Chapter 32 Solutions

***32.1** $|\overline{\mathcal{E}}| = L\dfrac{\Delta I}{\Delta t} = (3.00 \times 10^{-3}\text{ H})\left(\dfrac{1.50\text{ A} - 0.200\text{ A}}{0.200\text{ s}}\right) = 1.95 \times 10^{-2}\text{ V} = \boxed{19.5\text{ mV}}$

32.2 Treating the telephone cord as a solenoid, we have:

$$L = \frac{\mu_0 N^2 A}{\ell} = \frac{(4\pi \times 10^{-7}\text{ T}\cdot\text{ m / A})(70.0)^2(\pi)(6.50 \times 10^{-3}\text{ m})^2}{0.600\text{ m}} = \boxed{1.36\ \mu\text{H}}$$

32.3 $|\overline{\mathcal{E}}| = +L\left(\dfrac{\Delta I}{\Delta t}\right) = (2.00\text{ H})\left(\dfrac{0.500\text{ A}}{0.0100\text{ s}}\right) = \boxed{100\text{ V}}$

32.4 $L = \mu_0 n^2 A \ell$ so $n = \sqrt{\dfrac{L}{\mu_0 A \ell}} = \boxed{7.80 \times 10^3\text{ turns / m}}$

32.5 $L = \dfrac{N\Phi_B}{I} \rightarrow \Phi_B = \dfrac{LI}{N} = \boxed{240\text{ nT}\cdot\text{m}^2}$ (through each turn)

32.6 $|\mathcal{E}| = L\dfrac{dI}{dt}$ where $L = \dfrac{\mu_0 N^2 A}{\ell}$

Thus, $|\mathcal{E}| = \left(\dfrac{\mu_0 N^2 A}{\ell}\right)\dfrac{dI}{dt} = \dfrac{(4\pi \times 10^{-7}\text{ T}\cdot\text{m/A})(300)^2(\pi \times 10^{-4}\text{ m}^2)}{0.150\text{ m}}(10.0\text{ A/s}) = \boxed{2.37\text{ mV}}$

32.7 $\mathcal{E}_{\text{back}} = -\mathcal{E} = L\dfrac{dI}{dt} = L\dfrac{d}{dt}(I_{\max}\sin\omega t) = L\omega I_{\max}\cos\omega t = (10.0 \times 10^{-3})(120\pi)(5.00)\cos\omega t$

$\mathcal{E}_{\text{back}} = (6.00\pi)\cos(120\pi t) = \boxed{(18.8\text{ V})\cos(377t)}$

***32.8** From $|\mathcal{E}| = L\left(\frac{\Delta I}{\Delta t}\right)$, we have $L = \frac{\mathcal{E}}{\left(\frac{\Delta I}{\Delta t}\right)} = \frac{24.0 \times 10^{-3}\text{ V}}{10.0\text{ A/s}} = 2.40 \times 10^{-3}\text{ H}$

From $L = \frac{N\Phi_B}{I}$, we have $\Phi_B = \frac{LI}{N} = \frac{(2.40 \times 10^{-3}\text{ H})(4.00\text{ A})}{500} = \boxed{19.2\ \mu\text{T}\cdot\text{m}^2}$

32.9 $L = \frac{\mu_0 N^2 A}{l} = \frac{\mu_0(420)^2(3.00 \times 10^{-4})}{0.160} = 4.16 \times 10^{-4}\text{ H}$

$\mathcal{E} = -L\frac{dI}{dt} \to \frac{dI}{dt} = \frac{-\mathcal{E}}{L} = \frac{-175 \times 10^{-6}\text{ V}}{4.16 \times 10^{-4}\text{ H}} = \boxed{-0.421\text{ A/s}}$

32.10 The induced emf is $\mathcal{E} = -L\frac{dI}{dt}$, where the self-inductance of a solenoid is given by $L = \frac{\mu_0 N^2 A}{l}$.

Thus, $\frac{dI}{dt} = -\frac{\mathcal{E}}{L} = \boxed{-\frac{\mathcal{E} l}{\mu_0 N^2 A}}$

32.11 $|\mathcal{E}| = L\frac{dI}{dt} = (90.0 \times 10^{-3})\frac{d}{dt}(t^2 - 6t)$ V

(a) At $t = 1.00$ s, $\mathcal{E} = \boxed{360\text{ mV}}$

(b) At $t = 4.00$ s, $\mathcal{E} = \boxed{180\text{ mV}}$

(c) $\mathcal{E} = (90.0 \times 10^{-3})(2t - 6) = 0$ when $\boxed{t = 3.00\text{ s}}$

32.12 (a) $B = \mu_0 nI = \mu_0\left(\frac{450}{0.120}\right)(0.0400\text{ mA}) = \boxed{188\ \mu\text{T}}$

(b) $\Phi_B = BA = \boxed{3.33 \times 10^{-8}\text{ T}\cdot\text{m}^2}$

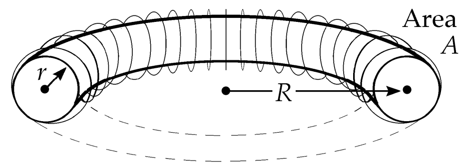
(c) $L = \frac{N\Phi_B}{I} = \boxed{0.375\text{ mH}}$

(d) $\boxed{B \text{ and } \Phi_B \text{ are proportional to current; } L \text{ is independent of current}}$

32.13 (a) $L = \dfrac{\mu_0 N^2 A}{\ell} = \dfrac{\mu_0(120)^2\pi(5.00\times 10^{-3})^2}{0.0900} = \boxed{15.8\ \mu\text{H}}$

(b) $\Phi_B' = \dfrac{\mu_m}{\mu_0}\Phi_B \rightarrow L = \dfrac{\mu_m N^2 A}{\ell} = 800(1.58\times 10^{-5}\ \text{H}) = \boxed{12.6\ \text{mH}}$

32.14 $L = \dfrac{N\Phi_B}{I} = \dfrac{NBA}{I} \approx \dfrac{NA}{I}\cdot\dfrac{\mu_0 NI}{2\pi R} = \boxed{\dfrac{\mu_0 N^2 A}{2\pi R}}$



32.15 $\mathcal{E} = \mathcal{E}_0 e^{-kt} = -L\dfrac{dI}{dt}$

$dI = -\dfrac{\mathcal{E}_0}{L}e^{-kt}\,dt$

If we require $I \rightarrow 0$ as $t \rightarrow \infty$, the solution is $I = \dfrac{\mathcal{E}_0}{kL}e^{-kt} = \dfrac{dq}{dt}$

$Q = \int I\,dt = \int_0^\infty \dfrac{\mathcal{E}_0}{kL}e^{-kt}\,dt = -\dfrac{\mathcal{E}_0}{k^2L}$ $\boxed{|Q| = \dfrac{\mathcal{E}_0}{k^2L}}$

32.16 $I = \dfrac{\mathcal{E}}{R}(1 - e^{-Rt/L})$

$0.900\dfrac{\mathcal{E}}{R} = \dfrac{\mathcal{E}}{R}\left[1 - e^{-R(3.00\ \text{s})/2.50\ \text{H}}\right]$

