Chapter 39 Solutions

39.1 In the rest frame,

$$p_i = m_1 v_{1i} + m_2 v_{2i} = (2000\text{ kg})(20.0\text{ m/s}) + (1500\text{ kg})(0\text{ m/s}) = 4.00 \times 10^4\text{ kg}\cdot\text{m/s}$$

$$p_f = (m_1 + m_2)v_f = (2000\text{ kg} + 1500\text{ kg})v_f$$

Since $p_i = p_f$, $$v_f = \frac{4.00 \times 10^4\text{ kg}\cdot\text{m/s}}{2000\text{ kg} + 1500\text{ kg}} = 11.429\text{ m/s}$$

In the moving frame, these velocities are all reduced by +10.0 m/s.

$$v'_{1i} = v_{1i} - v' = 20.0\text{ m/s} - (+10.0\text{ m/s}) = 10.0\text{ m/s}$$

$$v'_{2i} = v_{2i} - v' = 0\text{ m/s} - (+10.0\text{ m/s}) = -10.0\text{ m/s}$$

$$v'_f = 11.429\text{ m/s} - (+10.0\text{ m/s}) = 1.429\text{ m/s}$$

Our initial momentum is then

$$p'_i = m_1 v'_{1i} + m_2 v'_{2i} = (2000\text{ kg})(10.0\text{ m/s}) + (1500\text{ kg})(-10.0\text{ m/s}) = 5000\text{ kg}\cdot\text{m/s}$$

and our final momentum is

$$p'_f = (2000\text{ kg} + 1500\text{ kg})\,v'_f = (3500\text{ kg})(1.429\text{ m/s}) = 5000\text{ kg}\cdot\text{m/s}$$

39.2 (a) $v = v_T + v_B = \boxed{60.0\text{ m/s}}$

(b) $v = v_T - v_B = \boxed{20.0\text{ m/s}}$

(c) $v = \sqrt{v_T^2 + v_B^2} = \sqrt{20^2 + 40^2} = \boxed{44.7\text{ m/s}}$

39.3 The first observer watches some object accelerate under applied forces. Call the instantaneous velocity of the object v_1. The second observer has constant velocity v_{21} relative to the first, and measures the object to have velocity $v_2 = v_1 - v_{21}$.

The second observer measures an acceleration of $$a_2 = \frac{dv_2}{dt} = \frac{dv_1}{dt}$$

This is the same as that measured by the first observer. In this nonrelativistic case, they measure the same forces as well. Thus, the second observer also confirms that $\Sigma F = ma$.

39.4 The laboratory observer notes Newton's second law to hold: $F_1 = ma_1$

(where the subscript 1 implies the measurement was made in the laboratory frame of reference). The observer in the accelerating frame measures the acceleration of the mass as $a_2 = a_1 - a'$

(where the subscript 2 implies the measurement was made in the accelerating frame of reference, and the primed acceleration term is the acceleration of the accelerated frame with respect to the laboratory frame of reference). If Newton's second law held for the accelerating frame, that observer would then find valid the relation

$F_2 = ma_2$ or $F_1 = ma_2$

(since $F_1 = F_2$ and the mass is unchanged in each). But, instead, the accelerating frame observer will find that $F_2 = ma_2 - ma'$ which is *not* Newton's second law.

***39.5** $L = L_p\sqrt{1 - v^2/c^2} \Rightarrow v = c\sqrt{1 - (L/L_p)^2}$

Taking $L = L_p/2$ where $L_p = 1.00$ m gives $v = c\sqrt{1 - \left(\frac{L_p/2}{L_p}\right)^2} = c\sqrt{1 - \frac{1}{4}} = \boxed{0.866c}$

39.6 $\Delta t = \dfrac{\Delta t_p}{\left[1 - (v/c)^2\right]^{1/2}}$ so $v = c\left[1 - \left(\dfrac{\Delta t_p}{\Delta t}\right)^2\right]^{1/2}$

For $\Delta t = 2\Delta t_p \Rightarrow v = c\left[1 - \left(\dfrac{\Delta t_p}{2\Delta t_p}\right)^2\right]^{1/2} = c\left[1 - \dfrac{1}{4}\right]^{1/2} = \boxed{0.866c}$

***39.7** (a) $\gamma = \dfrac{1}{\sqrt{1 - (v/c)^2}} = \dfrac{1}{\sqrt{1 - (0.500)^2}} = \dfrac{2}{\sqrt{3}}$

The time interval between pulses as measured by the Earth observer is

$$\Delta t = \gamma\Delta t_p = \frac{2}{\sqrt{3}}\left(\frac{60.0\text{ s}}{75.0}\right) = 0.924\text{ s}$$

Thus, the Earth observer records a pulse rate of $\dfrac{60.0\text{ s/min}}{0.924\text{ s}} = \boxed{64.9/\text{min}}$

(b) At a relative speed $v = 0.990c$, the relativistic factor γ increases to 7.09 and the pulse rate recorded by the Earth observer decreases to $\boxed{10.6/\text{min}}$. That is, the life span of the astronaut (reckoned by the total number of his heartbeats) is much longer as measured by an Earth clock than by a clock aboard the space vehicle.

39.8 The observed length of an object moving at speed v is $L = L_p\sqrt{1 - v^2/c^2}$ with L_p as the proper length. For the two ships, we know $L_2 = L_1$, $L_{2p} = 3L_{1p}$, and $v_1 = 0.350c$

Thus, $L_2^2 = L_1^2$ and $9L_{1p}{}^2\left(1 - \frac{v_2{}^2}{c^2}\right) = L_{1p}{}^2\left[1 - (0.350)^2\right]$

giving $9 - 9\frac{v_2^2}{c^2} = 0.878$, or $v_2 = \boxed{0.950c}$

***39.9** $\Delta t = \gamma\Delta t_p = \frac{\Delta t_p}{\sqrt{1 - v^2/c^2}}$ so $\Delta t_p = \left(\sqrt{1 - v^2/c^2}\right)\Delta t \approx \left(1 - \frac{v^2}{2c^2}\right)\Delta t$ and $\Delta t - \Delta t_p = \left(\frac{v^2}{2c^2}\right)\Delta t$

If $v = 1000 \text{ km/h} = \frac{1.00 \times 10^6 \text{ m}}{3600 \text{ s}} = 277.8 \text{ m/s}$, then $\frac{v}{c} = 9.26 \times 10^{-7}$

and $\left(\Delta t - \Delta t_p\right) = (4.28 \times 10^{-13})(3600 \text{ s}) = 1.54 \times 10^{-9} \text{ s} = \boxed{1.54 \text{ ns}}$

39.10 $\gamma^{-1} = \sqrt{1 - \frac{v^2}{c^2}} = \sqrt{1 - (0.950)^2} = 0.312$

(a) astronauts' time: $\Delta t_p = \gamma^{-1}\Delta t = (0.312)(4.42 \text{ yr}) = \boxed{1.38 \text{ yr}}$

(b) astronauts' distance: $L = \gamma^{-1}\Delta L_p = (0.312)(4.20 \text{ ly}) = \boxed{1.31 \text{ ly}}$

39.11 The spaceship appears length-contracted to the Earth observer as given by

$L = L_p\sqrt{1 - v^2/c^2}$ or $L^2 = L_p{}^2\left(1 - v^2/c^2\right)$

Also, the contracted length is related to the time required to pass overhead by:

$L = vt$ or $L^2 = v^2t^2 = \frac{v^2}{c^2}(ct)^2$

Equating these two expressions gives $L_p{}^2 - L_p{}^2\frac{v^2}{c^2} = (ct)^2\frac{v^2}{c^2}$ or $\left[L_p{}^2 + (ct)^2\right]\frac{v^2}{c^2} = L_p{}^2$

Using the given values: $L_p = 300 \text{ m}$ and $t = 7.50 \times 10^{-7} \text{ s}$

this becomes $(1.41 \times 10^5 \text{ m}^2)\frac{v^2}{c^2} = 9.00 \times 10^4 \text{ m}^2$

giving $v = \boxed{0.800c}$

Goal Solution

A spaceship with a proper length of 300 m takes 0.750 μs seconds to pass an Earth observer. Determine its speed as measured by the Earth observer.

G: We should first determine if the spaceship is traveling at a relativistic speed: classically, v = (300m)/(0.750 μs) = 4.00×10^8 m/s, which is faster than the speed of light (impossible)! Quite clearly, the relativistic correction must be used to find the correct speed of the spaceship, which we can guess will be close to the speed of light.

O: We can use the contracted length equation to find the speed of the spaceship in terms of the proper length and the time. The time of 0.750 μs is the **proper time** measured by the Earth observer, because it is the time interval between two events that she sees as happening at the same point in space. The two events are the passage of the front end of the spaceship over her stopwatch, and the passage of the back end of the ship.

