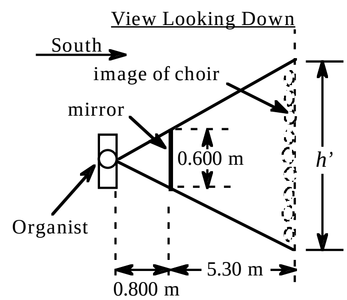
Chapter 36 Solutions

***36.1** I stand 40 cm from my bathroom mirror. I scatter light which travels to the mirror and back to me in time

$$\frac{0.8 \text{ m}}{3 \times 10^8 \text{ m/s}} \boxed{\sim 10^{-9} \text{ s}}$$

showing me a view of myself as I was at that look-back time. I'm no Dorian Gray!

***36.2** The virtual image is as far behind the mirror as the choir is in front of the mirror. Thus, the image is 5.30 m behind the mirror.



The image of the choir is $0.800 \text{ m} + 5.30 \text{ m} = 6.10 \text{ m}$ from the organist. Using similar triangles:

$$\frac{h'}{0.600 \text{ m}} = \frac{6.10 \text{ m}}{0.800 \text{ m}} \quad \text{or} \quad h' = (0.600 \text{ m})\left(\frac{6.10 \text{ m}}{0.800 \text{ m}}\right) = \boxed{4.58 \text{ m}}$$

36.3 The flatness of the mirror is described by $R = \infty$, $f = \infty$, and $1/f = 0$. By our general mirror equation,

$$\frac{1}{p} + \frac{1}{q} = \frac{1}{f}, \quad \text{or} \quad q = -p$$

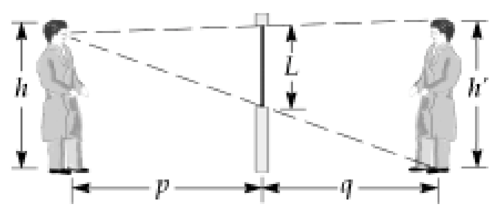


Figure for Goal Solution

Thus, the image is as far behind the mirror as the person is in front. The magnification is then

$$M = \frac{-q}{p} = 1 = \frac{h'}{h} \quad \text{so} \quad h' = h = 70.0"$$

The required height of the mirror is defined by the triangle from the person's eyes to the top and bottom of his image, as shown. From the geometry of the triangle, we see that the mirror height must be:

$$h'\left(\frac{p}{p-q}\right) = h'\left(\frac{p}{2p}\right) = \frac{h'}{2}$$

Thus, the mirror must be $\boxed{\text{at least 35.0" high}}$.

Goal Solution

Determine the minimum height of a vertical flat mirror in which a person 5'10" in height can see his or her full image. (A ray diagram would be helpful.)

G: A diagram with the optical rays that create the image of the person is shown above. From this diagram, it appears that the mirror only needs to be about half the height of the person.

O: The required height of the mirror can be found from the mirror equation, where this flat mirror is described by

$$R = \infty,\ f = \infty, \text{ and } 1/f = 0.$$

A: The general mirror equation is $\frac{1}{p} + \frac{1}{q} = \frac{1}{f}$, so with $f = \infty$, $q = -p$

Thus, the image is as far behind the mirror as the person is in front. The magnification is then

$$M = \frac{-q}{p} = 1 = \frac{h'}{h}$$

so $h' = h = 70.0$ in.

The required height of the mirror is defined by the triangle from the person's eyes to the top and bottom of the image, as shown. From the geometry of the similar triangles, we see that the length of the mirror must be:

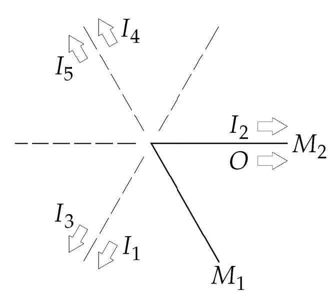
$$L = h'\left(\frac{p}{p-q}\right) = h'\left(\frac{p}{2p}\right) = \frac{h'}{2} = \frac{70.0 \text{ in}}{2} = 35.0 \text{ in}.$$

Thus, the mirror must be at least 35.0 in high.

L: Our result agrees with our prediction from the ray diagram. Evidently, a full-length mirror only needs to be a half-height mirror! On a practical note, the vertical positioning of such a mirror is also important for the person to be able to view his or her full image. To allow for some variation in positioning and viewing by persons of different heights, most full-length mirrors are about 5' in length.

36.4 A graphical construction produces 5 images, with images I_1 and I_2 directly into the mirrors from the object O,

and (O, I_3, I_4) and (I_1, I_2, I_5) forming the vertices of equilateral triangles.



***36.5** (1) The first image in the left mirror is 5.00 ft behind the mirror, or $\boxed{10.0 \text{ ft}}$ from the position of the person.

(2) The first image in the right mirror is located 10.0 ft behind the right mirror, but this location is 25.0 ft from the left mirror. Thus, the second image in the left mirror is 25.0 ft behind the mirror, or $\boxed{30.0 \text{ ft}}$ from the person.

(3) The first image in the left mirror forms an image in the right mirror. This first image is 20.0 ft from the right mirror, and, thus, an image 20.0 ft behind the right mirror is formed. This image in the right mirror also forms an image in the left mirror. The distance from this image in the right mirror to the left mirror is 35.0 ft. The third image in the left mirror is, thus, 35.0 ft behind the mirror, or $\boxed{40.0 \text{ ft}}$ from the person.

***36.6** For a concave mirror, both R and f are positive. We also know that $f = \dfrac{R}{2} = 10.0 \text{ cm}$

(a) $\dfrac{1}{q} = \dfrac{1}{f} - \dfrac{1}{p} = \dfrac{1}{10.0 \text{ cm}} - \dfrac{1}{40.0 \text{ cm}} = \dfrac{3}{40.0 \text{ cm}}$, and $\boxed{q = 13.3 \text{ cm}}$

$M = \dfrac{q}{p} = -\dfrac{13.3 \text{ cm}}{40.0 \text{ cm}} = \boxed{-0.333}$

The image is 13.3 cm in front of the mirror, is $\boxed{\text{real, and inverted}}$.

(b) $\dfrac{1}{q} = \dfrac{1}{f} - \dfrac{1}{p} = \dfrac{1}{10.0 \text{ cm}} - \dfrac{1}{20.0 \text{ cm}} = \dfrac{1}{20.0 \text{ cm}}$, and $\boxed{q = 20.0 \text{ cm}}$

$M = \dfrac{q}{p} = -\dfrac{20.0 \text{ cm}}{20.0 \text{ cm}} = \boxed{-1.00}$

The image is 20.0 cm in front of the mirror, is $\boxed{\text{real, and inverted}}$.

(c) $\dfrac{1}{q} = \dfrac{1}{f} - \dfrac{1}{p} = \dfrac{1}{10.0 \text{ cm}} - \dfrac{1}{10.0 \text{ cm}} = 0$ Thus, q = infinity.

$\boxed{\text{No image is formed.}}$ The rays are reflected parallel to each other.

***36.7** $\dfrac{1}{q} = \dfrac{1}{f} - \dfrac{1}{p} = -\dfrac{1}{0.275 \text{ m}} - \dfrac{1}{10.0 \text{ m}}$ gives $\boxed{q = -0.267 \text{ m}}$

Thus, the image is $\boxed{\text{virtual}}$.

$M = \dfrac{-q}{p} = -\dfrac{-0.267}{10.0 \text{ m}} = \boxed{0.0267}$

Thus, the image is $\boxed{\text{upright}}$ $(+M)$ and $\boxed{\text{diminished}}$ $(|M| < 1)$

***36.8** With radius 2.50 m, the cylindrical wall is a highly efficient mirror for sound, with focal length

$$f = \frac{R}{2} = 1.25 \text{ m}$$

In a vertical plane the sound disperses as usual, but that radiated in a horizontal plane is concentrated in a sound image at distance q from the back of the niche, where

$$\frac{1}{p} + \frac{1}{q} = \frac{1}{f} \quad \text{so} \quad \frac{1}{2.00 \text{ m}} + \frac{1}{q} = \frac{1}{1.25 \text{ m}} \quad q = \boxed{3.33 \text{ m}}$$

36.9 (a) $\frac{1}{p} + \frac{1}{q} = \frac{2}{R}$ gives $\frac{1}{(30.0 \text{ cm})} + \frac{1}{q} = \frac{2}{(-40.0 \text{ cm})}$

$$\frac{1}{q} = -\frac{2}{(40.0 \text{ cm})} - \frac{1}{(30.0 \text{ cm})} = -0.0833 \text{ cm}^{-1} \quad \text{so} \quad q = \boxed{-12.0 \text{ cm}}$$

$$M = \frac{-q}{p} = -\frac{(-12.0 \text{ cm})}{(30.0 \text{ cm})} = \boxed{0.400}$$

(b) $\frac{1}{p} + \frac{1}{q} = \frac{2}{R}$ gives $\frac{1}{(60.0 \text{ cm})} + \frac{1}{q} = -\frac{2}{(40.0 \text{ cm})}$

$$\frac{1}{q} = -\frac{2}{(40.0 \text{ cm})} - \frac{1}{(60.0 \text{ cm})} = -0.0666 \text{ cm}^{-1} \quad \text{so} \quad q = \boxed{-15.0 \text{ cm}}$$

$$M = -\frac{q}{p} = -\frac{(-15.0 \text{ cm})}{(60.0 \text{ cm})} = \boxed{0.250}$$

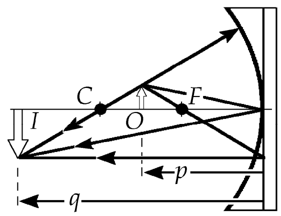
(c) Since $M > 0$, the images are $\boxed{\text{upright}}$.