$\exp\left(-\dfrac{R(3.00\ \text{s})}{2.50\ \text{H}}\right) = 0.100$

$R = \dfrac{2.50\ \text{H}}{3.00\ \text{s}}\ln 10.0 = \boxed{1.92\ \Omega}$

32.17 $\tau = \frac{L}{R} = 0.200$ s: $\frac{I}{I_{\max}} = 1 - e^{-t/\tau}$

(a) $0.500 = 1 - e^{-t/0.200} \rightarrow t = \tau \ln 2.00 = \boxed{0.139 \text{ s}}$

(b) $0.900 = 1 - e^{-t/0.200} \rightarrow t = \tau \ln 10.0 = \boxed{0.461 \text{ s}}$

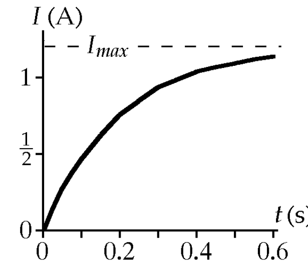


Figure for Goal Solution

Goal Solution

A 12.0-V battery is about to be connected to a series circuit containing a 10.0-Ω resistor and a 2.00-H inductor. How long will it take the current to reach (a) 50.0% and (b) 90.0% of its final value?

G: The time constant for this circuit is $\tau = L/R = 0.2$ s, which means that in 0.2 s, the current will reach $1/e$ = 63% of its final value, as shown in the graph to the right. We can see from this graph that the time to reach 50% of $I_{\max}$ should be slightly less than the time constant, perhaps about 0.15 s, and the time to reach $0.9I_{\max}$ should be about 2.5τ = 0.5 s.

O: The exact times can be found from the equation that describes the rising current in the above graph and gives the current as a function of time for a known emf, resistance, and time constant. We set time $t = 0$ to be the moment the circuit is first connected.

A: At time t, $I(t) = \frac{\mathcal{E}(1 - e^{-t/\tau})}{R}$

where, after a long time, $I_{\max} = \frac{\mathcal{E}(1 - e^{-\infty})}{R} = \frac{\mathcal{E}}{R}$

At $I(t) = 0.500 I_{\max}$, $(0.500)\frac{\mathcal{E}}{R} = \frac{\mathcal{E}(1 - e^{-t/0.200 \text{ s}})}{R}$ so $0.500 = 1 - e^{-t/0.200 \text{ s}}$

Isolating the constants on the right, $\ln\left(e^{-t/2.00 \text{ s}}\right) = \ln(0.500)$

and solving for t, $-\frac{t}{0.200 \text{ s}} = -0.693$ or $t = 0.139$ s

(b) Similarly, to reach 90% of $I_{\max}$, $0.900 = 1 - e^{-t/\tau}$ and $t = -\tau \ln(1 - 0.900)$

Thus, $t = -(0.200 \text{ s})\ln(0.100) = 0.461$ s

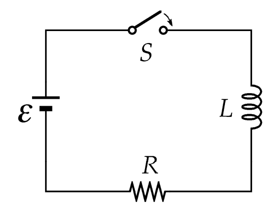
L: The calculated times agree reasonably well with our predictions. We must be careful to avoid confusing the equation for the rising current with the similar equation for the falling current. Checking our answers against predictions is a safe way to prevent such mistakes.

32.18 Taking $\tau = L/R$, $I = I_0 e^{-t/\tau}$: $\dfrac{dI}{dt} = I_0 e^{-t/\tau}\left(-\dfrac{1}{\tau}\right)$

$IR + L\dfrac{dI}{dt} = 0$ will be true if $I_0 R e^{-t/\tau} + L\left(I_0 e^{-t/\tau}\right)\left(-\dfrac{1}{\tau}\right) = 0$

Because $\tau = L/R$, we have agreement with 0 = 0

***32.19** (a) $\tau = L/R = 2.00 \times 10^{-3}\text{ s} = \boxed{2.00\text{ ms}}$



(b) $I = I_{\max}\left(1 - e^{-t/\tau}\right) = \left(\dfrac{6.00\text{ V}}{4.00\ \Omega}\right)\left(1 - e^{-0.250/2.00}\right) = \boxed{0.176\text{ A}}$

(c) $I_{\max} = \dfrac{\mathcal{E}}{R} = \dfrac{6.00\text{ V}}{4.00\ \Omega} = \boxed{1.50\text{ A}}$

(d) $0.800 = 1 - e^{-t/2.00\text{ ms}} \rightarrow t = -(2.00\text{ ms})\ln(0.200) = \boxed{3.22\text{ ms}}$

***32.20** $I = \dfrac{\mathcal{E}}{R}\left(1 - e^{-t/\tau}\right) = \dfrac{120}{9.00}\left(1 - e^{-1.80/7.00}\right) = 3.02\text{ A}$

$\Delta V_R = IR = (3.02)(9.00) = 27.2\text{ V}$

$\Delta V_L = \mathcal{E} - \Delta V_R = 120 - 27.2 = \boxed{92.8\text{ V}}$

32.21 (a) $\Delta V_R = IR = (8.00\ \Omega)(2.00\text{ A}) = 16.0\text{ V}$ and
$\Delta V_L = \mathcal{E} - \Delta V_R = 36.0\text{ V} - 16.0\text{ V} = 20.0\text{ V}$

Therefore, $\dfrac{\Delta V_R}{\Delta V_L} = \dfrac{16.0\text{ V}}{20.0\text{ V}} = \boxed{0.800}$

(b) $\Delta V_R = IR = (4.50\text{ A})(8.00\ \Omega) = 36.0\text{ V}$

$\Delta V_L = \mathcal{E} - \Delta V_R = \boxed{0}$

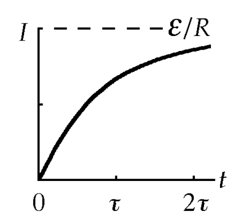


Figure for Goal Solution

Goal Solution

For the *RL* circuit shown in Figure P32.19, let L = 3.00 H, R = 8.00 Ω, and $\mathcal{E}$ = 36.0 V. (a) Calculate the ratio of the potential difference across the resistor to that across the inductor when I = 2.00 A. (b) Calculate the voltage across the inductor when I = 4.50 A.

G: The voltage across the resistor is proportional to the current, $\Delta V_R = IR$, while the voltage across the inductor is proportional to the **rate of change** in the current, $\mathcal{E}_L = -L\,dI/dt$. When the switch is first closed, the voltage across the inductor will be large as it opposes the sudden change in current. As the current approaches its steady state value, the voltage across the resistor increases and the inductor's emf decreases. The maximum current will be $\mathcal{E}/R$ = 4.50 A, so when I = 2.00 A, the resistor and inductor will share similar voltages at this mid-range current, but when I = 4.50 A, the entire circuit voltage will be across the resistor, and the voltage across the inductor will be zero.