A: $L = L_p / \gamma$, with $L = v\Delta t$: $\quad v\Delta t = L_p\left(1 - v^2/c^2\right)^{1/2}$

Squaring both sides, $\quad v^2\Delta t^2 = L_p{}^2\left(1 - v^2/c^2\right)$

$$v^2c^2 = L_p{}^2c^2/\Delta t^2 - v^2L_p{}^2/\Delta t^2$$

Solving for the velocity, $\quad v = \dfrac{cL_p/\Delta t}{\sqrt{c^2 + L_p{}^2/\Delta t^2}}$

So

$$v = \frac{\left(3.00\times10^8\right)(300\text{ m})/\left(0.750\times10^{-6}\text{ s}\right)}{\sqrt{\left(3.00\times10^8\right)^2 + (300\text{ m})^2/\left(0.750\times10^{-6}\text{ s}\right)^2}} = 2.40\times10^8\text{ m/s}$$

L: The spaceship is traveling at $0.8c$. We can also verify that the general equation for the speed reduces to the classical relation $v = L_p/\Delta t$ when the time is relatively large.

39.12 The spaceship appears to be of length L to Earth observers,

where $\quad L = L_p\left(1 - \dfrac{v^2}{c^2}\right)^{1/2} \quad$ and $\quad L = vt$

$$vt = L_p\left(1 - \frac{v^2}{c^2}\right)^{1/2} \quad \text{so} \quad v^2t^2 = L_p{}^2\left(1 - \frac{v^2}{c^2}\right)$$

Solving for v, $\quad v^2\left(t^2 + \dfrac{L_p{}^2}{c^2}\right) = L_p{}^2 \qquad \boxed{\dfrac{v}{c} = L_p\left(c^2t^2 + L_p{}^2\right)^{-1/2}}$

***39.13** For $\frac{v}{c} = 0.990, \quad \gamma = 7.09$

(a) The muon's lifetime as measured in the Earth's rest frame is $\Delta t = \frac{4.60\text{ km}}{0.990c}$

and the lifetime measured in the muon's rest frame is

$$\Delta t_p = \frac{\Delta t}{\gamma} = \frac{1}{7.09}\left[\frac{4.60 \times 10^3\text{ m}}{0.990\left(3.00 \times 10^8\text{ m/s}\right)}\right] = \boxed{2.18\ \mu\text{s}}$$

(b) $L = L_p\sqrt{1-\left(v/c\right)^2} = \frac{L_p}{\gamma} = \frac{4.60 \times 10^3\text{ m}}{7.09} = \boxed{649\text{ m}}$

39.14 We find Carpenter's speed: $\frac{GMm}{r^2} = \frac{mv^2}{r}$

$$v = \left[\frac{GM}{(R+h)}\right]^{1/2} = \left[\frac{(6.67 \times 10^{-11})(5.98 \times 10^{24})}{(6.37 \times 10^6 + 0.160 \times 10^6)}\right]^{1/2} = 7.82\text{ km / s}$$

Then the time period of one orbit, $T = \frac{2\pi(R+h)}{v} = \frac{2\pi(6.53 \times 10^6)}{7.82 \times 10^3} = 5.25 \times 10^3\text{ s}$

(a) The time difference for 22 orbits is $\Delta t - \Delta t_p = (\gamma - 1)\Delta t_p = \left[\left(1 - v^2/c^2\right)^{-1/2} - 1\right]\left(22T\right)$

$$\Delta t - \Delta t_p \approx \left(1 + \frac{1}{2}\frac{v^2}{c^2} - 1\right)(22T) = \frac{1}{2}\left(\frac{7.82 \times 10^3\text{ m / s}}{3 \times 10^8\text{ m / s}}\right)^2 22\left(5.25 \times 10^3\text{ s}\right) = \boxed{39.2\ \mu\text{s}}$$

(b) For one orbit, $\Delta t - \Delta t_p = \frac{39.2\ \mu\text{s}}{22} = 1.78\ \mu\text{s}$. The press report is $\boxed{\text{accurate to one digit}}$.

39.15 For pion to travel 10.0 m in Δt in our frame, $10.0\text{ m} = v\Delta t = v(\gamma\Delta t_p) = \frac{v(26.0 \times 10^{-9}\text{ s})}{\sqrt{1 - v^2/c^2}}$

Solving for the velocity,

$$(3.85 \times 10^8\text{ m / s})^2(1 - v^2/c^2) = v^2$$

$$1.48 \times 10^{17}\text{ m}^2/\text{s}^2 = v^2(1 + 1.64)$$

$$v = 2.37 \times 10^8\text{ m / s} = \boxed{0.789c}$$

***39.16** $\gamma = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}} = 1.01$ so $\boxed{v = 0.140c}$

***39.17** (a) Since your ship is identical to his, and you are at rest with respect to your own ship, its length is $\boxed{20.0\text{ m}}$.

(b) His ship is in motion relative to you, so you see its length contracted to $\boxed{19.0\text{ m}}$.

(c) We have $L = L_p\sqrt{1-v^2/c^2}$

from which $\dfrac{L}{L_p} = \dfrac{19.0\text{ m}}{20.0\text{ m}} = 0.950 = \sqrt{1-\dfrac{v^2}{c^2}}$ and $\boxed{v = 0.312c}$

***39.18** (a) $\Delta t = \gamma\Delta t_p = \dfrac{\Delta t_p}{\sqrt{1-(v/c)^2}} = \dfrac{15.0\text{ yr}}{\sqrt{1-(0.700)^2}} = \boxed{21.0\text{ yr}}$

(b) $d = v(\Delta t) = [0.700c](21.0\text{ yr}) = [(0.700)(1.00\text{ ly/yr})](21.0\text{ yr}) = \boxed{14.7\text{ ly}}$

(c) The astronauts see Earth flying out the back window at $0.700c$:

$d = v(\Delta t_p) = [0.700c](15.0\text{ yr}) = [(0.700)(1.00\text{ ly/yr})](15.0\text{ yr}) = \boxed{10.5\text{ ly}}$

(d) Mission control gets signals for 21.0 yr while the battery is operating, and then for 14.7 years after the battery stops powering the transmitter, 14.7 ly away: $21.0\text{ yr} + 14.7\text{ yr} = \boxed{35.7\text{ yr}}$

***39.19** The orbital speed of the Earth is as described by $\Sigma F = ma$: $\dfrac{Gm_S m_E}{r^2} = \dfrac{m_E v^2}{r}$

$$v = \sqrt{\frac{Gm_S}{r}} = \sqrt{\frac{(6.67\times10^{-11}\text{ N}\cdot\text{m}^2/\text{kg}^2)(1.99\times10^{30}\text{ kg})}{1.496\times10^{11}\text{ m}}} = 2.98\times10^4\text{ m s}$$

The maximum frequency received by the extraterrestrials is

$$f_{\text{obs}} = f_{\text{source}}\sqrt{\frac{1+v/c}{1-v/c}} = (57.0\times10^6\text{ Hz})\sqrt{\frac{1+(2.98\times10^4\text{ m/s})/(3.00\times10^8\text{ m/s})}{1-(2.98\times10^4\text{ m/s})/(3.00\times10^8\text{ m/s})}} = 57.005\ 66\times10^6\text{ Hz}$$

The minimum frequency received is

$$f_{\text{obs}} = f_{\text{source}}\sqrt{\frac{1-v/c}{1+v/c}} = (57.0\times10^6\text{ Hz})\sqrt{\frac{1-(2.98\times10^4\text{ m/s})/(3.00\times10^8\text{ m/s})}{1+(2.98\times10^4\text{ m/s})/(3.00\times10^8\text{ m/s})}} = 56.994\ 34\times10^6\text{ Hz}$$

The difference, which lets them figure out the speed of our planet, is

$(57.005\ 66 - 56.994\ 34)\times10^6\text{ Hz} = \boxed{1.13\times10^4\text{ Hz}}$

39.20 (a) Let f_c be the frequency as seen by the car. Thus, $f_c = f_{\text{source}}\sqrt{\frac{c+v}{c-v}}$

and, if f is the frequency of the reflected wave, $f = f_c\sqrt{\frac{c+v}{c-v}}$

Combining gives $\boxed{f = f_{\text{source}}\frac{(c+v)}{(c-v)}}$

(b) Using the above result, $f(c-v) = f_{\text{source}}(c+v)$

which gives $(f - f_{\text{source}})c = (f + f_{\text{source}})v \approx 2f_{\text{source}}v$

