\text{ eV})(2)^2}{(1)^2} = \boxed{54.4\text{ eV}}$$

40.50 $r = \dfrac{n^2\hbar^2}{Zm_e k_e e^2} = \dfrac{n^2}{Z}\left(\dfrac{\hbar^2}{m_e k_e e^2}\right)$; $n = 1$

$$r = \frac{1}{Z}\left[\frac{(1.055\times10^{-34}\ \text{J}\cdot\text{s})^2}{(9.11\times10^{-31}\ \text{kg})(8.99\times10^{9}\ \text{N}\cdot\text{m}^2/\text{C}^2)(1.602\times10^{-19}\ \text{C})^2}\right] = \frac{5.29\times10^{-11}\ \text{m}}{Z}$$

(a) For He^{+}, $Z = 2$ $\quad r = \dfrac{5.29\times10^{-11}\ \text{m}}{2} = 2.65\times10^{-11}\ \text{m} = \boxed{0.0265\ \text{nm}}$

(b) For Li^{2+}, $Z = 3$ $\quad r = \dfrac{5.29\times10^{-11}\ \text{m}}{3} = 1.77\times10^{-11}\ \text{m} = \boxed{0.0177\ \text{nm}}$

(c) For Be^{3+}, $Z = 4$ $\quad r = \dfrac{5.29\times10^{-11}\ \text{m}}{4} = 1.32\times10^{-11}\ \text{m} = \boxed{0.0132\ \text{nm}}$

40.51 Since $F = qvB = \dfrac{mv^2}{r}$ we have $qrB = mv$,

or $qr^2B = mvr = n\hbar$ so $\boxed{r_n = \sqrt{\dfrac{n\hbar}{qB}}}$

40.52 (a) The time for one complete orbit is: $T = \dfrac{2\pi r}{v}$

From Bohr's quantization postulate, $L = m_e vr = n\hbar$, we see that $v = \dfrac{n\hbar}{m_e r}$

Thus, the orbital period becomes:

$$T = \frac{2\pi m_e r^2}{n\hbar} = \frac{2\pi m_e (a_0 n^2)^2}{n\hbar} = \frac{2\pi m_e a_0^2}{\hbar} n^3 \quad \text{or} \quad T = t_0 n^3 \quad \text{where}$$

$$t_0 = \frac{2\pi m_e a_0^2}{\hbar} = \frac{2\pi(9.11\times10^{-31}\ \text{kg})(0.0529\times10^{-9}\ \text{m})^2}{(1.055\times10^{-34}\ \text{J}\cdot\text{s})} = \boxed{1.52\times10^{-16}\ \text{s}}$$

(b) With $n = 2$, we have $T = 8t_0 = 8(1.52\times10^{-16}\ \text{s}) = 1.21\times10^{-15}\ \text{s}$

Thus, if the electrons stay in the $n = 2$ state for 10 μs, it will make

$$\frac{10.0\times10^{-6}\ \text{s}}{1.21\times10^{-15}\ \text{s/rev}} = \boxed{8.23\times10^{9}\ \text{revolutions}}\ \text{of the nucleus}$$

(c) $\boxed{\text{Yes, for } 8.23\times10^{9} \text{ "electron years"}}$

***40.53** $\lambda = \dfrac{h}{p} = \dfrac{h}{mv} = \dfrac{6.626 \times 10^{-34}\ \text{J} \cdot \text{s}}{(1.67 \times 10^{-27}\ \text{kg})(1.00 \times 10^{6}\ \text{m/s})} = \boxed{3.97 \times 10^{-13}\ \text{m}}$

40.54 (a) $\dfrac{p^2}{2m} = (50.0)(1.60 \times 10^{-19}\ \text{J})$

$p = 3.81 \times 10^{-24}\ \text{kg} \cdot \text{m/s}$

$\lambda = \dfrac{h}{p} = \boxed{0.174\ \text{nm}}$

(b) $\dfrac{p^2}{2m} = (50.0 \times 10^{3})(1.60 \times 10^{-19}\ \text{J})$

$p = 1.20 \times 10^{-22}\ \text{kg} \cdot \text{m/s}$

$\lambda = \dfrac{h}{p} = \boxed{5.49 \times 10^{-12}\ \text{m}}$

The relativistic answer is slightly more precise:

$$\lambda = \frac{h}{p} = \frac{hc}{\left[(mc^2 + K)^2 - m^2c^4\right]^{1/2}} = 5.37 \times 10^{-12}\ \text{m}$$

***40.55** (a) Electron: $\lambda = \dfrac{h}{p}$ and $K = \frac{1}{2} m_e v^2 = \dfrac{m_e^2 v^2}{2m_e} = \dfrac{p^2}{2m_e}$

so $p = \sqrt{2m_e K}$

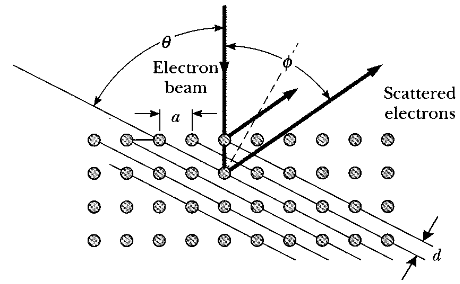
and $\lambda = \dfrac{h}{\sqrt{2m_e K}} = \dfrac{6.626 \times 10^{-34}\ \text{J} \cdot \text{s}}{\sqrt{2\left(9.11 \times 10^{-31}\ \text{kg}\right)(3.00)\left(1.60 \times 10^{-19}\ \text{J}\right)}}$

$\lambda = 7.09 \times 10^{-10}\ \text{m} = \boxed{0.709\ \text{nm}}$

(b) Photon: $\lambda = c/f$ and $E = hf$ so $f = E/h$ and

$$\lambda = \frac{hc}{E} = \frac{(6.626 \times 10^{-34}\ \text{J} \cdot \text{s})(3.00 \times 10^{8}\ \text{m/s})}{(3.00)(1.60 \times 10^{-19}\ \text{J})} = 4.14 \times 10^{-7}\ \text{m} = \boxed{414\ \text{nm}}$$