O: We can use the definition of resistance to calculate the voltage across the resistor for each current. We will find the voltage across the inductor by using Kirchhoff's loop rule.

A: (a) When $I = 2.00$ A, the voltage across the resistor is $\Delta V_R = IR = (2.00\text{ A})(8.00\ \Omega) = 16.0\text{ V}$

Kirchhoff's loop rule tells us that the sum of the changes in potential around the loop must be zero:

$$\mathcal{E} - \Delta V_R - \mathcal{E}_L = 36.0\text{ V} - 16.0\text{ V} - \mathcal{E}_L = 0 \qquad \text{so} \qquad \mathcal{E}_L = 20.0\text{ V} \quad \text{and} \quad \frac{\Delta V_R}{\mathcal{E}_L} = \frac{16.0\text{ V}}{20.0\text{ V}} = 0.800$$

(b) Similarly, for $I = 4.50$ A, $\Delta V_R = IR = (4.50\text{ A})(8.00\ \Omega) = 36.0\text{ V}$

$$\mathcal{E} - \Delta V_R - \mathcal{E}_L = 36.0\text{ V} - 36.0\text{ V} - \mathcal{E}_L = 0 \qquad \text{so} \qquad \mathcal{E}_L = 0$$

L: We see that when $I = 2.00$ A, $\Delta V_R < \mathcal{E}_L$, but they are similar in magnitude as expected. Also as predicted, the voltage across the inductor goes to zero when the current reaches its maximum value. A worthwhile exercise would be to consider the ratio of these voltages for several different times after the switch is reopened.

***32.22** After a long time, 12.0 V = (0.200 A)R Thus, R = 60.0 Ω. Now, $\tau = \dfrac{L}{R}$ gives

$$L = \tau R = (5.00 \times 10^{-4}\text{ s})(60.0\text{ V/A}) = \boxed{30.0\text{ mH}}$$

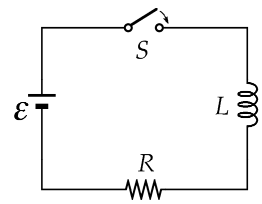
32.23 $I = I_{\max}\left(1 - e^{-t/\tau}\right)$: $\qquad \dfrac{dI}{dt} = -I_{\max}\left(e^{-t/\tau}\right)\left(-\dfrac{1}{\tau}\right)$

$$\tau = \frac{L}{R} = \frac{15.0\text{ H}}{30.0\ \Omega} = 0.500\text{ s}: \qquad \frac{dI}{dt} = \frac{R}{L} I_{\max} e^{-t/\tau} \quad \text{and} \quad I_{\max} = \frac{\mathcal{E}}{R}$$

(a) $t = 0$: $\dfrac{dI}{dt} = \dfrac{R}{L} I_{\max} e^0 = \dfrac{\mathcal{E}}{L} = \dfrac{100\text{ V}}{15.0\text{ H}} = \boxed{6.67\text{ A/s}}$

(b) $t = 1.50$ s: $\dfrac{dI}{dt} = \dfrac{\mathcal{E}}{L} e^{-t/\tau} = (6.67\text{ A/s})e^{-1.50/(0.500)} = (6.67\text{ A/s})e^{-3.00} = \boxed{0.332\text{ A/s}}$

32.24 $I = I_{max}\left(1 - e^{-t/\tau}\right)$



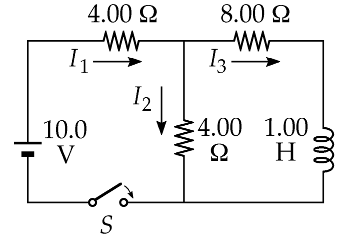
$0.980 = 1 - e^{-3.00\times10^{-3}/\tau}$

$0.0200 = e^{-3.00\times10^{-3}/\tau}$

$\tau = -\dfrac{3.00\times10^{-3}}{\ln(0.0200)} = 7.67\times10^{-4}\text{ s}$

$\tau = L/R$, so $L = \tau R = (7.67\times10^{-4})(10.0) = \boxed{7.67\text{ mH}}$

32.25 Name the currents as shown. By Kirchhoff's laws:



$I_1 = I_2 + I_3$ (1)

$+10.0\text{ V} - 4.00 I_1 - 4.00 I_2 = 0$ (2)

$+10.0\text{ V} - 4.00 I_1 - 8.00 I_3 - (1.00)\dfrac{dI_3}{dt} = 0$ (3)

From (1) and (2), $+10.0 - 4.00 I_1 - 4.00 I_1 + 4.00 I_3 = 0$ and $I_1 = 0.500 I_3 + 1.25\text{ A}$

Then (3) becomes $10.0\text{ V} - 4.00(0.500 I_3 + 1.25\text{ A}) - 8.00 I_3 - (1.00)\dfrac{dI_3}{dt} = 0$

$$(1.00\text{ H})(dI_3/dt) + (10.0\ \Omega)I_3 = 5.00\text{ V}$$

We solve the differential equation using Equations 32.6 and 32.7:

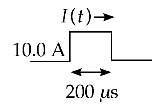
$I_3(t) = \dfrac{5.00\text{ V}}{10.0\ \Omega}\left[1 - e^{-(10.0\ \Omega)t/1.00\text{ H}}\right] = \boxed{(0.500\text{ A})\left[1 - e^{-10t/\text{s}}\right]}$

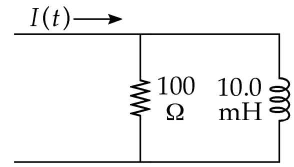
$I_1 = 1.25 + 0.500 I_3 = \boxed{1.50\text{ A} - (0.250\text{ A})e^{-10t/\text{s}}}$

32.26 (a) Using $\tau = RC = \dfrac{L}{R}$, we get $R = \sqrt{\dfrac{L}{C}} = \sqrt{\dfrac{3.00\text{ H}}{3.00\times10^{-6}\text{ F}}} = 1.00\times10^{3}\ \Omega = \boxed{1.00\text{ k}\Omega}$

(b) $\tau = RC = \left(1.00\times10^{3}\ \Omega\right)\left(3.00\times10^{-6}\text{ F}\right) = 3.00\times10^{-3}\text{ s} = \boxed{3.00\text{ ms}}$

32.27 For $t \le 0$, the current in the inductor is zero. At $t = 0$, it starts to grow from zero toward 10.0 A with time constant $\tau = L/R = (10.0\text{ mH})/(100\ \Omega) = 1.00 \times 10^{-4}\text{ s}$.