The beat frequency is then $f_b = f - f_{\text{source}} = \frac{2f_{\text{source}}v}{c} = \boxed{\frac{2v}{\lambda}}$

(c) $$f_b = \frac{(2)(30.0\text{ m/s})(10.0\times10^9\text{ Hz})}{3.00\times10^8\text{ m/s}} = \frac{(2)(30.0\text{ m/s})}{(0.0300\text{ m})} = 2000\text{ Hz} = \boxed{2.00\text{ kHz}}$$

$$\lambda = \frac{c}{f_{\text{source}}} = \frac{3.00\times10^8\text{ m/s}}{10.0\times10^9\text{ Hz}} = 3.00\text{ cm}$$

(d) $v = \frac{f_b\lambda}{2}$ so $\Delta v = \frac{\Delta f_b\lambda}{2} = \frac{(5\text{ Hz})(0.0300\text{ m})}{2} = \boxed{0.0750\text{ m/s} \approx 0.2\text{ mi / h}}$

39.21 (a) When the source moves away from an observer, the observed frequency is

$$f_{\text{obs}} = f_{\text{source}}\left(\frac{c-v_s}{c+v_s}\right)^{1/2} \qquad \text{where} \qquad v_s = v_{\text{source}}$$

When $v_s << c$, the binomial expansion gives

$$\left(\frac{c-v_s}{c+v_s}\right)^{1/2} = \left[1-\left(\frac{v_s}{c}\right)\right]^{1/2}\left[1+\left(\frac{v_s}{c}\right)\right]^{-1/2} \approx \left(1-\frac{v_s}{2c}\right)\left(1-\frac{v_s}{2c}\right) \approx \left(1-\frac{v_s}{c}\right)$$

So, $f_{\text{obs}} \approx f_{\text{source}}\left(1-\frac{v_s}{c}\right)$

The observed wavelength is found from $c = \lambda_{\text{obs}} f_{\text{obs}} = \lambda f_{\text{source}}$:

$$\lambda_{\text{obs}} = \frac{\lambda f_{\text{source}}}{f_{\text{obs}}} \approx \frac{\lambda f_{\text{source}}}{f_{\text{source}}(1-v_s/c)} = \frac{\lambda}{1-v_s/c}$$

$$\Delta\lambda = \lambda_{\text{obs}} - \lambda = \lambda\left(\frac{1}{1-v_s/c}-1\right) = \lambda\left(\frac{1}{1-v_s/c}-1\right) = \lambda\left(\frac{v_s/c}{1-v_s/c}\right)$$

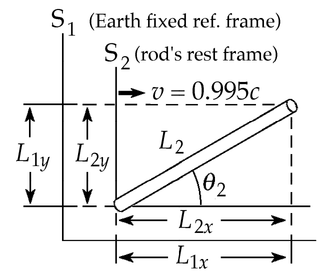
Since $1 - v_s/c \approx 1$, $\boxed{\frac{\Delta\lambda}{\lambda} \approx \frac{v_{\text{source}}}{c}}$

(b) $$v_{\text{source}} = c\left(\frac{\Delta\lambda}{\lambda}\right) = c\left(\frac{20.0\text{ nm}}{397\text{ nm}}\right) = \boxed{0.050\,4c}$$

39.22 $u'_x = \dfrac{u_x - v}{1 - u_x v / c^2} = \dfrac{0.950c - 0.750c}{1 - 0.950 \times 0.750} = \boxed{0.696c}$

39.23 $u'_x = \dfrac{u_x - v}{1 - u_x v/c^2} = \dfrac{-0.750c - 0.750c}{1 - (-0.750)(0.750)} = \boxed{-0.960c}$

***39.24** $\gamma = 10.0$ We are also given: L_1 = 2.00 m, and θ_1 = 30.0° (both measured in a reference frame moving relative to the rod).



Thus, $L_{1x} = L_1 \cos\theta_1 = (2.00 \text{ m})(0.867) = 1.73 \text{ m}$

and $L_{1y} = L_1 \sin\theta_1 = (2.00 \text{ m})(0.500) = 1.00 \text{ m}$

L_{2x} = a "proper length" is related to L_{1x}

by $L_{1x} = L_{2x}/\gamma$

Therefore, $L_{2x} = 10.0 L_{1x} = 17.3 \text{ m}$ and $L_{2y} = L_{1y} = 1.00 \text{ m}$

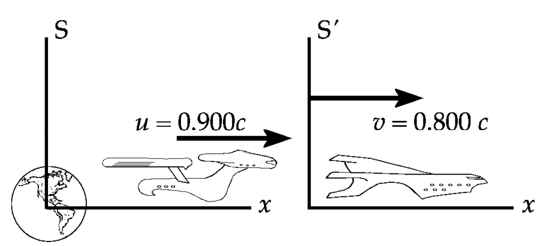
(Lengths perpendicular to the motion are unchanged).

(a) $L_2 = \sqrt{(L_{2x})^2 + (L_{2y})^2}$ gives $\boxed{L_2 = 17.4 \text{ m}}$

(b) $\theta_2 = \tan^{-1}\dfrac{L_{2y}}{L_{2x}}$ gives $\boxed{\theta_2 = 3.30°}$

39.25 u_x = Enterprise velocity

v = Klingon velocity



From Equation 39.16,

$u'_x = \dfrac{u_x - v}{1 - \dfrac{u_x v}{c^2}} = \dfrac{0.900c - 0.800c}{1 - (0.900)(0.800)} = \boxed{0.357c}$

***39.26** (a) From Equation 39.13, $\Delta x' = \gamma(\Delta x - v\,\Delta t)$,

$$0 = \gamma\left[2.00 \text{ m} - v\left(8.00\times10^{-9}\text{ s}\right)\right]$$

$$v = \frac{2.00 \text{ m}}{8.00\times10^{-9}\text{ s}} = \boxed{2.50\times10^{8}\text{ m/s}}$$

$$\gamma = \frac{1}{\sqrt{1-\left(2.50\times10^{8}\text{ m/s}\right)^2/\left(3.00\times10^{8}\text{ m/s}\right)^2}} = 1.81$$

(b) From Equation 39.11, $x' = \gamma(x - vt) = 1.81\left[3.00\text{ m} - \left(2.50\times10^{8}\text{ m/s}\right)\left(1.00\times10^{-9}\text{ s}\right)\right] = \boxed{4.97\text{ m}}$

(c)

$$t' = \gamma\left(t - \frac{v}{c^2}x\right) = 1.81\left[1.00\times10^{-9}\text{ s} - \frac{\left(2.50\times10^{8}\text{ m/s}\right)}{\left(3.00\times10^{8}\text{ m/s}\right)^2}(3.00\text{ m})\right]$$

$$t' = \boxed{-1.33\times10^{-8}\text{ s}}$$

39.27 $p = \gamma mu$

(a) For an electron moving at $0.0100c$, $\gamma = \dfrac{1}{\sqrt{1-(u/c)^2}} = \dfrac{1}{\sqrt{1-(0.0100)^2}} = 1.00005 \approx 1.00$

Thus, $p = 1.00\left(9.11\times10^{-31}\text{ kg}\right)(0.0100)\left(3.00\times10^{8}\text{ m / s}\right) = \boxed{2.73\times10^{-24}\text{ kg}\cdot\text{m/s}}$

(b) Following the same steps as used in part (a), we find at $0.500c$

$\gamma = 1.15$ and $p = \boxed{1.58\times10^{-22}\text{ kg}\cdot\text{m/s}}$

(c) At $0.900c$, $\gamma = 2.29$ and $p = \boxed{5.64\times10^{-22}\text{ kg}\cdot\text{m/s}}$

***39.28** Using the relativistic form, $p = \dfrac{mu}{\sqrt{1-(u/c)^2}} = \gamma mu$,

we find the difference Δp from the classical momentum, mu: $\Delta p = \gamma mu - mu = (\gamma - 1)mu$

(a) The difference is 1.00% when $(\gamma - 1)mu = 0.0100\,\gamma mu$:

$$\gamma = \frac{1}{0.990} = \frac{1}{\sqrt{1-(u/c)^2}} \Rightarrow 1-(u/c)^2 = (0.990)^2 \quad\text{or}\quad u = \boxed{0.141c}$$

(b) The difference is 10.0% when $(\gamma - 1)mu = 0.100\,\gamma mu$:

$$\gamma = \frac{1}{0.900} = \frac{1}{\sqrt{1-(u/c)^2}} \Rightarrow 1-(u/c)^2 = (0.900)^2 \quad\text{or}\quad u = \boxed{0.436c}$$