40.56 From the Bragg condition (Eq. 38.13),



$$m\lambda = 2d \sin\theta = 2d\cos(\phi/2)$$

But, $d = a\sin(\phi/2)$ where a is the lattice spacing.
Thus, with $m = 1$,

$$\lambda = 2a\sin(\phi/2)\cos(\phi/2) = a\sin\phi$$

$$\lambda = \frac{h}{p} = \frac{h}{\sqrt{2m_eK}} = \frac{6.626\times10^{-34}\text{ J}\cdot\text{s}}{\sqrt{2(9.11\times10^{-31}\text{ kg})(54.0\times1.60\times10^{-19}\text{ J})}} = 1.67\times10^{-10}\text{ m}$$

Therefore, the lattice spacing is

$$a = \frac{\lambda}{\sin\phi} = \frac{1.67\times10^{-10}\text{ m}}{\sin 50.0°} = 2.18\times10^{-10}\text{ m} = \boxed{0.218\text{ nm}}$$

***40.57** (a) $\lambda \sim 10^{-14}$ m or less.

$$p = \frac{h}{\lambda} \sim \frac{6.6\times10^{-34}\text{ J}\cdot\text{s}}{10^{-14}\text{ m}} = 10^{-19}\text{ kg}\cdot\text{m/s or more.}$$

The energy of the electron is

$$E = \sqrt{p^2c^2 + m_e^2c^4} \sim \left[(10^{-19})^2(3\times10^8)^2 + (9\times10^{-31})^2(3\times10^8)^4\right]^{1/2} \sim 10^{-11}\text{ J} \sim 10^8\text{ eV or more,}$$

so that $K = E - m_ec^2 \sim 10^8\text{ eV} - (0.5\times10^6\text{ eV})$ $\boxed{\sim 10^8\text{ eV}}$ or more.

(b) The electric potential energy of the electron would be

$$U_e = \frac{k_eq_1q_2}{r} \sim \frac{(9\times10^9\text{ N}\cdot\text{m}^2/\text{C}^2)(10^{-19}\text{ C})(-e)}{10^{-14}\text{ m}} \sim -10^5\text{ eV}$$

With its kinetic energy much larger than its negative potential energy, $\boxed{\text{the electron would immediately escape the nucleus}}$.

Goal Solution

The nucleus of an atom is on the order of 10^{-14} m in diameter. For an electron to be confined to a nucleus, its de Broglie wavelength would have to be of this order of magnitude or smaller. (a) What would be the kinetic energy of an electron confined to this region? (b) On the basis of this result, would you expect to find an electron in a nucleus? Explain.

G: The de Broglie wavelength of a normal ground-state orbiting electron is on the order 10^{-10} m (the diameter of a hydrogen atom), so with a shorter wavelength, the electron would have more kinetic energy if confined inside the nucleus. If the kinetic energy is much greater than the potential energy from its attraction with the positive nucleus, then the electron will escape from its electrostatic potential well.

O: If we try to calculate the velocity of the electron from the de Broglie wavelength, we find that

$$v = \frac{h}{m_e\lambda} = \frac{6.63\times10^{-34}\ \text{J}\cdot\text{s}}{\left(9.11\times10^{-31}\ \text{kg}\right)\left(10^{-14}\ \text{m}\right)} = 7.27\times10^{10}\ \text{m / s}$$

which is not possible since it exceeds the speed of light. Therefore, we must use the relativistic energy expression to find the kinetic energy of this fast-moving electron.

A: (a) The relativistic kinetic energy of a particle is $K = E - mc^2$, where $E^2 = (pc)^2 + \left(mc^2\right)^2$, and the momentum is $p = h/\lambda$:

$$p = \frac{6.63\times10^{-34}\ \text{J}\cdot\text{s}}{10^{-14}\ \text{m}} = 6.63\times10^{-20}\ \text{N}\cdot\text{s}$$

$$E = \sqrt{\left(1.99\times10^{-11}\ \text{J}\right)^2 + \left(8.19\times10^{-14}\ \text{J}\right)^2} = 1.99\times10^{-11}\ \text{J}$$

$$K = E - mc^2 = \frac{1.99\times10^{-11}\ \text{J} - 8.19\times10^{-14}\ \text{J}}{1.60\times10^{-19}\ \text{J / eV}} = 124\ \text{MeV} \sim 100\ \text{MeV}$$

(b) The electrostatic potential energy of the electron 10^{-14} m away from a positive proton is :

$$U = -k_e e^2/r = -\frac{\left(8.99\times10^{9}\ \dfrac{\text{N}\cdot\text{m}^2}{\text{C}^2}\right)\left(1.60\times10^{-19}\ \text{C}\right)^2}{10^{-14}\ \text{m}} = -2.30\times10^{-14}\ \text{J} \sim -0.1\ \text{MeV}$$

L: Since the kinetic energy is nearly 1000 times greater than the potential energy, the electron would immediately escape the proton's attraction and would not be confined to the nucleus.

It is also interesting to notice in the above calculations that the rest energy of the electron is negligible compared to the momentum contribution to the total energy.

40.58 (a) From $E = \gamma m_e c^2$ $\qquad \gamma = \dfrac{20.0 \times 10^3 \text{ MeV}}{0.511 \text{ MeV}} = \boxed{3.91 \times 10^4}$

(b) $p \approx \dfrac{E}{c}$ $\qquad$ (for $m_e c^2 << pc$)

$$p = \frac{(2.00 \times 10^4 \text{ MeV})(1.60 \times 10^{-13} \text{ J/MeV})}{3.00 \times 10^8 \text{ m/s}} = \boxed{1.07 \times 10^{-17} \text{ kg} \cdot \text{m/s}}$$

(c) $$\lambda = \frac{h}{p} = \frac{6.626 \times 10^{-34} \text{ J} \cdot \text{s}}{1.07 \times 10^{-17} \text{ kg} \cdot \text{m/s}} = \boxed{6.22 \times 10^{-17} \text{ m}}$$

Since the size of a nucleus is on the order of 10^{-14} m, the 20-GeV electrons would be small enough to go through the nucleus.