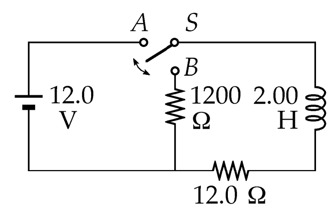
For $0 \le t \le 200\ \mu\text{s}$, $\quad I = I_{\text{max}}\left(1 - e^{-t/\tau}\right) = \boxed{(10.00\text{ A})\left(1 - e^{-10000t/\text{s}}\right)}$

At $t = 200\ \mu\text{s}$, $\quad I = (10.00\text{ A})\left(1 - e^{-2.00}\right) = 8.65\text{ A}$

Thereafter, it decays exponentially as $I = I_0 e^{-t'/\tau}$, so for $t \ge 200\ \mu\text{s}$,

$$I = (8.65\text{ A})e^{-10000(t - 200\ \mu\text{s})/\text{s}} = (8.65\text{ A})e^{-10000t/\text{s} + 2.00} = \left(8.65 e^{2.00}\text{ A}\right)e^{-10000t/\text{s}} = \boxed{(63.9\text{ A})e^{-10000t/\text{s}}}$$

32.28 (a) $I = \dfrac{\mathcal{E}}{R} = \dfrac{12.0\text{ V}}{12.0\ \Omega} = \boxed{1.00\text{ A}}$



(b) Initial current is 1.00 A, : $\quad \Delta V_{12} = (1.00\text{ A})(12.00\ \Omega) = \boxed{12.0\text{ V}}$

$\Delta V_{1200} = (1.00\text{ A})(1200\ \Omega) = \boxed{1.20\text{ kV}}$

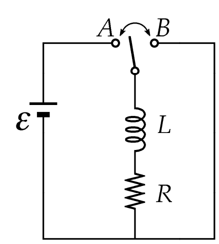
$\Delta V_L = \boxed{1.21\text{ kV}}$

(c) $I = I_{\text{max}} e^{-Rt/L}$: $\quad \dfrac{dI}{dt} = -I_{\text{max}} \dfrac{R}{L} e^{-Rt/L}$ $\quad$ and $\quad -L\dfrac{dI}{dt} = \Delta V_L = I_{\text{max}} R e^{-Rt/L}$

Solving $\quad 12.0\text{ V} = (1212\text{ V})e^{-1212t/2.00}$ $\quad$ so $\quad 9.90 \times 10^{-3} = e^{-606t}$

Thus, $\quad \boxed{t = 7.62\text{ ms}}$

32.29 $\tau = \dfrac{L}{R} = \dfrac{0.140}{4.90} = 28.6\text{ ms}; \quad I_{\text{max}} = \dfrac{\mathcal{E}}{R} = \dfrac{6.00\text{ V}}{4.90\ \Omega} = 1.22\text{ A}$



(a) $I = I_{\text{max}}\left(1 - e^{-t/\tau}\right)$ $\quad$ so $\quad 0.220 = 1.22\left(1 - e^{-t/\tau}\right)$

$e^{-t/\tau} = 0.820 \qquad t = -\tau \ln(0.820) = \boxed{5.66\text{ ms}}$

(b) $I = I_{\text{max}}\left(1 - e^{-\frac{10.0}{0.0286}}\right) = (1.22\text{ A})\left(1 - e^{-350}\right) = \boxed{1.22\text{ A}}$

(c) $I = I_{\text{max}} e^{-t/\tau}$ $\quad$ and $\quad 0.160 = 1.22 e^{-t/\tau}$ $\quad$ so $\quad t = -\tau \ln(0.131) = \boxed{58.1\text{ ms}}$

32.30 (a) For a series connection, both inductors carry equal currents at every instant, so dI/dt is the same for both. The voltage across the pair is

$$L_{eq}\frac{dI}{dt} = L_1\frac{dI}{dt} + L_2\frac{dI}{dt} \qquad \text{so} \qquad \boxed{L_{eq} = L_1 + L_2}$$

(b) $$L_{eq}\frac{dI}{dt} = L_1\frac{dI_1}{dt} = L_2\frac{dI_2}{dt} = \Delta V_L \qquad \text{where } I = I_1 + I_2 \quad \text{and} \quad \frac{dI}{dt} = \frac{dI_1}{dt} + \frac{dI_2}{dt}$$

Thus, $$\frac{\Delta V_L}{L_{eq}} = \frac{\Delta V_L}{L_1} + \frac{\Delta V_L}{L_2} \qquad \text{and} \qquad \boxed{\frac{1}{L_{eq}} = \frac{1}{L_1} + \frac{1}{L_2}}$$

(c) $$L_{eq}\frac{dI}{dt} + R_{eq}I = L_1\frac{dI}{dt} + IR_1 + L_2\frac{dI}{dt} + IR_2$$

Now I and dI/dt are separate quantities under our control, so functional equality requires both

$$\boxed{L_{eq} = L_1 + L_2 \quad \text{and} \quad R_{eq} = R_1 + R_2}$$

(d) $$\Delta V = L_{eq}\frac{dI}{dt} + R_{eq}I = L_1\frac{dI_1}{dt} + R_1I_1 = L_2\frac{dI_2}{dt} + R_2I_2 \qquad \text{where} \qquad I = I_1 + I_2 \quad \text{and} \quad \frac{dI}{dt} = \frac{dI_1}{dt} + \frac{dI_2}{dt}$$

We may choose to keep the currents constant in time. Then, $$\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2}$$

We may choose to make the current swing through 0. Then, $$\frac{1}{L_{eq}} = \frac{1}{L_1} + \frac{1}{L_2}$$

> This equivalent coil with resistance will be equivalent
> to the pair of real inductors for all other currents as well.

32.31 $$L = \frac{N\Phi_B}{I} = \frac{200(3.70\times10^{-4})}{1.75} = 42.3\text{ mH} \qquad \text{so} \qquad U = \frac{1}{2}LI^2 = \frac{1}{2}(0.423\text{ H})(1.75\text{ A})^2 = \boxed{0.0648\text{ J}}$$

32.32 (a) The magnetic energy density is given by

$$u = \frac{B^2}{2\mu_0} = \frac{(4.50\text{ T})^2}{2(1.26\times10^{-6}\text{ T}\cdot\text{m/A})} = \boxed{8.06\times10^6\text{ J/m}^3}$$

(b) The magnetic energy stored in the field equals u times the volume of the solenoid (the volume in which B is non-zero).

$$U = uV = (8.06\times10^6\text{ J/m}^3)\left[(0.260\text{ m})\pi(0.0310\text{ m})^2\right] = \boxed{6.32\text{ kJ}}$$

32.33 $L = \mu_0 \dfrac{N^2 A}{\ell} = \mu_0 \dfrac{(68.0)^2\ \pi(0.600\times10^{-2})^2}{0.0800} = 8.21\ \mu\text{H}$

$U = \frac{1}{2}LI^2 = \frac{1}{2}(8.21\times10^{-6}\ \text{H})(0.770\ \text{A})^2 = \boxed{2.44\ \mu\text{J}}$

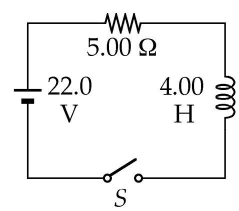
32.34 (a) $U = \frac{1}{2}\ LI^2 = \frac{1}{2}\ L\left(\dfrac{\mathcal{E}}{2R}\right)^2 = \dfrac{L\mathcal{E}^2}{8R^2} = \dfrac{(0.800)(500)^2}{8(30.0)^2} = \boxed{27.8\ \text{J}}$