***39.29** $\dfrac{p - mu}{mu} = \dfrac{\gamma mu - mu}{mu} = \gamma - 1$

$$\gamma - 1 = \frac{1}{\sqrt{1-(u/c)^2}} - 1 \approx 1 + \frac{1}{2}\left(\frac{u}{c}\right)^2 - 1 = \frac{1}{2}\left(\frac{u}{c}\right)^2$$

$$\frac{p - mu}{mu} = \frac{1}{2}\left(\frac{90.0\ \text{m/s}}{3.00\times 10^{8}\ \text{m/s}}\right)^2 = \boxed{4.50\times 10^{-14}}$$

39.30 $p = \dfrac{mu}{\sqrt{1-(u/c)^2}}$ becomes $1 - \dfrac{u^2}{c^2} = \dfrac{m^2u^2}{p^2}$

which gives: $1 = u^2\left(\dfrac{m^2}{p^2} + \dfrac{1}{c^2}\right)$

or $c^2 = u^2\left(\dfrac{m^2c^2}{p^2} + 1\right)$ and $\boxed{u = \dfrac{c}{\sqrt{\dfrac{m^2c^2}{p^2} + 1}}}$

***39.31** Relativistic momentum must be conserved:

For total momentum to be zero after as it was before, we must have, with subscript 2 referring to the heavier fragment, and subscript 1 to the lighter, $p_2 = p_1$

or $\gamma_2 m_2 u_2 = \gamma_1 m_1 u_1 = \dfrac{2.50\times 10^{-28}\ \text{kg}}{\sqrt{1-(0.893)^2}} \times (0.893c)$

or $\dfrac{(1.67\times 10^{-27}\ \text{kg})u_2}{\sqrt{1-(u_2/c)^2}} = (4.960\times 10^{-28}\ \text{kg})c$

and $u_2 = \boxed{0.285c}$

Goal Solution

An unstable particle at rest breaks into two fragments of **unequal** mass. The rest mass of the lighter fragment is 2.50×10^{-28} kg, and that of the heavier fragment is 1.67×10^{-27} kg. If the lighter fragment has a speed of $0.893c$ after the breakup, what is the speed of the heavier fragment?

G: The heavier fragment should have a speed less than that of the lighter piece since the momentum of the system must be conserved. However, due to the relativistic factor, the ratio of the speeds will not equal the simple ratio of the particle masses, which would give a speed of $0.134c$ for the heavier particle.

O: Relativistic momentum of the system must be conserved. For the total momentum to be zero after the fission, as it was before, $\mathbf{p}_1 + \mathbf{p}_2 = 0$, where we will refer to the lighter particle with the subscript '1', and to the heavier particle with the subscript '2.'

A: $\gamma_2 m_2 v_2 + \gamma_1 m_1 v_1 = 0$ so $\gamma_2 m_2 v_2 + \left(\dfrac{2.50 \times 10^{-28} \text{ kg}}{\sqrt{1 - 0.893^2}}\right)(0.893c) = 0$

Rearranging, $$\left(\frac{1.67 \times 10^{-27} \text{ kg}}{\sqrt{1 - v_2^2/c^2}}\right)\frac{v_2}{c} = -4.96 \times 10^{-28} \text{ kg}$$

Squaring both sides, $$\left(2.79 \times 10^{-54}\right)\left(\frac{v_2}{c}\right)^2 = \left(2.46 \times 10^{-55}\right)\left(1 - \frac{v_2^2}{c^2}\right)$$ and $v_2 = -0.285c$

We choose the negative sign only to mean that the two particles must move in opposite directions. The speed, then, is $|v_2| = 0.285c$

L: The speed of the heavier particle is less than the lighter particle, as expected. We can also see that for this situation, the relativistic speed of the heavier particle is about twice as great as was predicted by a simple non-relativistic calculation.

39.32 $\Delta E = (\gamma_1 - \gamma_2)mc^2$. For an electron, $mc^2 = 0.511$ MeV.

(a) $$\Delta E = \left(\sqrt{\frac{1}{(1 - 0.810)}} - \sqrt{\frac{1}{(1 - 0.250)}}\right)mc^2 = \boxed{0.582 \text{ MeV}}$$

(b) $$\Delta E = \left(\sqrt{\frac{1}{1 - (0.990)^2}} - \sqrt{\frac{1}{1 - 0.810}}\right)mc^2 = \boxed{2.45 \text{ MeV}}$$

39.33 $E = \gamma mc^2 = 2mc^2$, or $\gamma = 2$

Thus, $\dfrac{u}{c} = \sqrt{1 - (1/\gamma)^2} = \dfrac{\sqrt{3}}{2}$, or $u = \dfrac{c\sqrt{3}}{2}$.

The momentum is then $$p = \gamma mu = 2m\left(\frac{c\sqrt{3}}{2}\right) = \left(\frac{mc^2}{c}\right)\sqrt{3} = \left(\frac{938.3 \text{ MeV}}{c}\right)\sqrt{3} = \boxed{1.63 \times 10^3 \ \frac{\text{MeV}}{c}}$$

***39.34** The relativistic kinetic energy of an object of mass m and speed u is $K_r = \left(\frac{1}{\sqrt{1-u^2/c^2}} - 1\right)mc^2$

For $u = 0.100c$, $K_r = \left(\frac{1}{\sqrt{1-0.0100}} - 1\right)mc^2 = 0.005038\, mc^2$

The classical equation $K_c = \frac{1}{2}mu^2$ gives $K_c = \frac{1}{2}m(0.100c)^2 = 0.005000\, mc^2$

different by $\frac{0.005038 - 0.005000}{0.005038} = 0.751\%$

For still smaller speeds the agreement will be still better.

39.35 (a) $E_R = mc^2 = (1.67\times10^{-27}\text{ kg})(2.998\times10^8\text{ m/s})^2 = 1.50\times10^{-10}\text{ J} = \boxed{938\text{ MeV}}$

(b) $E = \gamma mc^2 = \frac{1.50\times10^{-10}\text{ J}}{[1-(0.95c/c)^2]^{1/2}} = 4.81\times10^{-10}\text{ J} = \boxed{3.00\times10^3\text{ MeV}}$

(c) $K = E - mc^2 = 4.81\times10^{-10}\text{ J} - 1.50\times10^{-10}\text{ J} = 3.31\times10^{-10}\text{ J} = \boxed{2.07\times10^3\text{ MeV}}$

***39.36** (a) $KE = E - E_R = 5E_R$

$E = 6E_R = 6(9.11\times10^{-31}\text{ kg})(3.00\times10^8\text{ m/s})^2 = 4.92\times10^{-13}\text{ J} = \boxed{3.07\text{ MeV}}$

(b) $E = \gamma mc^2 = \gamma E_R$

Thus, $\gamma = \frac{E}{E_r} = 6 = \frac{1}{\sqrt{1-u^2/c^2}}$ which yields $\boxed{u = 0.986c}$

39.37 The relativistic density is

$$\frac{E_R}{c^2 V} = \frac{mc^2}{c^2 V} = \frac{m}{V} = \frac{m}{(L_p)(L_p)\left[L_p\sqrt{1-(u/c)^2}\right]} = \frac{8.00\text{ g}}{(1.00\text{ cm})^3\sqrt{1-(0.900)^2}} = \boxed{18.4\text{ g/cm}^3}$$

***39.38** We must conserve both mass-energy and relativistic momentum. With subscript 1 referring to the $0.868c$ particle and subscript 2 to the $0.987c$ particle,

$$\gamma_1 = \frac{1}{\sqrt{1-(0.868)^2}} = 2.01 \qquad \text{and} \qquad \gamma_2 = \frac{1}{\sqrt{1-(0.987)^2}} = 6.22$$

Conservation of mass-energy gives $E_1 + E_2 = E_{\text{total}}$ which is $\gamma_1 m_1 c^2 + \gamma_2 m_2 c^2 = m_{\text{total}} c^2$

or $2.01 m_1 + 6.22 m_2 = 3.34 \times 10^{-27}$ kg

This reduces to: $m_1 + 3.09 m_2 = 1.66 \times 10^{-27}$ kg [1]

Since the momentum after must equal zero, $p_1 = p_2$ gives $\gamma_1 m_1 u_1 = \gamma_2 m_2 u_2$

or $(2.01)(0.868c)m_1 = (6.22)(0.987c)m_2$

which becomes $m_1 = 3.52 m_2$ [2]

Solving [1] and [2] simultaneously, $m_1 = \boxed{8.84 \times 10^{-28}\ \text{kg}}$ and $m_2 = \boxed{2.51 \times 10^{-28}\ \text{kg}}$