36.10 (a) $M = -\frac{q}{p}$. For a real image, $q > 0$ so in this case $M = -4.00$

$q = -pM = 120$ cm and from $\frac{1}{p} + \frac{1}{q} = \frac{2}{R}$

$$R = \frac{2pq}{(p+q)} = \frac{2(30.0 \text{ cm})(120 \text{ cm})}{(150 \text{ cm})} = \boxed{48.0 \text{ cm}}$$

(b)



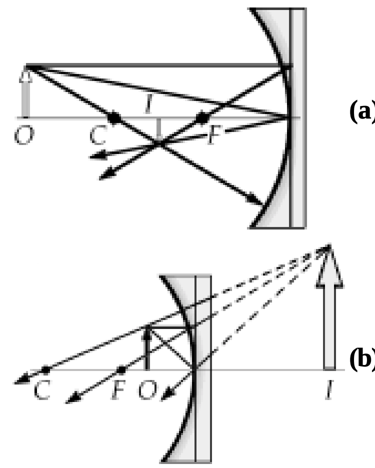
36.11 (a) $\frac{1}{p}+\frac{1}{q}=\frac{2}{R}$ becomes $\frac{1}{q}=\frac{2}{(60.0\text{ cm})}-\frac{1}{(90.0\text{ cm})}$

$q=\boxed{45.0\text{ cm}}$ and $M=\frac{-q}{p}=-\frac{(45.0\text{ cm})}{(90.0\text{ cm})}=\boxed{-0.500}$

(b) $\frac{1}{p}+\frac{1}{q}=\frac{2}{R}$ becomes $\frac{1}{q}=\frac{2}{(60.0\text{ cm})}-\frac{1}{(20.0\text{ cm})}$,

$q=\boxed{-60.0\text{ cm}}$ and $M=-\frac{q}{p}=-\frac{(-60.0\text{ cm})}{(20.0\text{ cm})}=\boxed{3.00}$

(c) The image in (a) is real, inverted and diminished. That of (b) is virtual, upright, and enlarged. The ray diagrams are similar to Figures 36.15(a) and 36.15(b), respectively.



Figures for Goal Solution

Goal Solution

A concave mirror has a radius of curvature of 60.0 cm. Calculate the image position and magnification of an object placed in front of the mirror (a) at a distance of 90.0 cm and (b) at a distance of 20.0 cm. (c) In each case, draw ray diagrams to obtain the image characteristics.

G: It is always a good idea to first draw a ray diagram for any optics problem. This gives a qualitative sense of how the image appears relative to the object. From the ray diagrams above, we see that when the object is 90 cm from the mirror, the image will be real, inverted, diminished, and located about 45 cm in front of the mirror, midway between the center of curvature and the focal point. When the object is 20 cm from the mirror, the image is be virtual, upright, magnified, and located about 50 cm behind the mirror.

O: The mirror equation can be used to find precise quantitative values.

A: (a) The mirror equation is applied using the sign conventions listed in the text.

$$\frac{1}{p}+\frac{1}{q}=\frac{2}{R} \quad \text{or} \quad \frac{1}{90.0\text{ cm}}+\frac{1}{q}=\frac{2}{60.0\text{ cm}} \quad \text{so} \quad q = 45.0\text{ cm (real, in front of the mirror)}$$

$$M=\frac{-q}{p}=-\frac{45.0\text{ cm}}{90.0\text{ cm}}=-0.500 \text{ (inverted)}$$

(b)
$$\frac{1}{p}+\frac{1}{q}=\frac{2}{R} \quad \text{or} \quad \frac{1}{20.0\text{ cm}}+\frac{1}{q}=\frac{2}{60.0\text{ cm}} \quad \text{so} \quad q = -60.0\text{ cm (virtual, behind the mirror)}$$

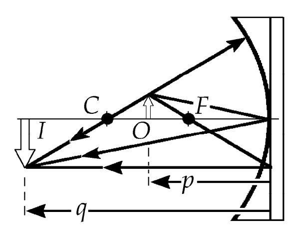
$$M=-\frac{q}{p}=-\frac{-60.0\text{ cm}}{20.0\text{ cm}}=3.00 \text{ (upright)}$$

L: The calculated image characteristics agree well with our predictions. It is easy to miss a minus sign or to make a computational mistake when using the mirror-lens equation, so the qualitative values obtained from the ray diagrams are useful for a check on the reasonableness of the calculated values.

36.12 For a concave mirror, R and f are positive. Also, for an erect image, M is positive. Therefore, $M=-\frac{q}{p}=4$ and $q=-4p$.

$$\frac{1}{f}=\frac{1}{p}+\frac{1}{q} \quad \text{becomes} \quad \frac{1}{40.0\text{ cm}}=\frac{1}{p}-\frac{1}{4p}=\frac{3}{4p}; \quad \text{from which, } p=\boxed{30.0\text{ cm}}$$

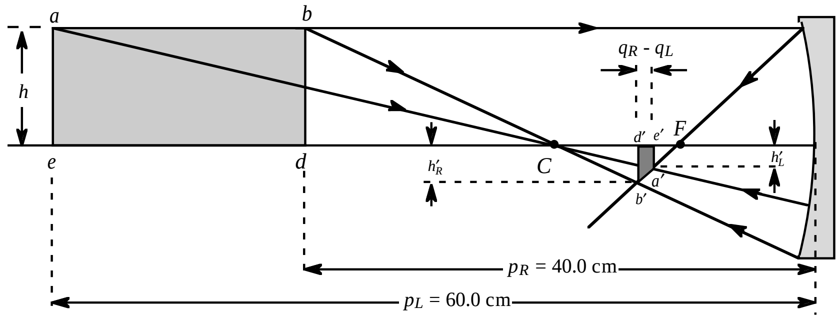
36.13 (a) $q=(p+5.00\text{ m})$ and, since the image must be real, $M=-\frac{q}{p}=-5$ or $q=5p$. Therefore, $p+5.00=5p$ or $p=1.25$ m and $q=6.25$ m.



From $\frac{1}{p}+\frac{1}{q}=\frac{2}{R}$, $R=\frac{2pq}{(q+p)}=\frac{2(1.25)(6.25)}{(6.25+1.25)}=\boxed{2.08\text{ m (concave)}}$

(b) From part (a), $p=1.25$ m; the mirror should be $\boxed{1.25\text{ m}}$ in front of the object.

36.14 (a) The image is the trapezoid $a'b'd'e'$ as shown in the ray diagram.



(b) To find the area of the trapezoid, the image distances, q_R and q_L, along with the heights h'_R and h'_L, must be determined. The mirror equation,

$$\frac{1}{p}+\frac{1}{q}=\frac{2}{R} \quad \text{becomes} \quad \frac{1}{40.0\text{ cm}}+\frac{1}{q_R}=\frac{2}{20.0\text{ cm}} \quad \text{or} \quad q_R = 13.3\text{ cm}$$

$$h_R = hM_R = h\left(\frac{-q_R}{p_R}\right) = (10.0\text{ cm})\left(\frac{-13.3\text{ cm}}{40.0\text{ cm}}\right) = -3.33\text{ cm}$$

Also $$\frac{1}{60.0\text{ cm}}+\frac{1}{q_L}=\frac{2}{20.0\text{ cm}} \quad \text{or} \quad q_L = 12.0\text{ cm}$$

$$h_L = hM_L = (10.0\text{ cm})\left(\frac{-12.0\text{ cm}}{60.0\text{ cm}}\right) = -2.00\text{ cm}$$

The area of the trapezoid is the sum of the area of a square plus the area of a triangle:

$$A_t = A_1 + A_2 = (q_R - q_L)h_L + \frac{1}{2}(q_R - q_L)(h_R - h_L) = \boxed{3.56\text{ cm}^2}$$

36.15 Assume that the object distance is the same in both cases (i.e., her face is the same distance from the hubcap regardless of which way it is turned). Also realize that the near image (q = – 10.0 cm) occurs when using the convex side of the hubcap. Applying the mirror equation to both cases gives:

(concave side: $R = |R|$, $q = -30.0$ cm) $$\frac{1}{p}-\frac{1}{30.0}=\frac{2}{|R|}, \quad \text{or} \quad \frac{2}{|R|}=\frac{30.0\text{ cm}-p}{(30.0\text{ cm})p} \qquad [1]$$

(convex side: $R = -|R|$, $q = -10.0$ cm) $$\frac{1}{p}-\frac{1}{10.0}=-\frac{2}{|R|}, \quad \text{or} \quad \frac{2}{|R|}=\frac{p-10.0\text{ cm}}{(10.0\text{ cm})p} \qquad [2]$$

(a) Equating Equations (1) and (2) gives: $$\frac{30.0\text{ cm}-p}{3.00}=p-10.0\text{ cm} \quad \text{or} \quad p = 15.0\text{ cm}$$ Thus, her face is $\boxed{15.0\text{ cm}}$ from the hubcap.

(b) Using the above result ($p = 15.0$ cm) in Equation [1] gives:

$$\frac{2}{|R|}=\frac{30.0\text{ cm} - 15.0\text{ cm}}{(30.0\text{ cm})(15.0\text{ cm})} \quad \text{or} \quad \frac{2}{|R|}=\frac{1}{30.0\text{ cm}}, \quad \text{and} \quad |R| = 60.0\text{ cm}$$

The radius of the hubcap is $\boxed{60.0\text{ cm}}$.