(b) $I = \left(\dfrac{\mathcal{E}}{R}\right)\left[1 - e^{-(R/L)t}\right]$ so $\dfrac{\mathcal{E}}{2R} = \left(\dfrac{\mathcal{E}}{R}\right)\left[1 - e^{-(R/L)t}\right] \rightarrow e^{-(R/L)t} = \dfrac{1}{2}$

$\dfrac{R}{L}t = \ln 2$ so $t = \dfrac{L}{R}\ln 2 = \dfrac{0.800}{30.0}\ln 2 = \boxed{18.5\ \text{ms}}$

32.35 $u = \epsilon_0 \dfrac{E^2}{2} = \boxed{44.2\ \text{nJ/m}^3}$ $u = \dfrac{B^2}{2\mu_0} = \boxed{995\ \mu\text{J/m}^3}$

***32.36** (a) $U = \frac{1}{2}\ LI^2 = \frac{1}{2}(4.00\ \text{H})(0.500\ \text{A})^2 = \boxed{0.500\ \text{J}}$



(b) $\dfrac{dU}{dt} = LI = (4.00\ \text{H})(1.00\ \text{A}) = 4.00\ \text{J/s} = \boxed{4.00\ \text{W}}$

(c) $\mathcal{P} = (\Delta V)I = (22.0\ \text{V})(0.500\ \text{A}) = \boxed{11.0\ \text{W}}$

32.37 From Equation 32.7, $I = \dfrac{\mathcal{E}}{R}\left(1 - e^{-Rt/L}\right)$

(a) The maximum current, after a long time t, is $I = \dfrac{\mathcal{E}}{R} = 2.00\ \text{A}.$

At that time, the inductor is fully energized and $\mathcal{P} = I(\Delta V) = (2.00\ \text{A})(10.0\ \text{V}) = \boxed{20.0\ \text{W}}$

(b) $\mathcal{P}_{\text{lost}} = I^2R = (2.00\ \text{A})^2(5.00\ \Omega) = \boxed{20.0\ \text{W}}$

(c) $\mathcal{P}_{\text{inductor}} = I(\Delta V_{\text{drop}}) = \boxed{0}$

(d) $U = \dfrac{LI^2}{2} = \dfrac{(10.0\ \text{H})(2.00\ \text{A})^2}{2} = \boxed{20.0\ \text{J}}$

32.38 We have $u = \epsilon_0 \dfrac{E^2}{2}$ and $u = \dfrac{B^2}{2\mu_0}$

Therefore $\epsilon_0 \dfrac{E^2}{2} = \dfrac{B^2}{2\mu_0}$ so $B^2 = \epsilon_0 \mu_0 E^2$

$$B = E\sqrt{\epsilon_0\mu_0} = \frac{6.80 \times 10^5 \text{ V / m}}{3.00 \times 10^8 \text{ m / s}} = \boxed{2.27 \times 10^{-3} \text{ T}}$$

32.39 The total magnetic energy is the volume integral of the energy density, $u = \dfrac{B^2}{2\mu_0}$

Because B changes with position, u is not constant. For $B = B_0(R / r)^2$, $u = \left(\dfrac{B_0{}^2}{2\mu_0}\right)\left(\dfrac{R}{r}\right)^4$

Next, we set up an expression for the magnetic energy in a spherical shell of radius r and thickness dr. Such a shell has a volume $4\pi r^2\, dr$, so the energy stored in it is

$$dU = u\left(4\pi r^2 dr\right) = \left(\frac{2\pi B_0{}^2 R^4}{\mu_0}\right)\frac{dr}{r^2}$$

We integrate this expression for $r = R$ to $r = \infty$ to obtain the total magnetic energy outside the sphere. This gives

$$U = \boxed{\frac{2\pi B_0^2 R^3}{\mu_0}} = \frac{2\pi(5.00 \times 10^{-5} \text{ T})^2 (6.00 \times 10^6 \text{ m})^3}{(1.26 \times 10^{-6} \text{ T} \cdot \text{m / A})} = \boxed{2.70 \times 10^{18} \text{ J}}$$

32.40 $I_1(t) = I_{\text{max}} e^{-\alpha t} \sin \omega t$ with $I_{\text{max}} = 5.00$ A, $\alpha = 0.0250 \text{ s}^{-1}$, and $\omega = 377$ rad/s.

$$\frac{dI_1}{dt} = I_{\text{max}} e^{-\alpha t}\left(-\alpha \sin \omega t + \omega \cos \omega t\right)$$

At $t = 0.800$ s, $$\frac{dI_1}{dt} = \left(5.00 \text{ A/s}\right)e^{-0.0200}\left[-(0.0250)\sin\left(0.800(377)\right) + 377\cos\left(0.800(377)\right)\right]$$

$$\frac{dI_1}{dt} = 1.85 \times 10^3 \text{ A/s}$$

Thus, $\mathcal{E}_2 = -M\dfrac{dI_1}{dt}$: $M = \dfrac{-\mathcal{E}_2}{dI_1/dt} = \dfrac{+3.20 \text{ V}}{1.85 \times 10^3 \text{ A/s}} = \boxed{1.73 \text{ mH}}$

32.41 $\mathcal{E}_2 = -M\dfrac{dI_1}{dt} = -(1.00\times10^{-4}\text{ H})(1.00\times10^{4}\text{ A / s})\cos(1000t)$

$(\mathcal{E}_2)_{\max} = \boxed{1.00\text{ V}}$

32.42 $M = \left|\dfrac{\mathcal{E}_2}{dI_1/dt}\right| = \dfrac{96.0\text{ mV}}{1.20\text{ A / s}} = \boxed{80.0\text{ mH}}$

32.43 (a) $M = \dfrac{N_B\Phi_{BA}}{I_A} = \dfrac{700(90.0\times10^{-6})}{3.50} = \boxed{18.0\text{ mH}}$

(b) $L_A = \dfrac{\Phi_A}{I_A} = \dfrac{400(300\times10^{-6})}{3.50} = \boxed{34.3\text{ mH}}$

(c) $\mathcal{E}_B = -M\dfrac{dI_A}{dt} = -(18.0\text{ mH})(0.500\text{ A / s}) = \boxed{-9.00\text{ mV}}$

32.44 $M = \dfrac{N_2\Phi_{12}}{I_1} = \dfrac{N_2(B_1A_1)}{I_1} = \dfrac{N_2\left[(\mu_0 n_1 I_1)A_1\right]}{I_1} = N_2\mu_0 n_1 A_1$

$M = (1.00)\left(4\pi\times10^{-7}\text{ T}\cdot\text{m/A}\right)\left(\dfrac{70.0}{0.0500\text{ m}}\right)\left[\pi\left(5.00\times10^{-3}\text{ m}\right)^2\right] = \boxed{138\text{ nH}}$

32.45 B at center of (larger) loop: $B_1 = \dfrac{\mu_0 I_1}{2R}$

(a) $M = \dfrac{\Phi_2}{I_1} = \dfrac{B_1A_2}{I_1} = \dfrac{(\mu_0 I_1 / 2R)(\pi r^2)}{I_1} = \boxed{\dfrac{\mu_0\pi r^2}{2R}}$