39.39 $E = \gamma mc^2$, $p = \gamma mu$; $E^2 = (\gamma mc^2)^2$; $p^2 = (\gamma mu)^2$;

$$E^2 - p^2c^2 = (\gamma mc^2)^2 - (\gamma mu)^2 c^2 = \gamma^2\left((mc^2)^2 - (mc)^2 u^2\right) = (mc^2)^2\left(1 - \frac{u^2}{c^2}\right)\left(1 - \frac{u^2}{c^2}\right)^{-1} = (mc^2)^2 \qquad \text{Q.E.D.}$$

39.40 (a) $K = 50.0$ GeV

$$mc^2 = \left(1.67 \times 10^{-27}\ \text{kg}\right)\left(2.998 \times 10^8\ \text{m/s}\right)^2\left(\frac{1}{1.60 \times 10^{-10}\ \text{J/GeV}}\right) = 0.938\ \text{GeV}$$

$$E = K + mc^2 = 50.0\ \text{GeV} + 0.938\ \text{GeV} = 50.938\ \text{GeV}$$

$$E^2 = p^2c^2 + \left(mc^2\right)^2 \quad \Rightarrow \quad p = \sqrt{\frac{E^2 - \left(mc^2\right)^2}{c^2}} = \sqrt{\frac{(50.938\ \text{GeV})^2 - (0.938\ \text{GeV})^2}{c^2}}$$

$$p = 50.9\ \frac{\text{GeV}}{c} = \left(\frac{50.9\ \text{GeV}}{3.00 \times 10^8\ \text{m/s}}\right)\left(\frac{1.60 \times 10^{-10}\ \text{J}}{1\ \text{GeV}}\right) = \boxed{2.72 \times 10^{-17}\ \text{kg} \cdot \text{m/s}}$$

(b) $$E = \gamma mc^2 = \frac{mc^2}{\sqrt{1-(u/c)^2}} \quad \Rightarrow \quad u = c\sqrt{1 - \left(mc^2/E\right)^2}$$

$$v = \left(3.00 \times 10^8\ \text{m/s}\right)\sqrt{1 - \left(\frac{0.938\ \text{GeV}}{50.938\ \text{GeV}}\right)^2} = \boxed{2.9995 \times 10^8\ \text{m/s}}$$

39.41 (a) $q(\Delta V) = K = (\gamma - 1)m_e c^2$

Thus, $\gamma = \frac{1}{\sqrt{1-(u/c)^2}} = 1 + \frac{q(\Delta V)}{m_e c^2}$ from which $\boxed{u = 0.302c}$

(b) $K = (\gamma - 1)m_e c^2 = q(\Delta V) = \left(1.60\times 10^{-19}\ \text{C}\right)\left(2.50\times 10^4\ \text{J/C}\right) = \boxed{4.00\times 10^{-15}\ \text{J}}$

39.42 (a) $E = \gamma mc^2 = 20.0\ \text{GeV}$ with $mc^2 = 0.511\ \text{MeV}$ for electrons. Thus, $\gamma = \frac{20.0\times 10^9\ \text{eV}}{0.511\times 10^6\ \text{eV}} = \boxed{3.91\times 10^4}$

(b) $\gamma = \frac{1}{\sqrt{1-(u/c)^2}} = 3.91\times 10^4$ from which $\boxed{u = 0.999\,999\,999\,7c}$

(c) $L = L_p\sqrt{1-(u/c)^2} = \frac{L_p}{\gamma} = \frac{3.00\times 10^3\ \text{m}}{3.91\times 10^4} = 7.67\times 10^{-2}\ \text{m} = \boxed{7.67\ \text{cm}}$

39.43 Conserving total momentum, $p_{\text{Before decay}} = p_{\text{after decay}} = 0$: $p_\nu = p_\mu = \gamma m_\mu u = \gamma\left(206\,m_e\right)u$

Conservation of mass-energy gives: $E_\mu + E_\nu = E_\pi$

$$\gamma m_\mu c^2 + p_\nu c = m_\pi c^2$$

$$\gamma\left(206\,m_e\right) + \frac{p_\nu}{c} = 270\,m_e$$

Substituting from the momentum equation above, $\gamma\left(206\,m_e\right) + \gamma\left(206\,m_e\right)\frac{u}{c} = 270\,m_e$

or $\gamma\left(1 + \frac{u}{c}\right) = \frac{270}{206} = 1.31 \quad \Rightarrow \quad \frac{u}{c} = 0.264$

Then, $K_\mu = (\gamma - 1)m_\mu c^2 = (\gamma - 1)206\left(m_e c^2\right) = \left(\frac{1}{\sqrt{1-(0.264)^2}} - 1\right)206(0.511\ \text{MeV}) = \boxed{3.88\ \text{MeV}}$

Also, $E_\nu = E_\pi - E_\mu = m_\pi c^2 - \gamma m_\mu c^2 = \left(270 - 206\gamma\right)m_e c^2$

$$E_\nu = \left(270 - \frac{206}{\sqrt{1-(0.264)^2}}\right)(0.511\ \text{MeV}) = \boxed{28.8\ \text{MeV}}$$

***39.44** Let a 0.3-kg flag be run up a flagpole 7 m high.

We put into it energy $mgh = 0.3\ \text{kg}(9.8\ \text{m/s}^2)\ 7\ \text{m} \approx 20\ \text{J}$

So we put into it extra mass $\Delta m = \dfrac{E}{c^2} = \dfrac{20\text{ J}}{(3\times 10^8\text{ m/s})^2} = 2\times 10^{-16}\text{ kg}$

for a fractional increase of $\dfrac{2\times 10^{16}\text{ kg}}{0.3\text{ kg}}$ $\boxed{\sim 10^{-15}}$

***39.45** $E = 2.86\times 10^5$ J. Also, the mass-energy relation says that $E = mc^2$.

Therefore, $m = \dfrac{E}{c^2} = \dfrac{2.86\times 10^5\text{ J}}{(3.00\times 10^8\text{ m/s})^2} = \boxed{3.18\times 10^{-12}\text{ kg}}$

No, a mass loss of this magnitude (out of a total of 9.00 g) $\boxed{\text{could not be detected}}$.

39.46 (a) $K = (\gamma - 1)mc^2 = \left(\dfrac{1}{\sqrt{1-u^2/c^2}} - 1\right)mc^2 = 0.25\,mc^2 = \boxed{2.25\times 10^{22}\text{ J}}$

(b) $E = m_{\text{fuel}}\,c^2$ so $m_{\text{fuel}} = \dfrac{2.25\times 10^{22}}{9.00\times 10^{16}} = \boxed{2.50\times 10^5\text{ kg}}$

39.47 $\Delta m = \dfrac{E}{c^2} = \dfrac{\mathcal{P}\,t}{c^2} = \dfrac{0.800\left(1.00\times 10^9\text{ J/s}\right)\left(3.00\text{ yr}\right)\left(3.16\times 10^7\text{ s/yr}\right)}{\left(3.00\times 10^8\text{ m/s}\right)^2} = \boxed{0.842\text{ kg}}$

39.48 Since the total momentum is zero before decay, it is necessary that after the decay

$$p_{\text{nucleus}} = p_{\text{photon}} = \frac{E_\gamma}{c} = \frac{14.0\text{ keV}}{c}$$

Also, for the recoiling nucleus, $E^2 = p^2c^2 + \left(mc^2\right)^2$ with $mc^2 = 8.60\times 10^{-9}\text{ J} = 53.8\text{ GeV}$

Thus, $\left(mc^2 + K\right)^2 = (14.0\text{ keV})^2 + \left(mc^2\right)^2$ or $\left(1 + \dfrac{K}{mc^2}\right)^2 = \left(\dfrac{14.0\text{ keV}}{mc^2}\right)^2 + 1$

So $1 + \dfrac{K}{mc^2} = \sqrt{1 + \left(\dfrac{14.0\text{ keV}}{mc^2}\right)^2} \approx 1 + \dfrac{1}{2}\left(\dfrac{14.0\text{ keV}}{mc^2}\right)^2$ (Binomial Theorem)

and $K \approx \dfrac{(14.0\text{ keV})^2}{2\,mc^2} = \dfrac{\left(14.0\times 10^3\text{ eV}\right)^2}{2\left(53.8\times 10^9\text{ eV}\right)} = \boxed{1.82\times 10^{-3}\text{ eV}}$

39.49 $\mathcal{P} = \dfrac{dE}{dt} = \dfrac{d\left(mc^2\right)}{dt} = c^2\dfrac{dm}{dt} = 3.77\times 10^{26}\text{ W}$

Thus, $\frac{dm}{dt} = \frac{3.77 \times 10^{26} \text{ J/s}}{\left(3.00 \times 10^{8} \text{ m/s}\right)^2} = \boxed{4.19 \times 10^{9} \text{ kg/s}}$