36.16 $\frac{1}{p}+\frac{1}{q}=\frac{1}{f}$ $f=\frac{R}{2}=-1.50\text{ cm}$

$\boxed{q=-\frac{15.0}{11.0}\text{ cm (behind mirror)}}$ $M=\frac{-q}{p}=\boxed{\frac{1}{11.0}}$

36.17 (a) The image starts from a point whose height above the mirror vertex is given by

$\frac{1}{p}+\frac{1}{q}=\frac{1}{f}=\frac{2}{R}$ $\frac{1}{3.00\text{ m}}+\frac{1}{q}=\frac{1}{0.500\text{ m}}$ Therefore, $q=0.600\text{ m}$

As the ball falls, p decreases and q increases. Ball and image pass when $q_1=p_1$. When this is true,

$\frac{1}{p_1}+\frac{1}{p_1}=\frac{1}{0.500\text{ m}}=\frac{2}{p_1}$ or $p_1=1.00\text{ m}$.

As the ball passes the focal point, the image switches from infinitely far above the mirror to infinitely far below the mirror. As the ball approaches the mirror from above, the virtual image approaches the mirror from below, reaching it together when $p_2=q_2=0$.

(b) The falling ball passes its real image when it has fallen

$3.00\text{ m}-1.00\text{ m}=2.00\text{ m}=\frac{1}{2}gt^2$, or when $t=\sqrt{\frac{2(2.00\text{ m})}{9.80\text{ m/s}^2}}=\boxed{0.639\text{ s}}$.

The ball reaches its virtual image when it has traversed

$3.00\text{ m}-0=3.00\text{ m}=\frac{1}{2}gt^2$, or at $t=\sqrt{\frac{2(3.00\text{ m})}{9.80\text{ m/s}^2}}=\boxed{0.782\text{ s}}$.

36.18 When $R\to\infty$, Equation 36.8 for a spherical surface becomes $q=-p(n_2/n_1)$. We use this to locate the final images of the two surfaces of the glass plate. First, find the image the glass forms of the *bottom* of the plate:

$$q_{B1}=-\left(\frac{1.33}{1.66}\right)(8.00\text{ cm})=-6.41\text{ cm}$$

This virtual image is 6.41 cm below the top surface of the glass or 18.41 cm below the water surface. Next, use this image as an object and locate the image the water forms of the bottom of the plate.

$$q_{B2} = -\left(\frac{1.00}{1.33}\right)(18.41\text{ cm}) = -13.84\text{ cm} \quad \text{or} \quad 13.84\text{ cm below the water surface.}$$

Now find image the water forms of the *top* surface of the glass.

$$q_3 = -\left(\frac{1}{1.33}\right)(12.0\text{ cm}) = -9.02\text{ cm},\text{ or }9.02\text{ cm below the water surface.}$$

Therefore, the apparent thickness of the glass is $\Delta t = 13.84\text{ cm} - 9.02\text{ cm} = \boxed{4.82\text{ cm}}$

36.19 $\frac{n_1}{p} + \frac{n_2}{q} = \frac{n_2 - n_1}{R} = 0$ and $R \to \infty$

$$q = -\frac{n_2}{n_1}p = -\frac{1}{1.309}(50.0\text{ cm}) = -38.2\text{ cm}$$

Thus, the virtual image of the dust speck is $\boxed{\text{38.2 cm below the top surface}}$ of the ice.

***36.20** $\frac{n_1}{p} + \frac{n_2}{q} = \frac{n_2 - n_1}{R}$ so $\frac{1.00}{\infty} + \frac{1.40}{21.0\text{ mm}} = \frac{1.40 - 1.00}{6.00\text{ mm}}$ and $0.0667 = 0.0667$

They agree. $\boxed{\text{The image is inverted, real and diminished.}}$

36.21 From Equation 36.8, $\frac{n_1}{p} + \frac{n_2}{q} = \frac{(n_2 - n_1)}{R}$

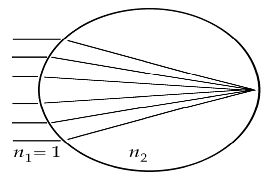
Solve for q to find $q = \frac{n_2 R p}{p(n_2 - n_1) - n_1 R}$

In this case, $n_1 = 1.50$, $n_2 = 1.00$, $R = -15.0$ cm, and $p = 10.0$ cm,

So $q = \frac{(1.00)(-15.0\text{ cm})(10.0\text{ cm})}{(10.0\text{ cm})(1.00 - 1.50) - (1.50)(-15.0\text{ cm})} = -8.57$ cm

Therefore, the $\boxed{\text{apparent depth is 8.57 cm}}$.

36.22 $p = \infty$ and $q = +2R$



$$\frac{1.00}{p} + \frac{n_2}{q} = \frac{n_2 - 1.00}{R}$$

$0 + \frac{n_2}{2R} = \frac{n_2 - 1.00}{R}$ so $\boxed{n_2 = 2.00}$

36.23 $\frac{n_1}{p} + \frac{n_2}{q} = \frac{(n_2 - n_1)}{R}$ because $\frac{1.00}{p} + \frac{1.50}{q} = \frac{1.50 - 1.00}{6.00\text{ cm}} = \frac{1.00}{12.0\text{ cm}}$

(a) $\frac{1}{20.0\text{ cm}} + \frac{1.50}{q} = \frac{1.00}{12.0\text{ cm}}$ or $q = \frac{1.50}{\left[\frac{1.00}{12.0\text{ cm}} - \frac{1.00}{20.0\text{ cm}}\right]} = \boxed{45.0\text{ cm}}$

(b) $\frac{1.00}{10.0\text{ cm}} + \frac{1.50}{q} = \frac{1.00}{12.0\text{ cm}}$ or $q = \frac{1.50}{\left[\frac{1.00}{12.0\text{ cm}} - \frac{1.00}{10.0\text{ cm}}\right]} = \boxed{-90.0\text{ cm}}$

(c) $\frac{1.00}{3.00\text{ cm}} + \frac{1.50}{q} = \frac{1.00}{12.0\text{ cm}}$ or $q = \frac{1.50}{\left[\frac{1.00}{12.0\text{ cm}} - \frac{1.00}{3.00\text{ cm}}\right]} = \boxed{-6.00\text{ cm}}$

36.24 For a plane surface, $\frac{n_1}{p} + \frac{n_2}{q} = \frac{n_2 - n_1}{R}$ becomes $q = -\frac{n_2 p}{n_1}$.

Thus, the magnitudes of the rate of change in the image and object positions are related by

$$\left|\frac{dq}{dt}\right| = \frac{n_2}{n_1}\left|\frac{dp}{dt}\right|$$

If the fish swims toward the wall with a speed of 2.00 cm/s, the speed of the image is given by

$$v_{\text{image}} = \left|\frac{dq}{dt}\right| = \frac{1.00}{1.33}(2.00\text{ cm/s}) = \boxed{1.50\text{ cm/s}}$$

36.25 $\frac{n_1}{p} + \frac{n_2}{q} = \frac{n_2 - n_1}{R}$ $\qquad n_1 = 1.33 \qquad n_2 = 1.00 \qquad p = +10.0 \text{ cm} \qquad R = -15.0 \text{ cm}$

$q = -9.01 \text{ cm}$, or the fish appears to be $\boxed{9.01 \text{ cm inside the bowl}}$

***36.26** Let R_1 = outer radius and R_2 = inner radius

$$\frac{1}{f} = (n-1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right) = (1.50 - 1)\left(\frac{1}{2.00 \text{ m}} - \frac{1}{2.50 \text{ cm}}\right) = \frac{0.0500}{\text{cm}} \qquad \text{so} \qquad f = \boxed{20.0 \text{ cm}}$$

36.27 (a) $\frac{1}{f} = (n-1)\left[\frac{1}{R_1} - \frac{1}{R_2}\right] = (0.440)\left[\frac{1}{(12.0 \text{ cm})} - \frac{1}{(-18.0 \text{ cm})}\right]$: $f = \boxed{16.4 \text{ cm}}$

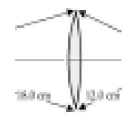


Figure for Goal Solution

(b) $\frac{1}{f} = (0.440)\left[\frac{1}{(18.0 \text{ cm})} - \frac{1}{(-12.0 \text{ cm})}\right]$: $f = \boxed{16.4 \text{ cm}}$

Goal Solution

The left face of a biconvex lens has a radius of curvature of magnitude 12.0 cm, and the right face has a radius of curvature of magnitude 18.0 cm. The index of refraction of the glass is 1.44. (a) Calculate the focal length of the lens. (b) Calculate the focal length if the radii of curvature of the two faces are interchanged.

G: Since this is a biconvex lens, the center is thicker than the edges, and the lens will tend to converge incident light rays. Therefore it has a positive focal length. Exchanging the radii of curvature amounts to turning the lens around so the light enters the opposite side first. However, this does not change the fact that the center of the lens is still thicker than the edges, so we should not expect the focal length of the lens to be different (assuming the thin-lens approximation is valid).

O: The lens makers' equation can be used to find the focal length of this lens.

A: The centers of curvature of the lens surfaces are on opposite sides, so the second surface has a negative radius:

(a) $\frac{1}{f} = (n-1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right) = (1.44 - 1.00)\left(\frac{1}{12.0 \text{ cm}} - \frac{1}{-18.0 \text{ cm}}\right)$ so $f = 16.4 \text{ cm}$

(b) $\frac{1}{f} = (0.440)\left(\frac{1}{18.0 \text{ cm}} - \frac{1}{-12.0 \text{ cm}}\right)$ so $f = 16.4 \text{ cm}$

L: As expected, reversing the orientation of the lens does not change what it does to the light, as long as the lens is relatively thin (variations may be noticed with a thick lens). The fact that light rays can be traced forward or backward through an optical system is sometimes referred to as the **principle of reversibility**. We can see that the focal length of this biconvex lens is about the same magnitude as the average radius of curvature. A few approximations, useful as checks, are that a symmetric biconvex lens with radii of magnitude R will have focal length $f \approx R$; a plano-convex lens with radius R will have $f \approx R/2$; and a symmetric biconcave lens has $f \approx -R$. These approximations apply when the lens has $n \approx 1.5$, which is typical of many types of clear glass and plastic.