(b) $M = \dfrac{\mu_0\pi(0.0200)^2}{2(0.200)} = \boxed{3.95\text{ nH}}$

***32.46** Assume the long wire carries current I. Then the magnitude of the magnetic field it generates at distance x from the wire is $B = \mu_0 I / 2\pi x$, and this field passes perpendicularly through the plane of the loop. The flux through the loop is

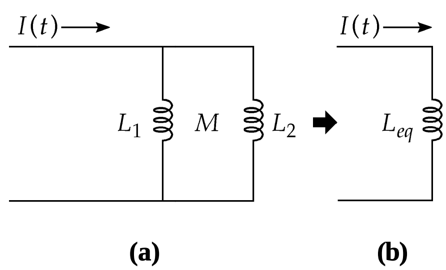
$$\Phi_B = \int \mathbf{B}\cdot d\mathbf{A} = \int B\,dA = \int B(\ell\,dx) = \frac{\mu_0 I \ell}{2\pi}\int_{0.400\text{ mm}}^{1.70\text{ mm}} \frac{dx}{x} = \frac{\mu_0 I \ell}{2\pi}\ln\left(\frac{1.70}{0.400}\right)$$

The mutual inductance between the wire and the loop is then

$$M = \frac{N_2\Phi_{12}}{I_1} = \frac{N_2\mu_0 I\ell}{2\pi I}\ln\left(\frac{1.70}{0.400}\right) = \frac{N_2\mu_0\ell}{2\pi}(1.45) = \frac{1(4\pi\times10^{-7}\text{ T}\cdot\text{m/A})(2.70\times10^{-3}\text{ m})}{2\pi}(1.45)$$

$$M = 7.81\times10^{-10}\text{ H} = \boxed{781\text{ pH}}$$

32.47 With $I = I_1 + I_2$, the voltage across the pair is:



$$\Delta V = -L_1\frac{dI_1}{dt} - M\frac{dI_2}{dt} = -L_2\frac{dI_2}{dt} - M\frac{dI_1}{dt} = -L_{\text{eq}}\frac{dI}{dt}$$

So, $$-\frac{dI_1}{dt} = \frac{\Delta V}{L_1} + \frac{M}{L_1}\frac{dI_2}{dt}$$

and $$-L_2\frac{dI_2}{dt} + \frac{M(\Delta V)}{L_1} + \frac{M^2}{L_1}\frac{dI_2}{dt} = \Delta V$$

$$(-L_1L_2 + M^2)\frac{dI_2}{dt} = \Delta V(L_1 - M) \qquad [1]$$

By substitution, $$-\frac{dI_2}{dt} = \frac{\Delta V}{L_2} + \frac{M}{L_2}\frac{dI_1}{dt}$$

leads to $$(-L_1L_2 + M^2)\frac{dI_1}{dt} = \Delta V(L_2 - M) \qquad [2]$$

Adding [1] to [2], $$(-L_1L_2 + M^2)\frac{dI}{dt} = \Delta V(L_1 + L_2 - 2M)$$

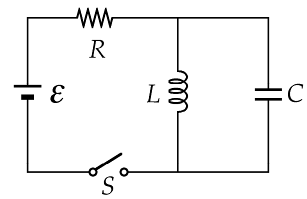
So, $$L_{\text{eq}} = -\frac{\Delta V}{dI/dt} = \boxed{\frac{L_1L_2 - M^2}{L_1 + L_2 - 2M}}$$

32.48 At different times, $(U_C)_{\max} = (U_L)_{\max}$ so $\left[\frac{1}{2}C(\Delta V)^2\right]_{\max} = \left(\frac{1}{2}LI^2\right)_{\max}$

$$I_{\max} = \sqrt{\frac{C}{L}}(\Delta V)_{\max} = \sqrt{\frac{1.00\times10^{-6}\text{ F}}{10.0\times10^{-3}\text{ H}}}(40.0\text{ V}) = \boxed{0.400\text{ A}}$$

32.49 $\left[\frac{1}{2}C(\Delta V)^2\right]_{max} = \left(\frac{1}{2}LI^2\right)_{max}$ so $(\Delta V_C)_{max} = \sqrt{\frac{L}{C}}\,I_{max} = \sqrt{\frac{20.0\times10^{-3}\text{ H}}{0.500\times10^{-6}\text{ F}}}\,(0.100\text{ A}) = \boxed{20.0\text{ V}}$

32.50 When the switch has been closed for a long time, battery, resistor, and coil carry constant current $I_{max} = \mathcal{E}/R$. When the switch is opened, current in battery and resistor drops to zero, but the coil carries this same current for a moment as oscillations begin in the LC loop.



We interpret the problem to mean that the voltage amplitude of these oscillations is ΔV, in $\frac{1}{2}C(\Delta V)^2 = \frac{1}{2}LI_{max}^2$.

Then, $L = \frac{C(\Delta V)^2}{I_{max}^2} = \frac{C(\Delta V)^2 R^2}{\mathcal{E}^2} = \frac{(0.500\times10^{-6}\text{ F})(150\text{ V})^2(250\ \Omega)^2}{(50.0\text{ V})^2} = \boxed{0.281\text{ H}}$

32.51 $C = \frac{1}{(2\pi f)^2 L} = \frac{1}{(2\pi\cdot 6.30\times10^6)^2(1.05\times10^6)} = \boxed{608\text{ pF}}$

Goal Solution

A fixed inductance $L = 1.05\ \mu$H is used in series with a variable capacitor in the tuning section of a radio. What capacitance tunes the circuit to the signal from a station broadcasting at 6.30 MHz?

G: It is difficult to predict a value for the capacitance without doing the calculations, but we might expect a typical value in the μF or pF range.

O: We want the resonance frequency of the circuit to match the broadcasting frequency, and for a simple RLC circuit, the resonance frequency only depends on the magnitudes of the inductance and capacitance.