39.50 $2m_e c^2 = 1.02 \text{ MeV}:$ $E_\gamma \geq \boxed{1.02 \text{ MeV}}$

39.51 The moving observer sees the charge as stationary, so she says it feels no magnetic force.

$q(\mathbf{E} + \mathbf{v} \times \mathbf{B}) = q(\mathbf{E}' + 0)$ and $\boxed{\mathbf{E}' = \mathbf{E} + \mathbf{v} \times \mathbf{B}}$

***39.52** (a) When $K_e = K_p$, $m_e c^2\left(\gamma_e - 1\right) = m_p c^2\left(\gamma_p - 1\right)$

In this case, $m_e c^2 = 0.511 \text{ MeV}$, $m_p c^2 = 938 \text{ MeV}$ and $\gamma_e = \left[1 - (0.750)^2\right]^{-1/2} = 1.5119$

Substituting, $\gamma_p = 1 + \frac{m_e c^2 (\gamma_e - 1)}{m_p c^2} = 1 + \frac{(0.511 \text{ MeV})(1.5119 - 1)}{938 \text{ MeV}} = 1.000279$

but $\gamma_p = \frac{1}{\left[1 - \left(u_p/c\right)^2\right]^{1/2}}$. Therefore, $u_p = c\sqrt{1 - \gamma_p^{-2}} = \boxed{0.0236c}$

(b) When $p_e = p_p$, $\gamma_p m_p u_p = \gamma_e m_e u_e$ or $\gamma_p u_p = \frac{\gamma_e m_e u_e}{m_p}$.

Thus, $\gamma_p u_p = \frac{(1.5119)\left(0.511 \text{ MeV}/c^2\right)\left(0.750c\right)}{938 \text{ MeV}/c^2} = 6.1772 \times 10^{-4} c$

and $\frac{u_p}{c} = 6.1772 \times 10^{-4} \sqrt{1 - \left(\frac{u_p}{c}\right)^2}$ which yields $u_p = \boxed{6.18 \times 10^{-4} c} = 185 \text{ km/s}$

39.53 (a) $10^{13} \text{ MeV} = (\gamma - 1) m_p c^2$ so $\gamma \approx 10^{10}$ $v_p \approx c$

$t' = \frac{t}{\gamma} = \frac{10^5 \text{ yr}}{10^{10}} = 10^{-5} \text{ yr} \sim \boxed{10^2 \text{ s}}$

(b) $d' = ct'$ $\boxed{\sim 10^{11} \text{ m}}$

Goal Solution

The cosmic rays of highest energy are protons, which have kinetic energy on the order of 10^{13} MeV. (a) How long would it take a proton of this energy to travel across the Milky Way galaxy, having a diameter on the order of ~10^5 light-years, as measured in the proton's frame? (b) From the point of view of the proton, how many kilometers across is the galaxy?

G: We can guess that the energetic cosmic rays will be traveling close to the speed of light, so the time it takes a proton to traverse the Milky Way will be much less in the proton's frame than 10^5 years. The galaxy will also appear smaller to the high-speed protons than the galaxy's proper diameter of 10^5 light-years.

O: The kinetic energy of the protons can be used to determine the relativistic γ-factor, which can then be applied to the time dilation and length contraction equations to find the time and distance in the proton's frame of reference.

A: The relativistic kinetic energy of a proton is $K = (\gamma - 1)mc^2 = 10^{13}$ MeV

Its rest energy is $$mc^2 = \left(1.67 \times 10^{-27}\text{ kg}\right)\left(2.998 \times 10^8 \ \frac{\text{m}}{\text{s}}\right)^2\left(\frac{1\text{ eV}}{1.60 \times 10^{-19}\text{ kg} \cdot \text{m}^2/\text{s}^2}\right) = 938\text{ MeV}$$

So $10^{13}\text{ MeV} = (\gamma - 1)(938\text{ MeV})$, and therefore $\gamma = 1.07 \times 10^{10}$

The proton's speed in the galaxy's reference frame can be found from $\gamma = 1\Big/\sqrt{1 - v^2/c^2}$:

$$1 - v^2/c^2 = 8.80 \times 10^{-21} \quad \text{and} \quad v = c\sqrt{1 - 8.80 \times 10^{-21}} = \left(1 - 4.40 \times 10^{-21}\right)c \approx 3.00 \times 10^8\text{ m/s}$$

The proton's speed is nearly as large as the speed of light. In the galaxy frame, the traversal time is $\Delta t = x/v = 10^5\text{ light-years}/c = 10^5\text{ years}$

(a) This is dilated from the proper time measured in the proton's frame. The proper time is found from $\Delta t = \gamma \Delta t_p$:

$$\Delta t_p = \Delta t/\gamma = 10^5\text{ yr}\big/1.07 \times 10^{10} = 9.38 \times 10^{-6}\text{ years} = 296\text{ s} \sim \text{a few hundred seconds}$$

(b) The proton sees the galaxy moving by at a speed nearly equal to c, passing in 296 s:

$$\Delta L_p = v\Delta t_p = \left(3.00 \times 10^8\right)(296\text{ s}) = 8.88 \times 10^7\text{ km} \sim 10^8\text{ km}$$

$$\Delta L_p = \left(8.88 \times 10^{10}\text{ m}\right)\left(9.46 \times 10^{15}\text{ m/ly}\right) = 9.39 \times 10^{-6}\text{ ly} \sim 10^{-5}\text{ ly}$$

L: The results agree with our predictions, although we may not have guessed that the protons would be traveling so close to the speed of light! The calculated results should be rounded to zero significant figures since we were given order of magnitude data. We should also note that the relative speed of motion v and the value of γ are the same in both the proton and galaxy reference frames.

39.54 Take the primed frame as:

(a) The mother ship: $u_x = \dfrac{u'_{x'} + v}{1 + u'_{x'}v/c^2} = \dfrac{v+v}{1+v^2/c^2} = \dfrac{2v}{1+v^2/c^2} = \dfrac{2(0.500c)}{1+(0.500)^2} = \boxed{0.800c}$

(b) The shuttle: $u_x = \dfrac{v + \dfrac{2v}{1+v^2/c^2}}{1 + \dfrac{v}{c^2}\left(\dfrac{2v}{1+v^2/c^2}\right)} = \dfrac{3v + v^3/c^2}{1+3v^2/c^2} = \dfrac{3(0.500c)+(0.500c)^3/c^2}{1+3(0.500)^2} = \boxed{0.929c}$

39.55 $\dfrac{\Delta mc^2}{mc^2} = \dfrac{4(938.78\text{ MeV}) - 3728.4\text{ MeV}}{4(938.78\text{ MeV})} \times 100\% = \boxed{0.712\%}$

39.56 $d_{\text{earth}} = vt_{\text{earth}} = v\gamma t_{\text{astro}}$ so $2.00\times10^6\text{ yr}\cdot c = v\dfrac{1}{\sqrt{1-v^2/c^2}}30.0\text{ yr}$

$\sqrt{1-v^2/c^2} = (v/c)(1.50\times10^{-5})$ $\qquad 1 - \dfrac{v^2}{c^2} = \dfrac{v^2}{c^2}(2.25\times10^{-10})$

$1 = \dfrac{v^2}{c^2}(1+2.25\times10^{-10})$ so $\dfrac{v}{c} = \left(1+2.25\times10^{-10}\right)^{-1/2} = 1 - \tfrac{1}{2}(2.25\times10^{-10})$

$\boxed{\dfrac{v}{c} = 1 - 1.12\times10^{-10}}$

***39.57** (a) Take the spaceship as the primed frame, moving toward the right at $v = +0.600c$. Then $u'_x = +0.800c$, and

$$u_x = \frac{u'_x + v}{1+(u'_x v)/c^2} = \frac{0.800c + 0.600c}{1+(0.800)(0.600)} = \boxed{0.946c}$$

(b) $L = \dfrac{L_p}{\gamma} = (0.200\text{ ly})\sqrt{1-(0.600)^2} = \boxed{0.160\text{ ly}}$

(c) The aliens observe the 0.160-ly distance closing because the probe nibbles into it from one end at $0.800c$ and the Earth reduces it at the other end at $0.600c$. Thus,

$$\text{time} = \frac{0.160\text{ ly}}{0.800c + 0.600c} = \boxed{0.114\text{ yr}}$$

(d) $K = \left(\dfrac{1}{\sqrt{1-u^2/c^2}} - 1\right)mc^2 = \left(\dfrac{1}{\sqrt{1-(0.946)^2}} - 1\right)(4.00\times10^5\text{ kg})(3.00\times10^8\text{ m/s})^2 = \boxed{7.50\times10^{22}\text{ J}}$

39.58 In this case, the proper time is T_0 (the time measured by the students on a clock at rest relative to them). The dilated time measured by the professor is: $\Delta t = \gamma T_0$

where $\Delta t = T + t$. Here T is the time she waits before sending a signal and t is the time required for the signal to reach the students.