40.59 (a) $E^2 = p^2c^2 + m^2c^4$

with $E = hf$, $\qquad p = \dfrac{h}{\lambda}$, $\qquad$ and $\qquad mc = \dfrac{h}{\lambda_C}$

so $\qquad h^2f^2 = \dfrac{h^2c^2}{\lambda^2} + \dfrac{h^2c^2}{\lambda_C^2}$ $\qquad$ and $\qquad \left(\dfrac{f}{c}\right)^2 = \dfrac{1}{\lambda^2} + \dfrac{1}{\lambda_C^2}$ $\qquad$ (Eq. 1)

(b) For a photon $\qquad f/c = 1/\lambda$.

The third term $1/\lambda_C$ in Equation 1 for electrons and other massive particles shows that $\boxed{\text{they will always have a different frequency from photons of the same wavelength}}$

40.60 (a) The wavelength of the student is $\lambda = h/p = h/mv$. If w is the width of the diffraction aperture, then we need $w \le 10.0\lambda = 10.0(h/mv)$, so that

$$v \le 10.0\frac{h}{mw} = 10.0\left(\frac{6.626 \times 10^{-34} \text{ J} \cdot \text{s}}{(80.0 \text{ kg})(0.750 \text{ m})}\right) = \boxed{1.10 \times 10^{-34} \text{ m/s}}$$

(b) Using $t = \dfrac{d}{v}$ we get: $\qquad t \ge \dfrac{0.150 \text{ m}}{1.10 \times 10^{-34} \text{ m/s}} = \boxed{1.36 \times 10^{33} \text{ s}}$

(c) $\boxed{\text{No}}$. The minimum time to pass through the door is over 10^{15} times the age of the Universe.

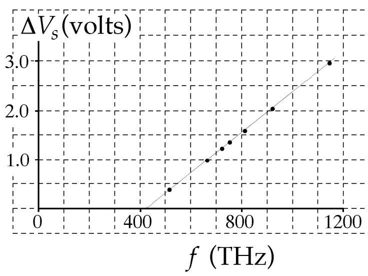
40.61 The de Broglie wavelength is: $\lambda = \dfrac{h}{\gamma m_e v}$

The Compton wavelength is: $\lambda_C = \dfrac{h}{m_e c}$

Therefore, we see that to have $\lambda = \lambda_C$, it is necessary that $\gamma v = c$.

This gives: $\dfrac{v}{\sqrt{1 - v^2/c^2}} = c$, or $\left(\dfrac{v}{c}\right)^2 = 1 - \left(\dfrac{v}{c}\right)^2$, yielding $v = \boxed{\dfrac{c}{\sqrt{2}}}$.

40.62 $\Delta V_S = \left(\dfrac{h}{e}\right) f - \dfrac{\phi}{e}$



From two points on the graph $0 = \left(\dfrac{h}{e}\right)\left(4.1 \times 10^{14}\ \text{Hz}\right) - \dfrac{\phi}{e}$

and $3.3\ \text{V} = \left(\dfrac{h}{e}\right)\left(12 \times 10^{14}\ \text{Hz}\right) - \dfrac{\phi}{e}$

Combining these two expressions we find:

(a) $\phi = \boxed{1.7\ \text{eV}}$

(b) $\dfrac{h}{e} = \boxed{4.2 \times 10^{-15}\ \text{V} \cdot \text{s}}$

(c) At the cutoff wavelength $\dfrac{hc}{\lambda_c} = \phi = \left(\dfrac{h}{e}\right)\dfrac{ec}{\lambda_c}$

$$\lambda_c = (4.2 \times 10^{-15}\ \text{V} \cdot \text{s})(1.6 \times 10^{-19}\ \text{C})\,\frac{(3.0 \times 10^8\ \text{m/s})}{(1.7\ \text{eV})(1.6 \times 10^{-19}\ \text{J/eV})} = \boxed{730\ \text{nm}}$$

40.63 $K_{\max} = \dfrac{q^2 B^2 R^2}{2m_e} = \dfrac{(1.60 \times 10^{-19}\ \text{C})^2 (2.00 \times 10^{-5}\ \text{T})^2 (0.200\ \text{m})^2}{2(9.11 \times 10^{-31}\ \text{kg})} = 2.25 \times 10^{-19}\ \text{J} = 1.40\ \text{eV} = hf - \phi$

$$\phi = hf - K_{\max} = \frac{hc}{\lambda} - K_{\max} = \frac{(4.14 \times 10^{-15}\ \text{eV} \cdot \text{s})(3.00 \times 10^8\ \text{m/s})}{450 \times 10^{-9}\ \text{m}} - 1.40\ \text{eV} = \boxed{1.36\ \text{eV}}$$

40.64 From the path the electrons follow in the magnetic field, the maximum kinetic energy is seen to be:

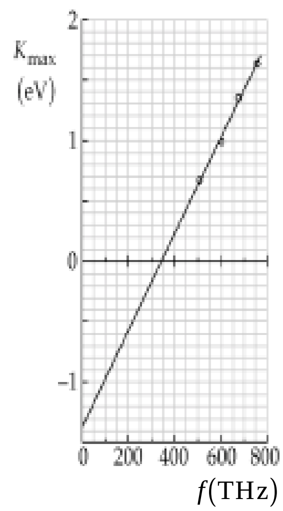
$$K_{\max} = \frac{e^2B^2R^2}{2m_e}$$

From the photoelectric equation, $K_{\max} = hf - \phi = \frac{hc}{\lambda} - \phi$

Thus, the work function is $\phi = \frac{hc}{\lambda} - K_{\max} = \boxed{\frac{hc}{\lambda} - \frac{e^2B^2R^2}{2m_e}}$

40.65 We want an Einstein plot of $K_{\max}$ versus f

λ, nm	f, 10^{14} Hz	$K_{\max}$, eV
588	5.10	0.67
505	5.94	0.98
445	6.74	1.35
399	7.52	1.63



(a) slope $= \frac{0.402 \text{ eV}}{10^{14} \text{ Hz}}$ ± 8%

(b) $e(\Delta V_S) = hf - \phi$

$h = (0.402)\frac{1.60\times10^{-19}\text{ J}\cdot\text{s}}{10^{14}} = \boxed{6.4\times10^{-34}\text{ J}\cdot\text{s} \pm 8\%}$