A: The resonance frequency is $f_0 = \frac{1}{2\pi\sqrt{LC}}$

Thus, $C = \frac{1}{(2\pi f_0)^2 L} = \frac{1}{\left[(2\pi)(6.30\times10^6\text{ Hz})\right]^2(1.05\times10^{-6}\text{ H})} = 608\text{ pF}$

L: This is indeed a typical capacitance, so our calculation appears reasonable. However, you probably would not hear any familiar music on this broadcast frequency. The frequency range for FM radio broadcasting is 88.0 – 108.0 MHz, and AM radio is 535 – 1605 kHz. The 6.30 MHz frequency falls in the Maritime Mobile SSB Radiotelephone range, so you might hear a ship captain instead of Top 40 tunes! This and other information about the radio frequency spectrum can be found on the National Telecommunications and Information Administration (NTIA) website, which at the time of this printing was at http://www.ntia.doc.gov/osmhome/allochrt.html

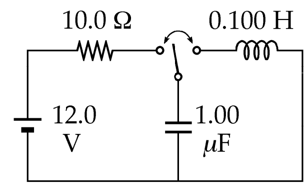
32.52 $f = \dfrac{1}{2\pi\sqrt{LC}}$: $L = \dfrac{1}{(2\pi f)^2 C} = \dfrac{1}{(2\pi \cdot 120)^2 (8.00\times10^{-6})} = \boxed{0.220 \text{ H}}$

32.53 (a) $f = \dfrac{1}{2\pi\sqrt{LC}} = \dfrac{1}{2\pi\sqrt{(0.0820 \text{ H})(17.0\times10^{-6} \text{ F})}} = \boxed{135 \text{ Hz}}$

(b) $Q = Q_{\max}\cos\omega t = (180\ \mu C)\cos(847\times0.00100) = \boxed{119\ \mu\text{C}}$

(c) $I = \dfrac{dQ}{dt} = -\omega Q_{\max}\sin\omega t = -(847)(180)\sin(0.847) = \boxed{-114 \text{ mA}}$

32.54 (a) $f = \dfrac{1}{2\pi\sqrt{LC}} = \dfrac{1}{2\pi\sqrt{(0.100 \text{ H})(1.00\times10^{-6} \text{ F})}} = \boxed{503 \text{ Hz}}$



(b) $Q = C\mathcal{E} = (1.00\times10^{-6} \text{ F})(12.0 \text{ V}) = \boxed{12.0\ \mu\text{C}}$

(c) $\frac{1}{2}C\mathcal{E}^2 = \frac{1}{2}LI_{\max}^2$

$$I_{\max} = \mathcal{E}\sqrt{\frac{C}{L}} = 12 \text{ V}\sqrt{\frac{1.00\times10^{-6} \text{ F}}{0.100 \text{ H}}} = \boxed{37.9 \text{ mA}}$$

(d) At all times $U = \frac{1}{2}C\mathcal{E}^2 = \frac{1}{2}(1.00\times10^{-6} \text{ F})(12.0 \text{ V})^2 = \boxed{72.0\ \mu\text{J}}$

32.55 $\omega = \dfrac{1}{\sqrt{LC}} = \dfrac{1}{\sqrt{(3.30 \text{ H})(840\times10^{-12} \text{ F})}} = 1.899\times10^4 \text{ rad s}$

$$Q = Q_{\max}\cos\omega t, \quad I = \frac{dQ}{dt} = -\omega Q_{\max}\sin\omega t$$

(a) $U_C = \dfrac{Q^2}{2C} = \dfrac{\left(\left[105\times10^{-6}\right]\cos\left[\left(1.899\times10^4 \text{ rad/s}\right)\left(2.00\times10^{-3} \text{ s}\right)\right]\right)^2}{2\left(840\times10^{-12}\right)} = \boxed{6.03 \text{ J}}$

(b) $U_L = \frac{1}{2}LI^2 = \frac{1}{2}L\omega^2 Q_{\max}^2\sin^2(\omega t) = \dfrac{Q_{\max}^2\sin^2(\omega t)}{2C}$

$$U_L = \frac{\left(105\times10^{-6} \text{ C}\right)^2\sin^2\left[\left(1.899\times10^4 \text{ rad/s}\right)\left(2.00\times10^{-3} \text{ s}\right)\right]}{2\left(840\times10^{-12} \text{ F}\right)} = \boxed{0.529 \text{ J}}$$

(c) $U_{\text{total}} = U_C + U_L = \boxed{6.56 \text{ J}}$

32.56 (a) $\omega_d = \sqrt{\frac{1}{LC} - \left(\frac{R}{2L}\right)^2} = \sqrt{\frac{1}{(2.20 \times 10^{-3})(1.80 \times 10^{-6})} - \left(\frac{7.60}{2(2.20 \times 10^{-3})}\right)^2} = 1.58 \times 10^4 \text{ rad / s}$

Therefore, $f_d = \frac{\omega_d}{2\pi} = \boxed{2.51 \text{ kHz}}$

(b) $R_c = \sqrt{\frac{4L}{C}} = \boxed{69.9\ \Omega}$

32.57 (a) $\omega_0 = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{(0.500)(0.100 \times 10^{-6})}} = \boxed{4.47 \text{ krad / s}}$

(b) $\omega_d = \sqrt{\frac{1}{LC} - \left(\frac{R}{2L}\right)^2} = \boxed{4.36 \text{ krad / s}}$

(c) $\frac{\Delta\omega}{\omega_0} = \boxed{2.53\% \text{ lower}}$

32.58 Choose to call positive current clockwise in Figure 32.19. It drains charge from the capacitor according to $I = -dQ/dt$. A clockwise trip around the circuit then gives

$$+\frac{Q}{C} - IR - L\frac{dI}{dt} = 0$$

$+\frac{Q}{C} + \frac{dQ}{dt}R + L\frac{d}{dt}\frac{dQ}{dt} = 0$, identical with Equation 32.29.

32.59 (a) $Q = Q_{\max} e^{-\frac{Rt}{2L}} \cos \omega_d t$ so $I_{\max} \propto e^{-\frac{Rt}{2L}}$

$0.500 = e^{-\frac{Rt}{2L}}$ and $\frac{Rt}{2L} = -\ln(0.500)$

$t = -\frac{2L}{R}\ln(0.500) = \boxed{0.693\left(\frac{2L}{R}\right)}$

(b) $U_0 \propto Q_{\max}^2$ and $U = 0.500 U_0$ so $Q = \sqrt{0.500}\, Q_{\max} = 0.707 Q_{\max}$

$t = -\frac{2L}{R}\ln(0.707) = \boxed{0.347\left(\frac{2L}{R}\right)}$ (half as long)

32.60 With $Q = Q_{\max}$ at $t = 0$, the charge on the capacitor at any time is $Q = Q_{\max}\cos\omega t$ where $\omega = 1/\sqrt{LC}$. The energy stored in the capacitor at time t is then

$$U = \frac{Q^2}{2C} = \frac{Q_{\max}^2}{2C}\cos^2\omega t = U_0\cos^2\omega t.$$

When $U = \frac{1}{4}U_0$, $\cos\omega t = \frac{1}{2}$ and $\omega t = \frac{1}{3}\pi$ rad

Therefore, $\dfrac{t}{\sqrt{LC}} = \dfrac{\pi}{3}$ or $\dfrac{t^2}{LC} = \dfrac{\pi^2}{9}$

The inductance is then: $L = \boxed{\dfrac{9t^2}{\pi^2 C}}$

32.61 (a) $\mathcal{E}_L = -L\dfrac{dI}{dt} = -(1.00\text{ mH})\dfrac{d(20.0t)}{dt} = \boxed{-20.0\text{ mV}}$

(b) $Q = \int_0^t I\,dt = \int_0^t (20.0t)\,dt = 10.0t^2$

$$\Delta V_C = \frac{-Q}{C} = \frac{-10.0t^2}{1.00\times10^{-6}\text{ F}} = \boxed{-\left(10.0\text{ MV/s}^2\right)t^2}$$