Thus, we have: $T + t = \gamma T_0$ (1)

To determine the travel time t, realize that the distance the students will have moved beyond the professor before the signal reaches them is: $d = v(T + t)$

The time required for the signal to travel this distance is: $t = \dfrac{d}{c} = \left(\dfrac{v}{c}\right)(T + t)$

Solving for t gives: $t = \dfrac{(v/c)T}{1-(v/c)}$

Substituting this into equation (1) yields: $T + \dfrac{(v/c)T}{1-(v/c)} = \gamma T_0$

or $T = (1 - v/c)^{-1} = \gamma T_0$

Then $T = T_0 \dfrac{1-(v/c)}{\sqrt{1-(v^2/c^2)}} = T_0 \dfrac{1-(v\,/\,c)}{\sqrt{[1+(v\,/\,c)][1-(v\,/\,c)]}} = \boxed{T_0\sqrt{\dfrac{1-(v/c)}{1+(v/c)}}}$

39.59 Look at the situation from the instructor's viewpoint since they are at rest relative to the clock, and hence measure the proper time. The Earth moves with velocity $v = -0.280c$ relative to the instructors while the students move with a velocity $u' = -0.600c$ relative to Earth. Using the velocity addition equation, the velocity of the students relative to the instructors (and hence the clock) is:

$$u = \frac{v + u'}{1 + vu'/c^2} = \frac{(-0.280c) - (0.600c)}{1 + (-0.280c)(-0.600c)/c^2} = -0.753c \text{ (students relative to clock)}$$

(a) With a proper time interval of $\Delta t_p = 50.0$ min, the time interval measured by the students is:

$\Delta t = \gamma \Delta t_p$ with $\gamma = \dfrac{1}{\sqrt{1-(0.753c)^2\,/\,c^2}} = 1.52$

Thus, the students measure the exam to last $T = 1.52(50.0 \text{ min}) = \boxed{76.0 \text{ minutes}}$

(b) The duration of the exam as measured by observers on Earth is:

$\Delta t = \gamma \Delta t_p$ with $\gamma = \dfrac{1}{\sqrt{1-(0.280c)^2/c^2}}$ so $T = 1.04(50.0 \text{ min}) = \boxed{52.1 \text{ minutes}}$

***39.60** The energy which arrives in one year is $E = \mathcal{P}t = \left(1.79\times10^{17}\text{ J / s}\right)\left(3.16\times10^{7}\text{ s}\right) = 5.66\times10^{24}\text{ J}$

Thus, $m = \dfrac{E}{c^2} = \dfrac{5.66\times10^{24}\text{ J}}{\left(3.00\times10^{8}\text{ m / s}\right)^2} = \boxed{6.28\times10^{7}\text{ kg}}$

***39.61** The observer sees the proper length of the tunnel, 50.0 m, but sees the train contracted to length

$L = L_p\sqrt{1 - v^2/c^2} = 100\text{ m}\sqrt{1-(0.950)^2} = 31.2\text{ m}$

shorter than the tunnel by $50.0 - 31.2 = \boxed{18.8\text{ m}}$ so it is completely within the tunnel.

***39.62** If the energy required to remove a mass m from the surface is equal to its mass energy mc^2, then

$$\frac{GM_sm}{R_g} = mc^2$$

and $$R_g = \frac{GM_s}{c^2} = \frac{(6.67\times10^{-11}\text{ N}\cdot\text{m}^2\text{ / kg}^2)(1.99\times10^{30}\text{ kg})}{(3.00\times10^{8}\text{ m / s})^2} = 1.47\times10^{3}\text{ m} = \boxed{1.47\text{ km}}$$

39.63 (a) At any speed, the momentum of the particle is given by $p = \gamma mu = \dfrac{mu}{\sqrt{1-(u/c)^2}}$

Since $F = qE = \dfrac{dp}{dt}$

$$qE = \frac{d}{dt}\left[mu\left(1-u^2/c^2\right)^{-1/2}\right]$$

$$qE = m\left(1-u^2/c^2\right)^{-1/2}\frac{du}{dt} + \frac{1}{2}mu\left(1-u^2/c^2\right)^{-3/2}\left(2u/c^2\right)\frac{du}{dt}$$

So $\dfrac{qE}{m} = \dfrac{du}{dt}\left[\dfrac{1-u^2/c^2+u^2/c^2}{\left(1-u^2/c^2\right)^{3/2}}\right]$ and $\boxed{a = \dfrac{du}{dt} = \dfrac{qE}{m}\left(1-\dfrac{u^2}{c^2}\right)^{3/2}}$

(b) As $u \to c$, $\boxed{a \to 0}$

(c) $\displaystyle\int_0^v \frac{du}{\left(1-u^2/c^2\right)^{3/2}} = \int_{t=0}^{t}\frac{qE}{m}dt$ so $\boxed{u = \dfrac{qEct}{\sqrt{m^2c^2+q^2E^2t^2}}}$

$$x = \int_0^t u\,dt = qEc\int_0^t \frac{t\,dt}{\sqrt{m^2c^2+q^2E^2t^2}} = \boxed{\frac{c}{qE}\left(\sqrt{m^2c^2+q^2E^2t^2} - mc\right)}$$

***39.64** (a) $f_{\text{observed}} = f_{\text{source}}\sqrt{\dfrac{1+v/c}{1-v/c}}$ implies $\dfrac{c}{\lambda+\Delta\lambda} = \dfrac{c}{\lambda}\sqrt{\dfrac{1+v/c}{1-v/c}}$,

or $$\sqrt{\frac{1-v/c}{1+v/c}} = \frac{\lambda+\Delta\lambda}{\lambda}$$

and $$\boxed{1+\frac{\Delta\lambda}{\lambda} = \sqrt{\frac{1-v/c}{1+v/c}}}$$

(b) $$1+\frac{550\text{ nm}-650\text{ nm}}{650\text{ nm}} = \sqrt{\frac{1-v/c}{1+v/c}} = 0.846$$

$$1-\frac{v}{c} = (0.846)^2\left(1+\frac{v}{c}\right) = 0.716+0.716\left(\frac{v}{c}\right)$$

$$v = 0.166c = \boxed{4.97\times10^{7}\text{ m/s}}$$

39.65 (a) An observer at rest relative to the mirror sees the light travel a distance

$$D = 2d - x = 2\left(1.80\times10^{12}\text{ m}\right) - (0.800c)t$$

where $x = (0.800c)t$ is the distance the ship moves toward the mirror in time t. Since this observer agrees that the speed of light is c, the time for it to travel distance D is:

$$t = \frac{D}{c} = \frac{2(1.80\times10^{12}\text{ m})}{3.00\times10^{8}\text{ m / s}} - 0.800t = \boxed{6.67\times10^{3}\text{ s}}$$

(b) The observer in the rocket sees a length-contracted initial distance to the mirror of:

$$L = d\sqrt{1-\frac{v^2}{c^2}} = \left(1.80\times10^{12}\text{ m}\right)\sqrt{1-\frac{(0.800c)^2}{c^2}} = 1.08\times10^{12}\text{ m},$$

and the mirror moving toward the ship at speed $v = 0.800c$. Thus, he measures the distance the light travels as:

$$D = 2\left(1.08\times10^{12}\text{ m} - y\right)$$

where $y = (0.800c)(t/2)$ is the distance the mirror moves toward the ship before the light reflects off it. This observer also measures the speed of light to be c, so the time for it to travel distance D is:

$$t = \frac{D}{c} = \frac{2}{c}\left[1.08\times10^{12}\text{ m} - (0.800c)\frac{t}{2}\right],\ \text{which gives } t = \boxed{4.00\times10^{3}\text{ s}}$$

39.66 (a) An observer at rest relative to the mirror sees the light travel a distance $D = 2d - x$, where $x = vt$ is the distance the ship moves toward the mirror in time t. Since this observer agrees that the speed of light is c, the time for it to travel distance D is

$$t = \frac{D}{c} = \frac{2d - vt}{c} = \boxed{\frac{2d}{c+v}}$$

(b) The observer in the rocket sees a length-contracted initial distance to the mirror of

$$L = d\sqrt{1 - \frac{v^2}{c^2}}$$

and the mirror moving toward the ship at speed v. Thus, he measures the distance the light travels as

$$D = 2(L - y)$$

where $y = vt/2$ is the distance the mirror moves toward the ship before the light reflects off it. This observer also measures the speed of light to be c, so the time for it to travel distance D is:

$$t = \frac{D}{c} = \frac{2}{c}\left(d\sqrt{1 - \frac{v^2}{c^2}} - \frac{vt}{2}\right) \qquad \text{so} \qquad (c+v)t = \frac{2d}{c}\sqrt{(c+v)(c-v)} \qquad \text{or} \qquad \boxed{t = \frac{2d}{c}\sqrt{\frac{c-v}{c+v}}}$$

39.67 (a) Since Mary is in the same reference frame, S′, as Ted, she observes the ball to have the same speed Ted observes, namely $|u_x'| = \boxed{0.800c}$.