(c) $K_{\max} = 0$ at $f \approx 344\times10^{12}$ Hz

$\phi = hf = 2.32\times10^{-19}\text{ J} = \boxed{1.4 \text{ eV}}$

40.66 $\Delta\lambda = \frac{h}{m_pc}(1-\cos\theta) = \frac{(6.626\times10^{-34}\text{ J}\cdot\text{s})}{(1.67\times10^{-27}\text{ kg})(3.00\times10^{8}\text{ m / s})}(0.234) = 3.09\times10^{-16}\text{ m}$

$$\lambda_0 = \frac{hc}{E_0} = \frac{(6.626\times10^{-34}\text{ J}\cdot\text{s})(3.00\times10^{8}\text{ m / s})}{(200\text{ MeV})(1.60\times10^{-13}\text{ J / MeV})} = 6.20\times10^{-15}\text{ m}$$

$$\lambda' = \lambda_0 + \Delta\lambda = 6.51\times10^{-15}\text{ m}$$

(a) $E_\gamma = \frac{hc}{\lambda'} = \boxed{191\text{ MeV}}$

(b) $K_p = \boxed{9.20\text{ MeV}}$

40.67 M is the mass of the positron which equals m_e, the mass of the electron.

So $$\mu \equiv \text{reduced mass} = \frac{m_e M}{m_e + M} = \frac{m_e}{2}$$

$$r_{\text{pos}} = \frac{n^2\hbar^2}{Z\mu k_e e^2} = \frac{n^2\hbar^2}{Z(m_e/2)k_e e^2} = \frac{2n^2\hbar^2}{Z m_e k_e e^2} \quad \text{or} \quad r_{\text{pos}} = 2r_{\text{Hyd}} = \boxed{\left(1.06\times10^{-10}\text{ m}\right)n^2}$$

This is the separation of the two particles.

$$E_{\text{pos}} = -\frac{\mu k_e{}^2 e^4}{2\hbar^2}\frac{1}{n^2} = -\frac{m_e k_e{}^2 e^4}{4\hbar^2}\left(\frac{1}{n^2}\right); \quad n = 1, 2, 3, \ldots \quad \text{or} \quad E_{\text{pos}} = \frac{E_{\text{Hyd}}}{2} = \boxed{\frac{-6.80\text{ eV}}{n^2}}$$

Goal Solution

Positronium is a hydrogen-like atom consisting of a positron (a positively charged electron) and an electron revolving around each other. Using the Bohr model, find the allowed radii (relative to the center of mass of the two particles) and the allowed energies of the system.

G: Since we are told that positronium is like hydrogen, we might expect the allowed radii and energy levels to be about the same as for hydrogen: $r = a_0 n^2 = \left(5.29\times10^{-11}\text{ m}\right)n^2$ and $E_n = (-13.6\text{ eV})/n^2$.

O: Similar to the textbook calculations for hydrogen, we can use the quantization of angular momentum of positronium to find the allowed radii and energy levels.

A: Let r represent the distance between the electron and the positron. The two move in a circle of radius $r/2$ around their center of mass with opposite velocities. The total angular momentum is quantized according to

$$L_n = \frac{mvr}{2} + \frac{mvr}{2} = n\hbar, \quad \text{where} \quad n = 1, 2, 3, \ldots$$

For each particle, $\Sigma F = ma$ expands to $$\frac{k_e e^2}{r^2} = \frac{mv^2}{r/2}$$

We can eliminate $v = \dfrac{n\hbar}{mr}$ to find $$\frac{k_e e^2}{r} = \frac{2mn^2\hbar}{m^2 r^2}$$

So the separation distances are $$r = \frac{2n^2\hbar^2}{mk_e e^2} = 2a_0 n^2 = \left(1.06\times10^{-10}\text{ m}\right)n^2$$

The orbital radii are $r/2 = a_0 n^2$, the same as for the electron in hydrogen.

The energy can be calculated from $$E = K + U = \tfrac{1}{2}mv^2 + \tfrac{1}{2}mv^2 - \frac{k_e e^2}{r}$$

Since $mv^2 = \dfrac{k_e e^2}{2r}$, $$E = \frac{k_e e^2}{2r} - \frac{k_e e^2}{r} = -\frac{k_e e^2}{2r} = \frac{-k_e e^2}{4a_0 n^2} = -\frac{6.80\text{ eV}}{n^2}$$

L: It appears that the allowed radii for positronium are twice as large as for hydrogen, while the energy levels are half as big. One way to explain this is that in a hydrogen atom, the proton is much more massive than the electron, so the proton remains nearly stationary with essentially no kinetic energy. However, in positronium, the positron and electron have the same mass and therefore both have kinetic energy that separates them from each other and reduces their total energy compared with hydrogen.

40.68 Isolate the terms involving ϕ in Equations 40.12 and 40.13. Square and add to eliminate ϕ.

$$h^2\left[\frac{1}{\lambda_0^2}+\frac{1}{\lambda'^2}-\frac{2\cos\theta}{\lambda_0\lambda'}\right]=\gamma^2 m_e^2 v^2$$

Solve for $\dfrac{v^2}{c^2}=\dfrac{b}{\left(b+c^2\right)}$:

$$b=\frac{h^2}{m_e^2}\left[\frac{1}{\lambda_0^2}+\frac{1}{\lambda'^2}-\frac{2\cos\theta}{\lambda_0\lambda'}\right]$$

Substitute into Eq. 40.11:

$$1+\left(\frac{h}{m_e c}\right)\left[\frac{1}{\lambda_0}-\frac{1}{\lambda'}\right]=\gamma=\sqrt{1-\frac{b}{b+c^2}}$$

Square each side:

$$c^2+\frac{2hc}{m_e}\left[\frac{1}{\lambda_0}-\frac{1}{\lambda'}\right]+\frac{h^2}{m_e^2}\left[\frac{1}{\lambda_0}-\frac{1}{\lambda'}\right]^2=c^2+\left(\frac{h^2}{m_e^2}\right)\left[\frac{1}{\lambda_0^2}+\frac{1}{\lambda'^2}-\frac{2\cos\theta}{\lambda_0\lambda'}\right]$$