***36.28** For a converging lens, f is positive. We use $\frac{1}{p} + \frac{1}{q} = \frac{1}{f}$.

(a) $\frac{1}{q} = \frac{1}{f} - \frac{1}{p} = \frac{1}{20.0\text{ cm}} - \frac{1}{40.0\text{ cm}} = \frac{1}{40.0\text{ cm}}$ $\boxed{q = 40.0\text{ cm}}$

$M = -\frac{q}{p} = -\frac{40.0}{40.0} = \boxed{-1.00}$

The image is $\boxed{\text{real, inverted}}$, and located 40.0 cm past the lens.

(b) $\frac{1}{q} = \frac{1}{f} - \frac{1}{p} = \frac{1}{20.0\text{ cm}} - \frac{1}{20.0\text{ cm}} = 0$ $\boxed{q = \text{infinity}}$

$\boxed{\text{No image}}$ is formed. The rays emerging from the lens are parallel to each other.

(c) $\frac{1}{q} = \frac{1}{f} - \frac{1}{p} = \frac{1}{20.0\text{ cm}} - \frac{1}{10.0\text{ cm}} = -\frac{1}{20.0\text{ cm}}$ $\boxed{q = -20.0\text{ cm}}$

$M = \frac{-q}{p} = -\frac{-20.0}{10.0} = \boxed{2.00}$

The image is $\boxed{\text{upright, virtual}}$, and 20.0 cm in front of the lens.

***36.29** (a) $\frac{1}{q} = \frac{1}{f} - \frac{1}{p} = \frac{1}{25.0\text{ cm}} - \frac{1}{26.0\text{ cm}}$ $q = \boxed{650\text{ cm}}$

The image is $\boxed{\text{real, inverted, and enlarged}}$.

(b) $\frac{1}{q} = \frac{1}{f} - \frac{1}{p} = \frac{1}{25.0\text{ cm}} - \frac{1}{24.0\text{ cm}}$ $q = \boxed{-600\text{ cm}}$

The image is $\boxed{\text{virtual, upright, and enlarged}}$.

36.30 (a) $\frac{1}{p} + \frac{1}{q} = \frac{1}{f} \Rightarrow \frac{1}{(32.0\text{ cm})} + \frac{1}{(8.00\text{ cm})} = \frac{1}{f}$ so $\boxed{f = 6.40\text{ cm}}$

(b) $M = \frac{-q}{p} = \frac{-(8.00\text{ cm})}{(32.00\text{ cm})} = \boxed{-0.250}$

(c) Since $f > 0$, the lens is $\boxed{\text{converging}}$.

36.31 We are looking at an enlarged, upright, virtual image:

$$M = \frac{h'}{h} = 2 = -\frac{q}{p} \qquad \text{so} \qquad p = -\frac{q}{2} = -\frac{-2.84 \text{ cm}}{2} = +1.42 \text{ cm}$$

$$\frac{1}{p} + \frac{1}{q} = \frac{1}{f} \qquad \text{gives} \qquad \frac{1}{1.42 \text{ cm}} + \frac{1}{-2.84 \text{ cm}} = \frac{1}{f}$$

$f = \boxed{2.84 \text{ cm}}$

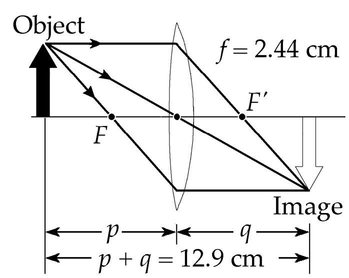
***36.32** To use the lens as a magnifying glass, we form an upright, virtual image:

$$M = +2.00 = \frac{-q}{p} \qquad \text{or} \qquad \frac{1}{p} + \frac{1}{q} = \frac{1}{f}$$

We eliminate $q = -2.00p$::
$$\frac{1}{p} + \frac{1}{-2.00p} = \frac{1}{15.0 \text{ cm}} \qquad \text{or} \qquad \frac{-2.00 + 1.00}{-2.00p} = \frac{1}{15.0 \text{ cm}}$$

Solving, $p = \boxed{7.50 \text{ cm}}$

36.33 (a) Note that $q = 12.9 \text{ cm} - p$



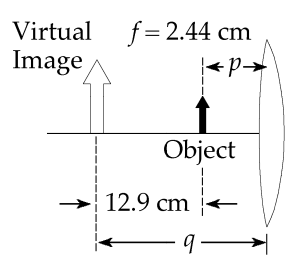
so
$$\frac{1}{p} + \frac{1}{12.9 - p} = \frac{1}{2.44}$$

which yields a quadratic in p: $-p^2 + 12.9p = 31.5$

which has solutions $\boxed{p = 9.63 \text{ cm or } p = 3.27 \text{ cm}}$

Both solutions are valid.

(b) For a virtual image, $-q = p + 12.9 \text{ cm}$



$$\frac{1}{p} - \frac{1}{12.9 + p} = \frac{1}{2.44}$$

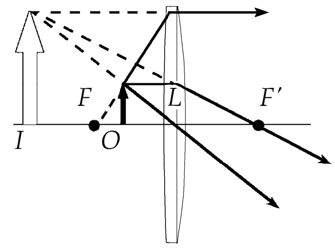
or $p^2 + 12.9p = 31.8$

from which $\boxed{p = 2.10 \text{ cm}}$ or $p = -15.0$ cm.

We must have a real object so the -15.0 cm solution must be rejected.

36.34 (a) $\frac{1}{p} + \frac{1}{q} = \frac{1}{f}:$ $\quad \frac{1}{p} + \frac{1}{-30.0\text{ cm}} = \frac{1}{12.5\text{ cm}}$

$p = 8.82$ cm $\quad M = -\frac{q}{p} = -\frac{(-30.0)}{8.82} = \boxed{3.40\text{, upright}}$



(b) See the figure to the right.

***36.35** $\frac{1}{p} + \frac{1}{q} = \frac{1}{f}:$ $\quad p^{-1} + q^{-1} = \text{constant}$

We may differentiate through with respect to p: $\quad -1p^{-2} - 1q^{-2}\frac{dq}{dp} = 0$

$$\frac{dq}{dp} = -\frac{q^2}{p^2} = -M^2$$

36.36 The image is inverted: $\quad M = \frac{h'}{h} = \frac{-1.80\text{ m}}{0.0240\text{ m}} = -75.0 = \frac{-q}{p}$ $\quad q = 75.0p$

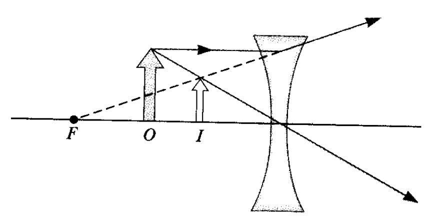
(b) $q + p = 3.00\text{ m} = 75.0p + p$ $\quad p = \boxed{39.5\text{ mm}}$

(a) $q = 2.96$ m $\quad \frac{1}{f} = \frac{1}{p} + \frac{1}{q} = \frac{1}{0.0395\text{ m}} + \frac{1}{2.96\text{ m}}$

$f = \boxed{39.0\text{ mm}}$

36.37 (a) $\frac{1}{p} + \frac{1}{q} = \frac{1}{f}$ $\quad \frac{1}{(20.0\text{ cm})} + \frac{1}{q} = \frac{1}{(-32.0\text{ cm})}$

so $\quad q = -\left(\frac{1}{20.0} + \frac{1}{32.0}\right)^{-1} = \boxed{-12.3\text{ cm}}$



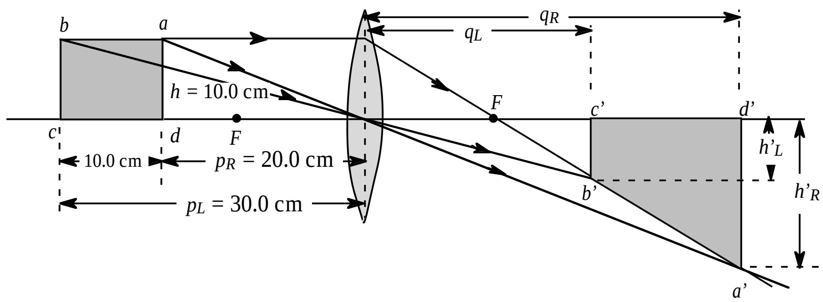
The image is 12.3 cm to the left of the lens.

(b) $M = -\frac{q}{p} = -\frac{(-12.3\text{ cm})}{(20.0\text{ cm})} = \boxed{0.615}$

(c) See the ray diagram above.

36.38 (a) $\frac{1}{f} = (n-1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right) = (1.50-1)\left[\frac{1}{15.0\text{ cm}} - \frac{1}{(-12.0\text{ cm})}\right]$, or $\boxed{f = 13.3\text{ cm}}$

(b) Ray Diagram:



(c) To find the area, first find q_R and q_L, along with the heights h'_R and h'_L, using the thin lens equation.