(b) $$\Delta t' = \frac{L_p}{|u_x'|} = \frac{1.80 \times 10^{12}\text{ m}}{0.800(3.00 \times 10^{8}\text{ m/s})} = \boxed{7.50 \times 10^{3}\text{ s}}$$

(c) $$L = L_p\sqrt{1 - \frac{v^2}{c^2}} = (1.80 \times 10^{12}\text{ m})\sqrt{1 - \frac{(0.600c)^2}{c^2}} = \boxed{1.44 \times 10^{12}\text{ m}}$$

Since $v = 0.600c$ and $u_x' = -0.800c$, the velocity Jim measures for the ball is

$$u_x = \frac{u_x' + v}{1 + u_x' v/c^2} = \frac{(-0.800c) + (0.600c)}{1 + (-0.800)(0.600)} = \boxed{-0.385c}$$

(d) Jim observes the ball and Mary to be initially separated by 1.44×10^{12} m. Mary's motion at $0.600c$ and the ball's motion at $0.385c$ nibble into thi distance from both ends. The gap closes at the rate $0.600c + 0.385c = 0.985c$, so the ball and catcher meet after a time

$$\Delta t = \frac{1.44 \times 10^{12}\text{ m}}{0.985(3.00 \times 10^{18}\text{ m / s})} = \boxed{4.88 \times 10^{3}\text{ s}}$$

39.68 (a) $L_0^2 = L_{0x}^2 + L_{0y}^2$ and $L^2 = L_x^2 + L_y^2$

The motion is in the x direction: $L_y = L_{0y} = L_0 \sin\theta_0$

$$L_x = L_{0x}\sqrt{1-(v/c)^2} = (L_0 \cos\theta_0)\sqrt{1-(v/c)^2}$$

Thus, $$L^2 = L_0{}^2 \cos^2\theta_0\left[1-\left(\frac{v}{c}\right)^2\right] + L_0{}^2 \sin^2\theta_0 = L_0{}^2\left[1-\left(\frac{v}{c}\right)^2\cos^2\theta_0\right]$$

or $$\boxed{L = L_0\left[1-(v/c)^2\cos^2\theta_0\right]^{1/2}}$$

(b) $$\tan\theta = \frac{L_y}{L_x} = \frac{L_{0y}}{L_{0x}\sqrt{1-(v/c)^2}} = \boxed{\gamma\tan\theta_0}$$

39.69 (a) First, we find the velocity of the stick relative to S′ using $L = L_p\sqrt{1-(u_x')^2/c^2}$

Thus $$u_x' = \pm\, c\sqrt{1-(L/L_p)^2}$$

Selecting the negative sign because the stick moves in the negative x direction in S′ gives:

$$u_x' = -c\sqrt{1-\left(\frac{0.500\text{ m}}{1.00\text{ m}}\right)^2} = -0.866c$$ so the speed is $|u_x'| = \boxed{0.866c}$

Now determine the velocity of the stick relative to S, using the measured velocity of the stick relative to S′ and the velocity of S′ relative to S. From the velocity addition equation, we have:

$$u_x = \frac{u_x' + v}{1+vu_x'/c^2} = \frac{(-0.866c)+(0.600c)}{1+(0.600c)(-0.866c)} = -0.554c$$ and the speed is $|u_x| = \boxed{0.554c}$

(b) Therefore, the contracted length of the stick as measured in S is:

$$L = L_p\sqrt{1-(u_x/c)^2} = (1.00\text{ m})\sqrt{1-(0.554)^2} = \boxed{0.833\text{ m}}$$

39.70 (b) Consider a hermit who lives on an asteroid halfway between the Sun and Tau Ceti, stationary with respect to both. Just as our spaceship is passing him, he also sees the blast waves from both explosions. Judging both stars to be stationary, this observer concludes that $\boxed{\text{the two stars blew up simultaneously}}$.

(a) We in the spaceship moving past the hermit do not calculate the explosions to be simultaneous. We see the distance we have traveled from the Sun as

$$L = L_p\sqrt{1-(v/c)^2} = (6.00\text{ ly})\sqrt{1-(0.800)^2} = 3.60\text{ ly}$$

We see the Sun flying away from us at $0.800c$ while the light from the Sun approaches at $1.00c$. Thus, the gap between the Sun and its blast wave has opened at $1.80c$, and the time we calculate to have elapsed since the Sun exploded is

$$3.60\text{ ly}/1.80c = 2.00\text{ yr}.$$

We see Tau Ceti as moving toward us at $0.800c$, while its light approaches at $1.00c$, only $0.200c$ faster. We see the gap between that star and its blast wave as 3.60 ly and growing at $0.200c$. We calculate that it must have been opening for

$$3.60\text{ ly}/0.200c = 18.0\text{ yr}$$

and conclude that $\boxed{\text{Tau Ceti exploded 16.0 years before the Sun}}$.

***39.71** The unshifted frequency is $f_{\text{source}} = \frac{c}{\lambda} = \frac{3.00\times10^8\text{ m/s}}{394\times10^{-9}\text{ m}} = 7.61\times10^{14}\text{ Hz}$

We observe frequency $f = \frac{3.00\times10^8\text{ m/s}}{475\times10^{-9}\text{ m}} = 6.32\times10^{14}\text{ Hz}$

Then $f = f_{\text{source}}\sqrt{\frac{1+v/c}{1-v/c}}$

gives: $6.32 = 7.61\sqrt{\frac{1+v/c}{1-v/c}}$

or $\frac{1+v/c}{1-v/c} = (0.829)^2$

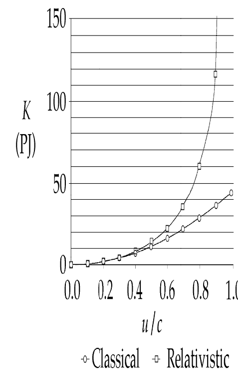
Solving for v yields: $v = -0.185c = \boxed{0.185c\ (\text{away})}$

39.72 Take m = 1.00 kg.

The classical kinetic energy is $K_c = \frac{1}{2}mu^2 = \frac{1}{2}mc^2\left(\frac{u}{c}\right)^2 = \left(4.50\times10^{16}\text{ J}\right)\left(\frac{u}{c}\right)^2$

and the actual kinetic energy is $K_r = \left(\frac{1}{\sqrt{1-(u/c)^2}} - 1\right)mc^2 = \left(9.00\times10^{16}\text{ J}\right)\left(\frac{1}{\sqrt{1-(u/c)^2}} - 1\right)$

u/c	K_c (J)	K_r (J)
0.000	0.000	0.000
0.100	0.045×10^{16}	0.0453×10^{16}
0.200	0.180×10^{16}	0.186×10^{16}
0.300	0.405×10^{16}	0.435×10^{16}
0.400	0.720×10^{16}	0.820×10^{16}
0.500	1.13×10^{16}	1.39×10^{16}
0.600	1.62×10^{16}	2.25×10^{16}
0.700	2.21×10^{16}	3.60×10^{16}
0.800	2.88×10^{16}	6.00×10^{16}
0.900	3.65×10^{16}	11.6×10^{16}
0.990	4.41×10^{16}	54.8×10^{16}



$K_c = 0.990K_r$ when $\frac{1}{2}(u/c)^2 = 0.990\left[\frac{1}{\sqrt{1-(u/c)^2}} - 1\right]$, yielding $u = \boxed{0.115c}$

Similarly, $K_c = 0.950K_r$ when $u = \boxed{0.257c}$

and $K_c = 0.500K_r$ when $u = \boxed{0.786c}$

39.73 $\Delta m = \frac{E}{c^2} = \frac{mc(\Delta T)}{c^2} = \frac{\rho Vc(\Delta T)}{c^2} = \frac{\left(1030\text{ kg / m}^3\right)\left(1.40\times10^9\right)\left(10^3\text{ m}\right)^3\left(4186\text{ J / kg}\cdot{}^\circ\text{C}\right)\left(10.0\ {}^\circ\text{C}\right)}{\left(3.00\times10^8\text{ m / s}\right)^2}$

$\Delta m = \boxed{6.71\times10^8\text{ kg}}$