From this we get Eq. 40.10:

$$\lambda'-\lambda_0=\left(h/m_e c\right)\left[1-\cos\theta\right]$$

40.69 $hf=\Delta E=\dfrac{4\pi^2 m_e k_e^2 e^4}{2h^2}\left(\dfrac{1}{(n-1)^2}-\dfrac{1}{n^2}\right)$ so $f=\dfrac{2\pi^2 m_e k_e^2 e^4}{h^3}\left(\dfrac{2n-1}{(n-1)^2 n^2}\right)$

As n approaches infinity, we have f approaching $\dfrac{2\pi^2 m_e k_e^2 e^4}{h^3}\dfrac{2}{n^3}$

The classical frequency is $f=\dfrac{v}{2\pi r}=\dfrac{1}{2\pi}\sqrt{\dfrac{k_e e^2}{m_e}}\dfrac{1}{r^{3/2}}$ where $r=\dfrac{n^2h^2}{4\pi m_e k_e e^2}$

Using this equation to eliminate r from the expression for f, $f=\dfrac{2\pi^2 m_e k_e^2 e^4}{h^3}\dfrac{2}{n^3}$

40.70 Show that if all of the energy of a photon is transmitted to an electron, momentum will not be conserved.

Energy: $\dfrac{hc}{\lambda_0}=\dfrac{hc}{\lambda'}+K_e=m_e c^2(\gamma-1)$ if $\dfrac{hc}{\lambda'}=0$ (1)

Momentum: $\dfrac{h}{\lambda_0}=\dfrac{h}{\lambda'}+\gamma m_e v=\gamma m_e v$ if $\lambda'=\infty$ (2)

From (1), $\gamma=\dfrac{h}{\lambda_0 m_e c}+1$ (3)

$$v=c\sqrt{1-\left(\frac{\lambda_0 m_e c}{h+\lambda_0 m_e c}\right)^2} \qquad (4)$$

Substitute (3) and (4) into (2) and show the inconsistency:

$$\frac{h}{\lambda_0}=\left(1+\frac{h}{\lambda_0 m_e c}\right)m_e c\sqrt{1-\left(\frac{\lambda_0 m_e c}{h+\lambda_0 m_e c}\right)^2}=\frac{\lambda_0 m_e c+h}{\lambda_0}\sqrt{\frac{h(h+2\lambda_0 m_e c)}{(h+\lambda_0 m_e c)^2}}=\frac{h}{\lambda_0}\sqrt{\frac{h+2\lambda_0 m_e c}{h}}$$

This is impossible, so all of the energy of a photon cannot be transmitted to an electron.

40.71 Begin with momentum expressions: $p=\dfrac{h}{\lambda}$, and $p=\gamma mv=\gamma mc\left(\dfrac{v}{c}\right)$.

Equating these expressions, $\gamma\left(\frac{v}{c}\right)=\left(\frac{h}{mc}\right)\frac{1}{\lambda}=\frac{\lambda_C}{\lambda}$

Thus, $\frac{(v/c)^2}{1-(v/c)^2}=\left(\frac{\lambda_C}{\lambda}\right)^2$

or $\left(\frac{v}{c}\right)^2=\left(\frac{\lambda_C}{\lambda}\right)^2-\left(\frac{\lambda_C}{\lambda}\right)^2\left(\frac{v}{c}\right)^2=\frac{(\lambda_C/\lambda)^2}{1+(\lambda_C/\lambda)^2}=\frac{1}{(\lambda/\lambda_C)^2+1}$

giving $\boxed{v=\frac{c}{\sqrt{1+(\lambda\ /\ \lambda_C)^2}}}$

40.72 (a) The energy of the ground state is: $E_1=-\frac{hc}{\lambda_{\text{series limit}}}=-\frac{1240\text{ eV}\cdot\text{nm}}{152.0\text{ nm}}=\boxed{-8.16\text{ eV}}$

From the wavelength of the L_α line, we see: $E_2-E_1=\frac{hc}{\lambda}=\frac{1240\text{ nm}\cdot\text{eV}}{202.6\text{ nm}}=6.12\text{ eV}$

$E_2=E_1+6.12\text{ eV}=\boxed{-2.04\text{ eV}}$

Using the wavelength of the L_β line gives: $E_3-E_1=\frac{1240\text{ nm}\cdot\text{eV}}{170.9\text{ nm}}=7.26\text{ eV}$

so $E_3=\boxed{-0.902\text{ eV}}$

Next, using the L_γ line gives: $E_4-E_1=\frac{1240\text{ nm}\cdot\text{eV}}{162.1\text{ nm}}=7.65\text{ eV}$

and $E_4=\boxed{-0.508\text{ eV}}$

From the L_δ line, $E_5-E_1=\frac{1240\text{ nm}\cdot\text{eV}}{158.3\text{ nm}}=7.83\text{ eV}$

so $E_5=\boxed{-0.325\text{ eV}}$

(b) For the Balmer series, $\frac{hc}{\lambda}=E_i-E_2$, or $\lambda=\frac{1240\text{ nm}\cdot\text{eV}}{E_i-E_2}$

For the α line, $E_i=E_3$ and so $\lambda_\alpha=\frac{1240\text{ nm}\cdot\text{eV}}{(-0.902\text{ eV})-(-2.04\text{ eV})}=\boxed{1090\text{ nm}}$

Similarly, the wavelengths of the β line, γ line, and the short wavelength limit are found to be: $\boxed{811\text{ nm}}$, $\boxed{724\text{ nm}}$, and $\boxed{609\text{ nm}}$.

(c) Computing 60.0% of the wavelengths of the spectral lines shown on the energy-level diagram gives:

0.600(202.6 nm) = $\boxed{122\text{ nm}}$, 0.600(170.9 nm) = $\boxed{103\text{ nm}}$, 0.600(162.1 nm) = $\boxed{97.3\text{ nm}}$,
0.600(158.3 nm) = $\boxed{95.0\text{ nm}}$, and 0.600(152.0 nm) = $\boxed{91.2\text{ nm}}$.