$\frac{1}{p_R} + \frac{1}{q_R} = \frac{1}{f}$ becomes: $\frac{1}{20.0\text{ cm}} + \frac{1}{q_R} = \frac{1}{13.3\text{ cm}}$ or $q_R = 40.0\text{ cm}$

$$h'_R = hM_R = h\left(\frac{-q_R}{p_R}\right) = (10.0\text{ cm})(-2.00) = -20.0\text{ cm}$$

$\frac{1}{30.0\text{ cm}} + \frac{1}{q_L} = \frac{1}{13.3\text{ cm}}$: $q_L = 24.0\text{ cm}$

$$h'_L = hM_L = (10.0\text{ cm})(-0.800) = -8.00\text{ cm}$$

Thus, the area of the image is: Area = $|q_R - q_L||h'_L| + \frac{1}{2}|q_R - q_L||h'_R - h'_L| = \boxed{224\text{ cm}^2}$

36.39 (a) The distance from the object to the lens is p, so the image distance is $q = 5.00\text{ m} - p$.

Thus, $\frac{1}{p} + \frac{1}{q} = \frac{1}{f}$ becomes: $\frac{1}{p} + \frac{1}{5.00\text{ m} - p} = \frac{1}{0.800\text{ m}}$

This reduces to a quadratic equation: $p^2 - (5.00\text{ m})p + (4.00\text{ m}) = 0$

which yields $\boxed{p = 4.00\text{ m, or } p = 1.00\text{ m}}$.

Thus, there are two possible object distances, both corresponding to real objects.

(b) For $p = 4.00$ m: $q = 5.00\text{ m} - 4.00\text{ m} = 1.00\text{ m}$: $M = -\frac{1.00\text{ m}}{4.00\text{ m}} = \boxed{-0.250}$.

For $p = 1.00$ m: $q = 5.00\text{ m} - 1.00\text{ m} = 4.00\text{ m}$: $M = -\frac{4.00\text{ m}}{1.00\text{ m}} = \boxed{-4.00}$.

Both images are $\boxed{\text{real and inverted}}$, but the magnifications are different, with one being larger than the object and the other smaller.

36.40 (a) The image distance is: $q = d - p$. Thus, $\frac{1}{p} + \frac{1}{q} = \frac{1}{f}$ becomes $\frac{1}{p} + \frac{1}{d-p} = \frac{1}{f}$

This reduces to a quadratic equation: $p^2 + (-d)p + (fd) = 0$

which yields: $\boxed{p = \frac{d \pm \sqrt{d^2 - 4fd}}{2} = \left(\frac{d}{2}\right) \pm \sqrt{\frac{d^2}{4} - fd}}$

Since $f < d/4$, both solutions are meaningful and the two solutions are not equal to each other. Thus, there are two distinct lens positions that form an image on the screen.

(b) The smaller solution for p gives a larger value for q, with a $\boxed{\text{real, enlarged, inverted image}}$.

The larger solution for p describes a $\boxed{\text{real, diminished, inverted image}}$.

***36.41** To properly focus the image of a distant object, the lens must be at a distance equal to the focal length from the film ($q_1 = 65.0$ mm). For the closer object:

$\frac{1}{p_2} + \frac{1}{q_2} = \frac{1}{f}$ becomes $\frac{1}{2000 \text{ mm}} + \frac{1}{q_2} = \frac{1}{65.0 \text{ mm}}$ and $q_2 = (65.0 \text{ mm})\left(\frac{2000}{2000 - 65.0}\right)$

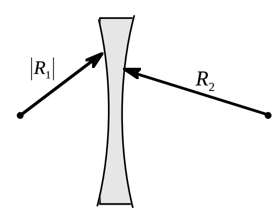
The lens must be moved $\boxed{\text{away from the film}}$ by a distance

$$D = q_2 - q_1 = (65.0 \text{ mm})\left(\frac{2000}{2000 - 65.0}\right) - 65.0 \text{ mm} = \boxed{2.18 \text{ mm}}$$

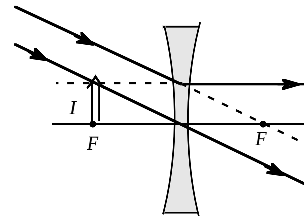
***36.42** (a) The focal length of the lens is given by

$$\frac{1}{f} = (n-1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right) = (1.53 - 1.00)\left(\frac{1}{-32.5 \text{ cm}} - \frac{1}{42.5 \text{ cm}}\right)$$

$f = -34.7$ cm



Note that R_1 is negative because the center of curvature of the first surface is on the virtual image side.



When $p = \infty$, the thin lens equation gives $q = f$. Thus, the violet image of a very distant object is formed at $\boxed{q = -34.7\text{ cm}}$. The image is $\boxed{\text{virtual, upright, and diminished}}$.

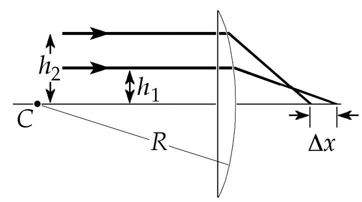
(b) The same ray diagram and image characteristics apply for red light. Again, $q = f$, and now

$$\frac{1}{f} = (1.51 - 1.00)\left(\frac{1}{-32.5\text{ cm}} - \frac{1}{42.5\text{ cm}}\right) \text{ giving } f = \boxed{-36.1\text{ cm}}.$$

36.43 Ray h_1 is undeviated at the plane surface and strikes the second surface at angle of incidence given by

$$\theta_1 = \sin^{-1}\left(\frac{h_1}{R}\right) = \sin^{-1}\left(\frac{0.500\text{ cm}}{20.0\text{ cm}}\right) = 1.43°$$

Then, $(1.00)\sin\theta_2 = (1.60)\sin\theta_1 = (1.60)\left(\frac{0.500}{20.0\text{ cm}}\right)$ so $\theta_2 = 2.29°$



The angle this emerging ray makes with the horizontal is

$$\theta_2 - \theta_1 = 0.860°$$

It crosses the axis at a point farther out by f_1 where

$$f_1 = \frac{h_1}{\tan(\theta_2 - \theta_1)} = \frac{0.500\text{ cm}}{\tan(0.860°)} = 33.3\text{ cm}$$

The point of exit for this ray is distant axially from the lens vertex by

$$20.0\text{ cm} - \sqrt{(20.0\text{ cm})^2 - (0.500\text{ cm})^2} = 0.00625\text{ cm}$$

so ray h_1 crosses the axis at this distance from the vertex:

$$x_1 = 33.3\text{ cm} - 0.00625\text{ cm} = 33.3\text{ cm}$$

Now we repeat this calculation for ray h_2: $\theta_1 = \sin^{-1}\left(\frac{12.0\text{ cm}}{20.0\text{ cm}}\right) = 36.9°$

$$(1.00)\sin\theta_2 = (1.60)\sin\theta_1 = (1.60)\left(\frac{12.00}{20.0}\right) \qquad \theta_2 = 73.7°$$

$$f_2 = \frac{h_2}{\tan(\theta_2 - \theta_1)} = \frac{12.0\text{ cm}}{\tan(36.8°)} = 16.0\text{ cm}$$

$$x_2 = (16.0\text{ cm})\left(20.0\text{ cm} - \sqrt{(20.0\text{ cm})^2 - (12.0\text{ cm})^2}\right) = 12.0\text{ cm}$$

Now $\Delta x = 33.3\text{ cm} - 12.0\text{ cm} = \boxed{21.3\text{ cm}}$

36.44 For starlight going through Nick's glasses, $\frac{1}{p}+\frac{1}{q}=\frac{1}{f}$

$$\frac{1}{\infty}+\frac{1}{(-0.800\text{ m})}=\frac{1}{f}=-1.25\text{ diopters}$$

For a nearby object, $\frac{1}{p}+\frac{1}{(-0.180\text{ m})}=-1.25\text{ m}^{-1}$, so $p=\boxed{23.2\text{ cm}}$

36.45 $P=\frac{1}{f}=\frac{1}{p}+\frac{1}{q}=\frac{1}{\infty}-\frac{1}{0.250\text{ m}}=-4.00\text{ diopters}=\boxed{-4.00\text{ diopters, a diverging lens}}$

36.46 Consider an object at infinity, imaged at the person's far point:

$$\frac{1}{p}+\frac{1}{q}=\frac{1}{f} \qquad \frac{1}{\infty}+\frac{1}{q}=-4.00\text{ m}^{-1} \qquad q=-25.0\text{ cm}$$

The person's far point is $25.0\text{ cm}+2.00\text{ cm}=27.0\text{ cm}$ from his eyes. For the contact lenses we want

$$\frac{1}{\infty}+\frac{1}{(-0.270\text{ m})}=\frac{1}{f}=\boxed{-3.70\text{ diopters}}$$

36. 47 First, we use the thin lens equation to find the object distance: $\frac{1}{p}+\frac{1}{(-25.0\text{ cm})}=\frac{1}{10.0\text{ cm}}$

Then, $p=7.14\text{ cm}$ and Then, $M=-\frac{q}{p}=-\frac{(-25.0\text{ cm})}{7.14\text{ cm}}=\boxed{3.50}$

36.48 (a) From the thin lens equation: $\frac{1}{p}+\frac{1}{(-25.0\text{ cm})}=\frac{1}{5.00\text{ cm}}$ or $p=\boxed{4.17\text{ cm}}$

(b) $M=-\frac{q}{p}=1+\frac{25.0\text{ cm}}{f}=1+\frac{25.0\text{ cm}}{5.00\text{ cm}}=\boxed{6.00}$

36.49 Using Equation 36.20, $M=-\left(\frac{L}{f_o}\right)\left(\frac{25.0\text{ cm}}{f_e}\right)=-\left(\frac{23.0\text{ cm}}{0.400\text{ cm}}\right)\left(\frac{25.0\text{ cm}}{2.50\text{ cm}}\right)=\boxed{-575}$

36.50 $$M = M_1 m_e = M_1\left(\frac{25.0\text{ cm}}{f_e}\right) \Rightarrow f_e = \left(\frac{M_1}{M}\right)(25.0\text{ cm}) = \left(\frac{-12.0}{-140}\right)(25.0\text{ cm}) = \boxed{2.14\text{ cm}}$$

36.51 $f_o = 20.0\text{ m} \quad f_e = 0.0250\text{ m}$

(a) The angular magnification produced by this telescope is: $m = -\frac{f_o}{f_e} = \boxed{-800}$

(b) Since $m < 0$, the image is $\boxed{\text{inverted}}$.

***36.52** (a) The lensmaker's equation $\frac{1}{p}+\frac{1}{q}=\frac{1}{f}$

gives $q=\frac{1}{1/f-1/p}=\frac{1}{\left(\frac{p-f}{fp}\right)}=\frac{fp}{p-f}$

Then, $M=\frac{h'}{h}=-\frac{q}{p}=-\frac{f}{p-f}$

gives $\boxed{h'=\frac{hf}{f-p}}$

(b) For $p>>f$, $f-p\approx -p$. Then, $h'=\boxed{-\frac{hf}{p}}$.

(c) Suppose the telescope observes the space station at the zenith:

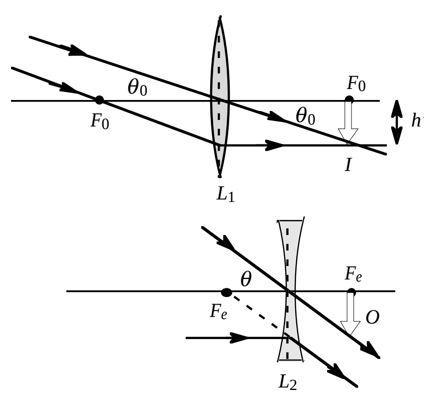
$$h'=-\frac{hf}{p}=-\frac{(108.6\text{ m})(4.00\text{ m})}{407\times 10^{3}\text{ m}}=\boxed{-1.07\text{ mm}}$$

***36.53** (b) Call the focal length of the objective f_o and that of the eyepiece $-|f_e|$. The distance between the lenses is $f_o-|f_e|$. The objective forms a real diminished inverted image of a very distant object at $q_1=f_o$. This image is a virtual object for the eyepiece at $p_2=-|f_e|$.

For it $\frac{1}{p}+\frac{1}{q}=\frac{1}{f}$ becomes $\frac{1}{-|f_e|}+\frac{1}{q}=\frac{1}{-|f_e|}$, $\frac{1}{q_2}=0$

and $\boxed{q_2=\infty}$

(a) The user views the image as $\boxed{\text{virtual}}$. Letting h' represent the height of the first image, $\theta_o=h'/f_o$ and $\theta=h'/|f_e|$. The angular magnification is



$$m=\frac{\theta}{\theta_o}=\frac{h'/|f_e|}{h'/f_o}=\frac{f_o}{|f_e|}$$

(c) Here, $f_o-|f_e|=10.0$ cm and $\frac{f_o}{|f_e|}=3.00$.

Thus, $|f_e|=\frac{f_o}{3.00}$ and $\frac{2}{3}f_o=10.0$ cm.

$f_o=\boxed{15.0\text{ cm}}$

$|f_e|=5.00$ cm and $f_e=\boxed{-5.00\text{ cm}}$

***36.54** Let I_0 represent the intensity of the light from the nebula and θ_o its angular diameter. With the first telescope, the image diameter h' on the film is given by $\theta_o = -h'/f_o$ as $h' = -\theta_o(2000 \text{ mm})$.

The light power captured by the telescope aperture is $\mathcal{P}_1 = I_0 A_1 = I_0\left[\pi(200 \text{ mm})^2/4\right]$, and the light energy focused on the film during the exposure is $E_1 = \mathcal{P}_1 t_1 = I_0\left[\pi(200 \text{ mm})^2/4\right](1.50 \text{ min})$.

Likewise, the light power captured by the aperture of the second telescope is $\mathcal{P}_2 = I_0 A_2 = I_0\left[\pi(60.0 \text{ mm})^2/4\right]$ and the light energy is $E_2 = I_0\left[\pi(60.0 \text{ mm})^2/4\right]t_2$. Therefore, to have the same light energy per unit area, it is necessary that

$$\frac{I_0\left[\pi(60.0 \text{ mm})^2/4\right]t_2}{\pi\left[\theta_o(900 \text{ mm})^2/4\right]} = \frac{I_0\left[\pi(200 \text{ mm})^2/4\right](1.50 \text{ min})}{\pi\left[\theta_o(2000 \text{ mm})^2/4\right]}$$

The required exposure time with the second telescope is

$$t_2 = \frac{(200 \text{ mm})^2(900 \text{ mm})^2}{(60.0 \text{ mm})^2(2000 \text{ mm})^2}(1.50 \text{ min}) = \boxed{3.38 \text{ min}}$$

36.55 Only a diverging lens gives an upright diminished image. The image is virtual and

$$d = p - |q| = p + q: \qquad M = -\frac{q}{p} \qquad \text{so} \qquad q = -Mp \qquad \text{and} \qquad d = p - Mp$$

$$p = \frac{d}{1-M}: \qquad \frac{1}{p} + \frac{1}{q} = \frac{1}{f} = \frac{1}{p} + \frac{1}{-Mp} = \frac{-M+1}{-Mp} = \frac{(1-M)^2}{-Md}$$

$$f = \frac{-Md}{(1-M)^2} = \frac{-(0.500)(20.0 \text{ cm})}{(1-0.500)^2} = \boxed{-40.0 \text{ cm}}$$

36.56 If $M < 1$, the lens is diverging and the image is virtual. $d = p - |q| = p + q$

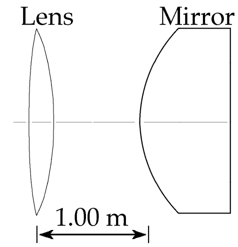
$$M = -\frac{q}{p} \qquad \text{so} \qquad q = -Mp \qquad \text{and} \qquad d = p - Mp$$

$$p = \frac{d}{1-M}: \qquad \frac{1}{p} + \frac{1}{q} = \frac{1}{f} = \frac{1}{p} + \frac{1}{-Mp} = \frac{-M+1}{-Mp} = \frac{(1-M)^2}{-Md} \qquad \boxed{f = \frac{-Md}{(1-M)^2}}$$

If $M > 1$, the lens is converging and the image is still virtual.

Now $d = -q - p$. We obtain in this case $\boxed{f = \frac{Md}{(M-1)^2}}$

***36.57** Start with the first pass through the lens.



$$\frac{1}{q_1} = \frac{1}{f_1} - \frac{1}{p_1} = \frac{1}{80.0\text{ cm}} - \frac{1}{100\text{ cm}} \qquad q_1 = 400\text{ cm to right of lens}$$

For the mirror, $p_2 = -300$ cm

$$\frac{1}{q_2} = \frac{1}{f_2} - \frac{1}{p_2} = \frac{1}{-50.0\text{ cm}} - \frac{1}{-300\text{ cm}} \qquad q_2 = -60.0\text{ cm}$$

For the second pass through the lens, $p_3 = 160$ cm

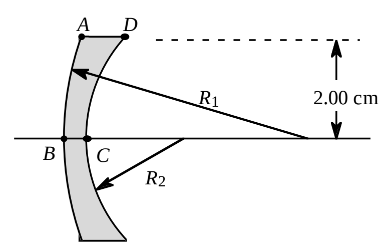
$$\frac{1}{q_3} = \frac{1}{f_1} - \frac{1}{p_3} = \frac{1}{80.0\text{ cm}} - \frac{1}{160\text{ cm}} \qquad q_3 = \boxed{160\text{ cm to left of lens}}$$

$$M_1 = -\frac{q_1}{p_1} = -\frac{400\text{ cm}}{100\text{ cm}} = -4.00 \qquad M_2 = -\frac{q_2}{p_2} = -\frac{-60.0\text{ cm}}{-300\text{ cm}} = -\frac{1}{5}$$

$$M_3 = -\frac{q_3}{p_3} = -\frac{160\text{ cm}}{160\text{ cm}} = -1 \qquad M = M_1M_2M_3 = \boxed{-0.800}$$

Since $M < 0$ the final image is $\boxed{\text{inverted}}$.

***36.58** (a) $\frac{1}{f} = (n-1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$



$$\frac{1}{-65.0\text{ cm}} = (1.66-1)\left(\frac{1}{50.0\text{ cm}} - \frac{1}{R_2}\right)$$

$$\frac{1}{R_2} = \frac{1}{50.0\text{ cm}} + \frac{1}{42.9\text{ cm}} \qquad \text{so} \qquad R_2 = \boxed{23.1\text{ cm}}$$

(b) The distance along the axis from B to A is

$$R_1 - \sqrt{R_1^2 - (2.00\text{ cm})^2} = 50.0\text{ cm} - \sqrt{(50.0\text{ cm})^2 - (2.00\text{ cm})^2} = 0.0400\text{ cm}$$

Similarly, the axial distance from C to D is

$$23.1\text{ cm} - \sqrt{(23.1\text{ cm})^2 - (2.00\text{ cm})^2} = 0.0868\text{ cm}$$

Then, $AD = 0.100\text{ cm} - 0.0400\text{ cm} + 0.0868\text{ cm} = \boxed{0.147\text{ cm}}$.