These are seen to be the wavelengths of the α, β, γ, and δ lines as well as the short wavelength limit for the Lyman series in Hydrogen.

(d) The observed wavelengths could be the result of Doppler shift when the source moves away from the Earth. The required speed of the source is found from

$$\frac{f'}{f} = \frac{\lambda}{\lambda'} = \sqrt{\frac{1-(v/c)}{1+(v/c)}} = 0.600 \qquad \text{yielding} \qquad \boxed{v = 0.471c}$$

40.73 (a) Starting with Planck's law,

$$I(\lambda, T) = \frac{2\pi hc^2}{\lambda^5\left[e^{hc/\lambda k_B T} - 1\right]}$$

the total power radiated per unit area

$$\int_0^\infty I(\lambda, T)\,d\lambda = \int_0^\infty \frac{2\pi hc^2}{\lambda^5\left[e^{hc/\lambda k_B T} - 1\right]}\,d\lambda.$$

Change variables by letting

$$x = \frac{hc}{\lambda k_B T}$$

and

$$dx = -\frac{hc\,d\lambda}{k_B T \lambda^2}$$

Note that as λ varies from $0 \to \infty$, x varies from $\infty \to 0$.

Then

$$\int_0^\infty I(\lambda, T)\,d\lambda = -\frac{2\pi k_B^4 T^4}{h^3 c^2}\int_\infty^0 \frac{x^3}{\left(e^x - 1\right)}\,dx = \frac{2\pi k_B^4 T^4}{h^3 c^2}\left(\frac{\pi^4}{15}\right)$$

Therefore,

$$\boxed{\int_0^\infty I(\lambda, T)\,d\lambda = \left(\frac{2\pi^5 k_B^4}{15 h^3 c^2}\right) T^4 = \sigma T^4}$$

(b) From part (a),

$$\sigma = \frac{2\pi^5 k_B^4}{15 h^3 c^2} = \frac{2\pi^5 \left(1.38 \times 10^{-23}\ \text{J/K}\right)^4}{15\left(6.626 \times 10^{-34}\ \text{J}\cdot\text{s}\right)^3 \left(3.00 \times 10^8\ \text{m/s}\right)^2}$$

$$\sigma = \boxed{5.67 \times 10^{-8}\ \text{W/m}^2 \cdot \text{K}^4}$$

***40.74** Planck's law states $I(\lambda,T)=\dfrac{2\pi hc^2}{\lambda^5\left[e^{hc/\lambda k_B T}-1\right]}=2\pi hc^2\lambda^{-5}\left[e^{hc/\lambda k_B T}-1\right]^{-1}$

To find the wavelength at which this distribution has a maximum, compute

$$\frac{dI}{d\lambda}=2\pi hc^2\left\{-5\lambda^{-6}\left[e^{hc/\lambda k_B T}-1\right]^{-1}-\lambda^{-5}\left[e^{hc/\lambda k_B T}-1\right]^{-2}e^{hc/\lambda k_B T}\left(-\frac{hc}{\lambda^2 k_B T}\right)\right\}=0$$

$$\frac{dI}{d\lambda}=\frac{2\pi hc^2}{\lambda^6\left[e^{hc/\lambda k_B T}-1\right]}\left\{-5+\frac{hc}{\lambda k_B T}\frac{e^{hc/\lambda k_B T}}{\left[e^{hc/\lambda k_B T}-1\right]}\right\}=0$$

Letting $x=\dfrac{hc}{\lambda k_B T}$, the condition for a maximum becomes $\dfrac{xe^x}{e^x-1}=5$.

We zero in on the solution to this transcendental equation by iterations as shown in the table below. The solution is found to be

x	$xe^x/(e^x-1)$
4.00000	4.0746294
4.50000	4.5505521
5.00000	5.0339183
4.90000	4.9367620
4.95000	4.9853130
4.97500	5.0096090
4.96300	4.9979452
4.96900	5.0037767
4.96600	5.0008609
4.96450	4.9994030
4.96550	5.0003749
4.96500	4.9998890
4.96525	5.0001320
4.96513	5.0000153
4.96507	4.9999570
4.96510	4.9999862
4.965115	5.0000008

$$x=\frac{hc}{\lambda_{\max}k_B T}=4.965115 \quad \text{and} \quad \lambda_{\max}T=\frac{hc}{4.965115\,k_B}$$

Thus, $\lambda_{\max}T=\dfrac{\left(6.626075\times10^{-34}\text{ J}\cdot\text{s}\right)\left(2.997925\times10^{8}\text{ m/s}\right)}{4.965115\left(1.380658\times10^{-23}\text{ J/K}\right)}=\boxed{2.897755\times10^{-3}\text{ m}\cdot\text{K}}$

This result is very close to Wien's experimental value of $\lambda_{\max}T=2.898\times10^{-3}\text{ m}\cdot\text{K}$

for this constant.

40.75

$$\Delta\lambda = \frac{h}{m_e c}(1-\cos\theta) = \lambda' - \lambda_0$$

$$E' = \frac{hc}{\lambda'} = \frac{hc}{\lambda_0 + \Delta\lambda} = hc\left[\lambda_0 + \frac{h}{m_e c}(1-\cos\theta)\right]^{-1}$$

$$E' = \frac{hc}{\lambda_0}\left[1 + \frac{hc}{m_e c^2 \lambda_0}(1-\cos\theta)\right]^{-1}$$

$$E' = \frac{hc}{\lambda_0}\left[1 + \frac{hc}{m_e c^2 \lambda_0}(1-\cos\theta)\right]^{-1} = E_0\left[1 + \frac{E_0}{m_e c^2}(1-\cos\theta)\right]^{-1}$$

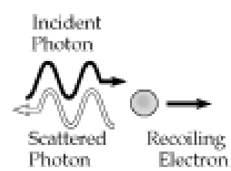
40.76

$$r_1 = \frac{(1)^2\hbar^2}{Z\mu k_e e^2} = \frac{\hbar^2}{(82)(207 m_e)k_e e^2} = \frac{a_0}{(82)(207)} = \frac{0.0529\text{ nm}}{(82)(207)} = \boxed{3.12\text{ fm}}$$

$$E_1 = \frac{-13.6\text{ eV}}{(1)^2}\left(\frac{207}{1}\right)\left(\frac{82}{1}\right)^2 = \boxed{-18.9\text{ MeV}}$$

40.77

This is a case of Compton scattering with a scattering angle of 180°.



$$\Delta\lambda = \lambda' - \lambda_0 = \frac{h}{m_e c}\left(1-\cos 180°\right) = \frac{2h}{m_e c}$$

$$E_0 = \frac{hc}{\lambda_0}, \quad \text{so} \quad \lambda_0 = \frac{hc}{E_0} \quad \text{and} \quad \lambda' = \lambda_0 + \Delta\lambda = \frac{hc}{E_0} + \frac{2h}{m_e c} = \frac{hc}{E_0}\left(1 + \frac{2E_0}{m_e c^2}\right)$$

The kinetic energy of the recoiling electron is then

$$K = E_0 - \frac{hc}{\lambda'} = E_0 - \frac{E_0}{\left(1 + 2E_0/m_e c^2\right)} = E_0\left(\frac{1 + 2E_0/m_e c^2 - 1}{1 + 2E_0/m_e c^2}\right) = \frac{2E_0^2/m_e c^2}{1 + 2E_0/m_e c^2}$$

Defining $a \equiv E_0/m_e c^2$, the kinetic energy can be written as

$$K = \frac{2E_0 a}{1+2a} = \frac{2(hf)a}{1+2a} = \boxed{2hfa(1+2a)^{-1}}$$

where f is the frequency of the incident photon.

40.78 (a) Planck's radiation law predicts maximum intensity at a wavelength $\lambda_{\max}$ we find from

$$\frac{dI}{d\lambda} = 0 = \frac{d}{d\lambda}\left\{2\pi hc^2\lambda^{-5}\left[e^{(hc/\lambda k_B T)} - 1\right]^{-1}\right\}$$

$$0 = 2\pi hc^2\lambda^{-5}(-1)\left[e^{(hc/\lambda k_B T)} - 1\right]^{-2} e^{(hc/\lambda k_B T)}\left(-hc/\lambda^2 k_B T\right) + 2\pi hc^2(-5)\lambda^{-6}\left[e^{(hc/\lambda k_B T)} - 1\right]^{-1}$$

or
$$\frac{-hc\,e^{(hc/\lambda k_B T)}}{\lambda^7 k_B T\left[e^{(hc/\lambda k_B T)} - 1\right]^2} + \frac{5}{\lambda^6\left[e^{(hc/\lambda k_B T)} - 1\right]} = 0$$

which reduces to
$$5\left(\lambda k_B T/hc\right)\left[e^{(hc/\lambda k_B T)} - 1\right] = e^{(hc/\lambda k_B T)}$$

Define $x = hc/\lambda k_B T$. Then we require $5e^x - 5 = xe^x$.

Numerical solution of this transcendental equation gives $x = 4.965$ to four digits. So $\lambda_{\max} = hc/4.965 k_B T$, in agreement with Wien's law.

The intensity radiated over all wavelengths is
$$\int_0^\infty I(\lambda, T)d\lambda = A + B = \int_0^\infty \frac{2\pi hc^2\, d\lambda}{\lambda^5\left[e^{(hc/\lambda k_B T)} - 1\right]}$$

Again, define $x = hc/\lambda k_B T$ so $\lambda = hc/xk_B T$ and $d\lambda = -\left(hc/x^2 k_B T\right)dx$

Then,
$$A + B = \int_{x=\infty}^{0} \frac{-2\pi hc^2 x^5 k_B^5 T^5\, hc\, dx}{h^5 c^5 x^2 k_B T\left(e^x - 1\right)} = \frac{2\pi k_B^4 T^4}{h^3 c^2}\int_0^\infty \frac{x^3 dx}{\left(e^x - 1\right)}$$

The integral is tabulated as $\pi^4/15$, so (in agreement with Stefan's law)
$$A + B = \frac{2\pi^5 k_B^4 T^4}{15 h^3 c^2}$$

The intensity radiated over wavelengths shorter than $\lambda_{\max}$ is

$$\int_0^{\lambda_{\max}} I(\lambda, T)d\lambda = A = \int_0^{\lambda_{\max}} \frac{2\pi hc^2\, d\lambda}{\lambda^5\left[e^{(hc/\lambda k_B T)} - 1\right]}$$

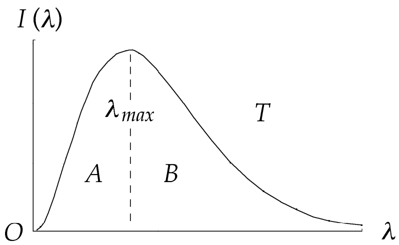
With $x = hc/\lambda k_B T$, this similarly becomes
$$A = \frac{2\pi k_B^4 T^4}{h^3 c^2}\int_{4.965}^{\infty} \frac{x^3\, dx}{e^x - 1}$$

So the fraction of power or of intensity radiated at wavelengths shorter than $\lambda_{\max}$ is

$$\frac{A}{A+B} = \frac{\dfrac{2\pi k_B^4 T^4}{h^3 c^2}\left(\dfrac{\pi^4}{15} - \displaystyle\int_0^{4.965}\frac{x^3\, dx}{e^x - 1}\right)}{\dfrac{2\pi^5 k_B^4 T^4}{15 h^3 c^2}} = \boxed{1 - \frac{15}{\pi^4}\int_0^{4.965}\frac{x^3\, dx}{e^x - 1}}$$

(b) Here are some sample values of the integrand, along with a sketch of the curve:

x	$x^3\left(e^x-1\right)^{-1}$
0.000	0.00
0.100	9.51×10^{-3}
0.200	3.61×10^{-2}
1.00	0.582
2.00	1.25
3.00	1.42
4.00	1.19
4.90	0.883
4.965	0.860



Approximating the integral by trapezoids gives $\dfrac{A}{A+B}\approx1-\dfrac{15}{\pi^4}(4.870)=\boxed{0.2501}$