***36.59** $\frac{1}{q_1} = \frac{1}{f_1} - \frac{1}{p_1} = \frac{1}{10.0\text{ cm}} - \frac{1}{12.5\text{ cm}}$ so $q_1 = 50.0\text{ cm}$ (to left of mirror)

This serves as an object for the lens (a virtual object), so

$$\frac{1}{q_2} = \frac{1}{f_2} - \frac{1}{p_2} = \frac{1}{-16.7\text{ cm}} - \frac{1}{-25.0\text{ cm}} \qquad q_2 = -50.3\text{ cm (to right of lens)}$$

Thus, the final image is located $\boxed{\text{25.3 cm to right of mirror}}$.

$$M_1 = -\frac{q_1}{p_1} = -\frac{50.0\text{ cm}}{12.5\text{ cm}} = -4.00$$

$$M_2 = -\frac{q_2}{p_2} = -\frac{-50.3\text{ cm}}{-25.0\text{ cm}} = -2.01$$

$$M = M_1 M_2 = \boxed{8.05}$$

Thus, the final image is $\boxed{\text{virtual, upright}}$, 8.05 times the size of object, and 25.3 cm to right of the mirror.

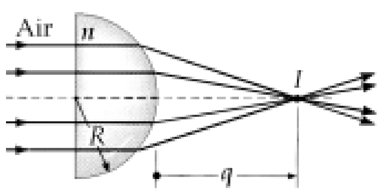
36.60 We first find the focal length of the mirror.

$$\frac{1}{f} = \frac{1}{p} + \frac{1}{q} = \frac{1}{10.0\text{ cm}} + \frac{1}{8.00\text{ cm}} = \frac{9}{40.0\text{ cm}} \qquad \text{and} \qquad f = 4.44\text{ cm}$$

Hence, if $p = 20.0$ cm, $\frac{1}{q} = \frac{1}{f} - \frac{1}{p} = \frac{1}{4.44\text{ cm}} - \frac{1}{20.0\text{ cm}} = \frac{15.56}{88.8\text{ cm}}$

Thus, $q = \boxed{5.71\text{ cm}}$, real

36.61 A hemisphere is too thick to be described as a thin lens. The light is undeviated on entry into the flat face. We next consider the light's exit from the second surface, for which $R = -6.00$ cm



The incident rays are parallel, so $p = \infty$.

Then, $\frac{n_1}{p} + \frac{n_2}{q} = \frac{n_2 - n_1}{R}$ becomes $0 + \frac{1}{q} = \frac{(1.00 - 1.56)}{-6.00\text{ cm}}$ and $\boxed{q = 10.7\text{ cm}}$

***36.62** (a) $I = \frac{\mathcal{P}}{4\pi r^2} = \frac{4.50 \text{ W}}{4\pi\left(1.60\times10^{-2} \text{ m}\right)^2} = \boxed{1.40 \text{ kW/m}^2}$

(b) $I = \frac{\mathcal{P}}{4\pi r^2} = \frac{4.50 \text{ W}}{4\pi(7.20 \text{ m})^2} = \boxed{6.91 \text{ mW/m}^2}$

(c) $\frac{1}{p}+\frac{1}{q}=\frac{1}{f} \Rightarrow \frac{1}{7.20 \text{ m}}+\frac{1}{q}=\frac{1}{0.350 \text{ m}}$ so $q = 0.368 \text{ m}$ and

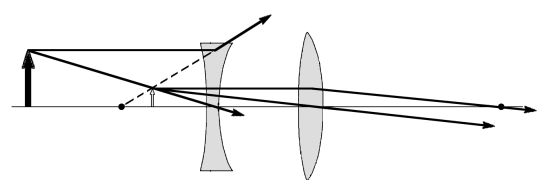
$M = \frac{h'}{3.20 \text{ cm}} = -\frac{q}{p} = -\frac{0.368 \text{ m}}{7.20 \text{ m}}$ $h' = \boxed{0.164 \text{ cm}}$

(d) The lens intercepts power given by $\mathcal{P} = IA = \left(6.91\times10^{-3} \text{ W/m}^2\right)\left[\frac{1}{4}\pi(0.150 \text{ m})^2\right]$

and puts it all onto the image where $I = \frac{\mathcal{P}}{A} = \frac{\left(6.91\times10^{-3} \text{ W/m}^2\right)\left[\pi(15.0 \text{ cm})^2/4\right]}{\pi(0.164 \text{ cm})^2/4} = \boxed{58.1 \text{ W/m}^2}$

36.63 From the thin lens equation, $q_1 = \frac{f_1 p_1}{p_1 - f_1} = \frac{(-6.00 \text{ cm})(12.0 \text{ cm})}{12.0 \text{ cm} - (-6.00 \text{ cm})} = -4.00 \text{ cm}$

When we require that $q_2 \to \infty$, the thin lens equation becomes $p_2 = f_2$;



In this case, $p_2 = d - (-4.00 \text{ cm})$

Therefore, $d + 4.00 \text{ cm} = f_2 = 12.0 \text{ cm}$

and $d = \boxed{8.00 \text{ cm}}$

***36.64** (a) For the light the mirror intercepts, $\mathcal{P} = I_0 A = I_0 \pi R_a^2$

$350 \text{ W} = \left(1000 \text{ W/m}^2\right)\pi R_a^2$ and $R_a = \boxed{0.334 \text{ m or larger}}$

(b) In $\frac{1}{p}+\frac{1}{q}=\frac{1}{f}=\frac{2}{R}$ we have $p \to \infty$ so $q = \frac{R}{2}$.

$M = \frac{h'}{h} = -\frac{q}{p}$, so $h' = -q(h/p) = -\left(\frac{R}{2}\right)\left[0.533° \left(\frac{\pi \text{ rad}}{180°}\right)\right] = -\left(\frac{R}{2}\right)(9.30 \text{ m rad})$

where h/p is the angle the Sun subtends. The intensity at the image is then

$$I = \frac{\mathcal{P}}{\pi h'^2/4} = \frac{4 I_0 \pi R_a^2}{\pi h'^2} = \frac{4 I_0 R_a^2}{(R/2)^2\left(9.30\times10^{-3} \text{ rad}\right)^2}$$

$120\times10^3 \text{ W/m}^2 = \frac{16\left(1000 \text{ W/m}^2\right)R_a^2}{R^2\left(9.30\times10^{-3} \text{ rad}\right)^2}$ so $\boxed{\frac{R_a}{R} = 0.0255 \text{ or larger}}$

36.65 For the mirror, $f = R/2 = +1.50$ m. In addition, because the distance to the Sun is so much larger than any other figures, we can take $p = \infty$. The mirror equation,

$$\frac{1}{p} + \frac{1}{q} = \frac{1}{f}, \text{ then gives } q = f = \boxed{1.50 \text{ m}}.$$

Now, in $M = -\frac{q}{p} = \frac{h'}{h}$, the magnification is nearly zero, but we can be more precise: $\frac{h}{p}$ is the angular diameter of the object. Thus, the image diameter is

$$h' = -\frac{hq}{p} = (-0.533°)\left(\frac{\pi}{180}\text{ rad/deg}\right)(1.50\text{ m}) = -0.140\text{ m} = \boxed{-1.40\text{ cm}}$$

36.66 (a) The lens makers' equation, $\frac{1}{f} = (n-1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$, becomes:

$$\frac{1}{5.00\text{ cm}} = (n-1)\left[\frac{1}{9.00\text{ cm}} - \frac{1}{(-11.0\text{ cm})}\right] \qquad \text{giving} \qquad n = \boxed{1.99}.$$

(b) As the light passes through the lens for the first time, the thin lens equation

$\frac{1}{p_1} + \frac{1}{q_1} = \frac{1}{f}$ becomes: $\frac{1}{8.00\text{ cm}} + \frac{1}{q_1} = \frac{1}{5.00\text{ cm}}$

or $q_1 = 13.3$ cm, and $M_1 = -\frac{q_1}{p_1} = -\frac{13.3\text{ cm}}{8.00\text{ cm}} = -1.67$

This image becomes the object for the concave mirror with:

$$p_m = 20.0\text{ cm} - q_1 = 20.0\text{ cm} - 13.3\text{ cm} = 6.67\text{ cm}, \text{ and } f = \frac{R}{2} = +4.00\text{ cm}.$$

The mirror equation becomes: $\frac{1}{6.67\text{ cm}} + \frac{1}{q_m} = \frac{1}{4.00\text{ cm}}$

giving $q_m = 10.0$ cm and $M_2 = -\frac{q_m}{p_m} = -\frac{10.0\text{ cm}}{6.67\text{ cm}} = -1.50$

The image formed by the mirror serves as a real object for the lens on the second pass of the light through the lens with:

$$p_3 = 20.0\text{ cm} - q_m = +10.0\text{ cm}$$

The thin lens equation yields: $\frac{1}{10.0\text{ cm}} + \frac{1}{q_3} = \frac{1}{5.00\text{ cm}}$

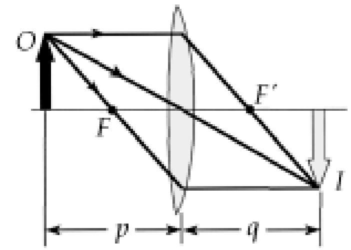
or $q_3 = 10.0$ cm, and $M_3 = -\frac{q_3}{p_3} = -\frac{10.0\text{ cm}}{10.0\text{ cm}} = -1.00.$

The final image is a real image located $\boxed{\text{10.0 cm to the left of the lens}}$.

The overall magnification is $M_{total} = M_1 M_2 M_3 = \boxed{-2.50}$.

(c) Since the total magnification is negative, this final image is $\boxed{\text{inverted}}$.

36.67 In the original situation, $p_1 + q_1 = 1.50 \text{ m}$



In the final situation, $p_2 = p_1 + 0.900 \text{ m}$

and $q_2 = q_1 - 0.900 \text{ m}$.

Our lens equation is $$\frac{1}{p_1} + \frac{1}{q_1} = \frac{1}{f} = \frac{1}{p_2} + \frac{1}{q_2}$$

Substituting, we have $$\frac{1}{p_1} + \frac{1}{1.50 \text{ m} - p_1} = \frac{1}{p_1 + 0.900} + \frac{1}{0.600 - p_1}$$

Adding the fractions, $$\frac{1.50 \text{ m} - p_1 + p_1}{p_1(1.50 \text{ m} - p_1)} = \frac{0.600 - p_1 + p_1 + 0.900}{(p_1 + 0.900)(0.600 - p_1)}$$

Simplified, this becomes $p_1(1.50 \text{ m} - p_1) = (p_1 + 0.900)(0.600 - p_1)$

(a) Thus, $$p_1 = \frac{0.540}{1.80} \text{ m} = \boxed{0.300 \text{ m}}$$

$$p_2 = p_1 + 0.900 = \boxed{1.20 \text{ m}}$$

(b) $$\frac{1}{f} = \frac{1}{0.300 \text{ m}} + \frac{1}{1.50 \text{ m} - 0.300 \text{ m}} \quad \text{and} \quad f = \boxed{0.240 \text{ m}}$$

(c) The second image is $\boxed{\text{real, inverted, and diminished}}$, with $M = -\frac{q_2}{p_2} = \boxed{-0.250}$

36.68 As the light passes through, the lens attempts to form an image at distance q_1 where

$$\frac{1}{q_1} = \frac{1}{f} - \frac{1}{p_1} \quad \text{or} \quad q_1 = \frac{fp_1}{p_1 - f}$$

This image serves as a virtual object for the mirror with $p_2 = -q_1$. The plane mirror then forms an image located at $q_2 = -p_2 = +q_1$ above the mirror and lens.

This second image serves as a virtual object ($p_3 = -q_2 = -q_1$) for the lens as the light makes a return passage through the lens. The final image formed by the lens is located at distance q_3 above the lens where

$$\frac{1}{q_3} = \frac{1}{f} - \frac{1}{p_3} = \frac{1}{f} + \frac{1}{q_1} = \frac{1}{f} + \frac{p_1 - f}{fp_1} = \frac{2p_1 - f}{fp_1} \quad \text{or} \quad q_3 = \frac{fp_1}{2p_1 - f}$$

If the final image coincides with the object, it is necessary to require $q_3 = p_1$, or $\frac{fp_1}{2p_1 - f} = p_1$.

This yields the solution $\boxed{p_1 = f}$ or $\boxed{\text{the object must be located at the focal point of the lens}}$.

36.69 For the objective: $\frac{1}{p}+\frac{1}{q}=\frac{1}{f}$ becomes $\frac{1}{3.40\text{ mm}}+\frac{1}{q}=\frac{1}{3.00\text{ mm}}$ so $q=25.5\text{ mm}$

The objective produces magnification $M_1=-q/p=-\frac{25.5\text{ mm}}{3.40\text{ mm}}=-7.50$

For the eyepiece as a simple magnifier, $m_e=\frac{25.0\text{ cm}}{f}=\frac{25.0\text{ cm}}{2.50\text{ cm}}=10.0$

and overall $M=M_1m_e=\boxed{-75.0}$

36.70 (a) Start with the second lens: This lens must form a virtual image located 19.0 cm to the left of it (i.e., $q_2=-19.0\text{ cm}$). The required object distance for this lens is then

$$p_2=\frac{q_2 f_2}{q_2-f_2}=\frac{(-19.0\text{ cm})(20.0\text{ cm})}{-19.0\text{ cm}-20.0\text{ cm}}=\frac{380\text{ cm}}{39.0}$$

The image formed by the first lens serves as the object for the second lens. Therefore, the image distance for the first lens is

$$q_1=50.0\text{ cm}-p_2=50.0\text{ cm}-\frac{380\text{ cm}}{39.0}=\frac{1570\text{ cm}}{39.0}$$

The distance the original object must be located to the left of the first lens is then given by

$$\frac{1}{p_1}=\frac{1}{f_1}-\frac{1}{q_1}=\frac{1}{10.0\text{ cm}}-\frac{39.0}{1570\text{ cm}}=\frac{157-39.0}{1570\text{ cm}}=\frac{118}{1570\text{ cm}} \quad \text{or} \quad p_1=\frac{1570\text{ cm}}{118}=\boxed{13.3\text{ cm}}$$

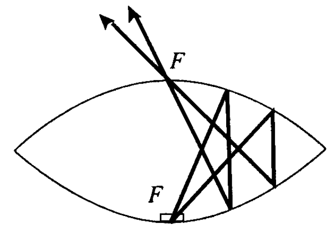
(b) $$M=M_1M_2=\left(-\frac{q_1}{p_1}\right)\left(-\frac{q_2}{p_2}\right)=\left[\left(\frac{1570\text{ cm}}{39.0}\right)\left(\frac{118}{1570\text{ cm}}\right)\right]\left[\frac{(-19.0\text{ cm})(39.0)}{380\text{ cm}}\right]=\boxed{-5.90}$$

(c) Since $M<0$, the final image is $\boxed{\text{inverted}}$.

36.71 (a) $$P=\frac{1}{f}=\frac{1}{p}+\frac{1}{q}=\frac{1}{(0.0224\text{ m})}+\frac{1}{\infty}=\boxed{44.6\text{ diopters}}$$

(b) $$P=\frac{1}{f}=\frac{1}{p}+\frac{1}{q}=\frac{1}{(0.330\text{ m})}+\frac{1}{\infty}=\boxed{3.03\text{ diopters}}$$

36.72 The object is located at the focal point of the upper mirror. Thus, the upper mirror creates an image at infinity (i.e., parallel rays leave this mirror).



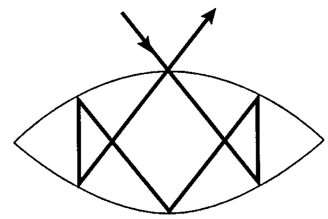
The lower mirror focuses these parallel rays at its focal point, located at the hole in the upper mirror. Thus, the image is real, inverted, and actual size .

For the upper mirror:

$$\frac{1}{p}+\frac{1}{q}=\frac{1}{f} \Rightarrow \frac{1}{7.50\text{ cm}}+\frac{1}{q_1}=\frac{1}{7.50\text{ cm}}: \; q_1=\infty$$

For the lower mirror:

$$\frac{1}{\infty}+\frac{1}{q_2}=\frac{1}{7.50\text{ cm}}: \qquad q_2=7.50\text{ cm}$$

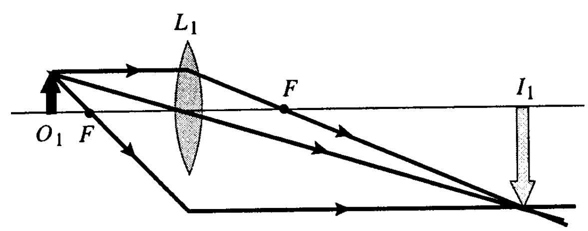


Light directed into the hole in the upper mirror reflects as shown, to behave as if it were reflecting from the hole.

36.73 (a) Lens one:

$$\frac{1}{40.0\text{ cm}}+\frac{1}{q_1}=\frac{1}{30.0\text{ cm}}: \qquad q_1=120\text{ cm}$$

$$M_1=-\frac{q_1}{p_1}=-\frac{120\text{ cm}}{40.0\text{ cm}}=-3.00$$

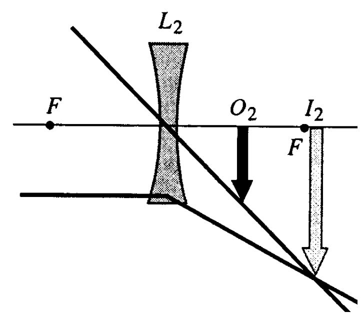


This real image is a virtual object for the second lens, at

$$p_2=110\text{ cm}-120\text{ cm}=-10.0\text{ cm}$$

$$\frac{1}{-10.0\text{ cm}}+\frac{1}{q_2}=\frac{1}{-20.0\text{ cm}}: \qquad q_2=\boxed{20.0\text{ cm}}$$

$$M_2=-\frac{q_2}{p_2}=-\frac{20.0\text{ cm}}{(-10.0\text{ cm})}=+2.00$$

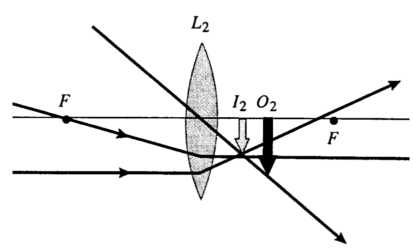


$$M_{\text{overall}}=M_1M_2=\boxed{-6.00}$$

(b) $M_{\text{overall}}<0$, so final image is inverted .

(c) Lens two converging: $\frac{1}{-10.0\text{ cm}}+\frac{1}{q_2}=\frac{1}{20.0\text{ cm}}$

$$q_2=\boxed{6.67\text{ cm}}$$



$$M_2=-\frac{6.67\text{ cm}}{(-10.0\text{ cm})}=+0.667$$

$$M_{\text{overall}}=M_1M_2=\boxed{-2.00}$$

Again, $M_{\text{overall}}<0$ and the final image is inverted .