40.79 $\lambda_C=\dfrac{h}{m_ec}$ and $\lambda=\dfrac{h}{p}$: $\dfrac{\lambda_C}{\lambda}=\dfrac{h/m_ec}{h/p}=\dfrac{p}{m_ec}$;

$E^2=c^2p^2+(m_ec^2)^2$: $p=\sqrt{\dfrac{E^2}{c^2}-(m_ec)^2}$

$$\frac{\lambda_C}{\lambda}=\frac{1}{m_ec}\sqrt{\frac{E^2}{c^2}-(m_ec)^2}=\sqrt{\frac{1}{(m_ec)^2}\left[\frac{E^2}{c^2}-(m_ec)^2\right]}=\sqrt{\left(\frac{E}{m_ec^2}\right)^2-1}$$

40.80 $p=mv=\sqrt{2mE}=\sqrt{2\left(1.67\times10^{-27}\text{ kg}\right)(0.0400\text{ eV})\left(1.60\times10^{-19}\text{ J/eV}\right)}$

$\lambda=\dfrac{h}{mv}=1.43\times10^{-10}\text{ m}=\boxed{0.143\text{ nm}}$

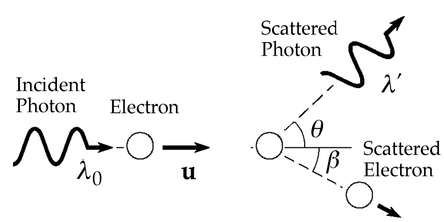
This is of the same order of magnitude as the spacing between atoms in a crystal so diffraction should appear.

40.81 Let u' represent the final speed of the electron and let $\gamma'=\left(1-u'^2/c^2\right)^{-1/2}$. We must eliminate β and u' from the three conservation equations:

$$\frac{hc}{\lambda_0}+\gamma m_ec^2=\frac{hc}{\lambda'}+\gamma' m_ec^2 \qquad [1]$$

$$\frac{h}{\lambda_0}+\gamma m_eu-\frac{h}{\lambda'}\cos\theta=\gamma' m_eu'\cos\beta \qquad [2]$$

$$\frac{h}{\lambda'}\sin\theta=\gamma' m_eu'\sin\beta \qquad [3]$$



(a) (b)

Square Equations [2] and [3] and add:

$$\frac{h^2}{\lambda_0^2}+\gamma^2 m_e^2 u^2+\frac{h^2}{\lambda'^2}+\frac{2h\gamma m_e u}{\lambda_0}-\frac{2h^2\cos\theta}{\lambda_0\lambda'}-\frac{2h\gamma m_e u\cos\theta}{\lambda'}=\gamma'^2 m_e^2 u'^2$$

$$\frac{h^2}{\lambda_0^2}+\frac{h^2}{\lambda'^2}+\gamma^2 m_e^2 u^2+\frac{2h\gamma m_e u}{\lambda_0}-\frac{2h\gamma m_e u\cos\theta}{\lambda'}-\frac{2h^2\cos\theta}{\lambda_0\lambda'}=\frac{m_e^2 u'^2}{1-u'^2/c^2}$$

Call the left-hand side b. Then $b-\frac{bu'^2}{c^2}=m_e^2u'^2$ and $u'^2=\frac{b}{m_e^2+b/c^2}=\frac{c^2b}{m_e^2c^2+b}$

Now square Equation [1] and substitute to eliminate γ':

$$\frac{h^2}{\lambda^2}+\gamma^2 m_e^2c^2+\frac{h^2}{\lambda'^2}+\frac{2h\gamma m_e c}{\lambda_0}-\frac{2h^2}{\lambda_0\lambda'}-\frac{2h\gamma m_e c}{\lambda'}=\frac{m_e^2c^2}{1-u'^2/c^2}=m_e^2c^2+b$$

So we have

$$\frac{h^2}{\lambda_0^2}+\frac{h^2}{\lambda'^2}+\gamma^2 m_e^2c^2+\frac{2h\gamma m_e c}{\lambda_0}-\frac{2h\gamma m_e c}{\lambda'}-\frac{2h^2}{\lambda_0\lambda'}$$
$$=m_ec^2+\frac{h^2}{\lambda_0^2}+\frac{h^2}{\lambda'^2}+\gamma^2 m_e^2u^2+\frac{2h\gamma m_e u}{\lambda_0}-\frac{2h\gamma m_e u\cos\theta}{\lambda'}-\frac{2h^2\cos\theta}{\lambda_0\lambda'}$$

Multiply through by $\lambda_0\lambda'/m_e^2c^2$

$$\lambda_0\lambda'\gamma^2+\frac{2h\lambda'\gamma}{m_ec}-\frac{2h\lambda_0\gamma}{m_ec}-\frac{2h^2}{m_e^2c^2}=\lambda_0\lambda'+\frac{\lambda_0\lambda'\gamma^2u^2}{c^2}+\frac{2h\lambda' u\gamma}{m_ec^2}-\frac{2h\gamma\lambda_0 u\cos\theta}{m_ec^2}-\frac{2h^2\cos\theta}{m_e^2c^2}$$

$$\lambda_0\lambda'\left(\gamma^2-1-\frac{\gamma^2u^2}{c^2}\right)+\frac{2h\gamma\lambda'}{m_ec}\left(1-\frac{u}{c}\right)=\frac{2h\gamma\lambda_0}{m_ec}\left(1-\frac{u\cos\theta}{c}\right)+\frac{2h^2}{m_e^2c^2}(1-\cos\theta)$$

The first term is zero. Then $$\lambda'=\lambda_0\left(\frac{1-(u\cos\theta)/c}{1-u/c}\right)+\frac{h\gamma^{-1}}{m_ec}\left(\frac{1}{1-u/c}\right)(1-\cos\theta)$$

Since $$\gamma^{-1}=\sqrt{1-(u/c)^2}=\sqrt{(1-u/c)(1+u/c)}$$

this result may be written as $$\boxed{\lambda'=\lambda_0\left(\frac{1-(u\cos\theta)/c}{1-u/c}\right)+\frac{h}{m_ec}\sqrt{\frac{1+u/c}{1-u/c}}(1-\cos\theta)}$$