(c) When $\dfrac{Q^2}{2C} \ge \dfrac{1}{2}LI^2$, or $\dfrac{\left(-10.0t^2\right)^2}{2\left(1.00\times10^{-6}\right)} \ge \dfrac{1}{2}\left(1.00\times10^{-3}\right)(20.0t)^2$,

then $100t^4 \ge \left(400\times10^{-9}\right)t^2$. The earliest time this is true is at $t = \sqrt{4.00\times10^{-9}}\text{ s} = \boxed{63.2\ \mu\text{s}}$

32.62 (a) $\mathcal{E}_L = -L\dfrac{dI}{dt} = -L\dfrac{d}{dt}(Kt) = \boxed{-LK}$

(b) $I = \dfrac{dQ}{dt}$, so $Q = \int_0^t I\,dt = \int_0^t Kt\,dt = \frac{1}{2}Kt^2$

and $\Delta V_C = \dfrac{-Q}{C} = \boxed{-\dfrac{Kt^2}{2C}}$

(c) When $\frac{1}{2}C(\Delta V_C)^2 = \frac{1}{2}LI^2$, $\frac{1}{2}C\left(\dfrac{K^2t^4}{4C^2}\right) = \frac{1}{2}L\left(K^2t^2\right)$

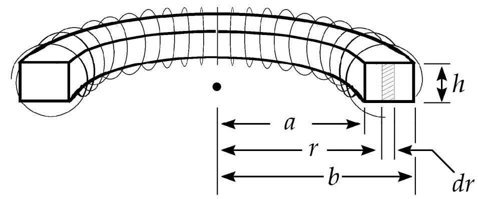
Thus $t = \boxed{2\sqrt{LC}}$

32.63 $\frac{1}{2}\frac{Q^2}{C} = \frac{1}{2C}\left(\frac{Q}{2}\right)^2 + \frac{1}{2}LI^2$ so $I = \sqrt{\frac{3Q^2}{4CL}}$

The flux through each turn of the coil is $\Phi_B = \frac{LI}{N} = \boxed{\frac{Q}{2N}\sqrt{\frac{3L}{C}}}$

where N is the number of turns.

32.64 Equation 30.16: $B = \frac{\mu_0 NI}{2\pi r}$



(a) $\Phi_B = \int B\,dA = \int_a^b \frac{\mu_0 NI}{2\pi r} h\,dr = \frac{\mu_0 NIh}{2\pi}\int_a^b \frac{dr}{r} = \frac{\mu_0 NIh}{2\pi}\ln\left(\frac{b}{a}\right)$

$L = \frac{N\Phi_B}{I} = \boxed{\frac{\mu_0 N^2 h}{2\pi}\ln\left(\frac{b}{a}\right)}$

(b) $L = \frac{\mu_0(500)^2(0.0100)}{2\pi}\ln\left(\frac{12.0}{10.0}\right) = \boxed{91.2\ \mu\text{H}}$

(c) $L_{\text{appx}} = \frac{\mu_0 N^2}{2\pi}\left(\frac{A}{R}\right) = \frac{\mu_0(500)^2}{2\pi}\left(\frac{2.00\times 10^{-4}\ \text{m}^2}{0.110}\right) = \boxed{90.9\ \mu\text{H}}$

***32.65** (a) At the center, $B = \frac{N\mu_0 IR^2}{2(R^2+0^2)^{3/2}} = \frac{N\mu_0 I}{2R}$

So the coil creates flux through itself $\Phi_B \approx BA\cos\theta = \frac{N\mu_0 I}{2R}\pi R^2\cos 0° = \frac{\pi}{2}N\mu_0 IR$

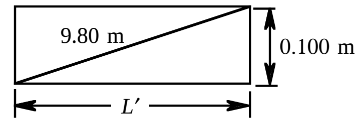
When the current it carries changes, $\mathcal{E}_L = -N\frac{d\Phi_B}{dt} \approx -N\frac{\pi}{2}N\mu_0 R\frac{dI}{dt} = -L\frac{dI}{dt}$

so $\boxed{L \approx \frac{\pi}{2}N^2\mu_0 R}$

(b) $2\pi r \approx 3(0.3\ \text{m})$, so $r \approx 0.14$ m; $L \approx \frac{\pi}{2}\ 1^2\left(4\pi\times 10^{-7}\frac{\text{T}\cdot\text{m}}{\text{A}}\right)\ 0.14\ \text{m} = 2.8\times 10^{-7}\ \text{H}\ \boxed{\sim 100\ \text{nH}}$

(c) $\frac{L}{R} \approx \frac{2.8\times 10^{-7}\ \text{V}\cdot\text{s/A}}{270\ \text{V/A}} = 1.0\times 10^{-9}\ \text{s}\ \boxed{\sim 1\ \text{ns}}$

32.66 (a) If unrolled, the wire forms the diagonal of a 0.100 m (10.0 cm) rectangle as shown. The length of this rectangle is



$$L' = \sqrt{(9.80\text{ m})^2 - (0.100\text{ m})^2}$$

The mean circumference of each turn is $C = 2\pi r'$, where $r' = \frac{24.0 + 0.644}{2}$ mm is the mean radius of each turn. The number of turns is then:

$$N = \frac{L'}{C} = \frac{\sqrt{(9.80\text{ m})^2 - (0.100\text{ m})^2}}{2\pi\left(\frac{24.0 + 0.644}{2}\right)\times 10^{-3}\text{ m}} = \boxed{127}$$

(b) $$R = \frac{\rho \ell}{A} = \frac{\left(1.70\times 10^{-8}\ \Omega\cdot\text{m}\right)(10.0\text{ m})}{\pi\left(0.322\times 10^{-3}\text{ m}\right)^2} = \boxed{0.522\ \Omega}$$

(c) $$L = \frac{\mu N^2 A}{\ell'} = \frac{800\mu_0}{\ell'}\left(\frac{L'}{C}\right)^2 \pi (r')^2$$

$$L = \frac{800\left(4\pi\times 10^{-7}\right)}{0.100\text{ m}}\left(\frac{\sqrt{(9.80\text{ m})^2 - (0.100\text{ m})^2}}{\pi(24.0 + 0.644)\times 10^{-3}\text{ m}}\right)^2 \pi\left[\left(\frac{24.0 + 0.644}{2}\right)\times 10^{-3}\text{ m}\right]^2$$

$$L = 7.68\times 10^{-2}\text{ H} = \boxed{76.8\text{ mH}}$$

32.67 From Ampere's law, the magnetic field at distance $r \le R$ is found as:

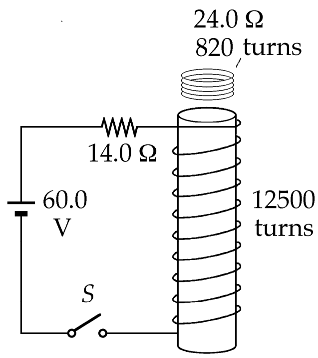
$$B(2\pi r) = \mu_0 J\left(\pi r^2\right) = \mu_0\left(\frac{I}{\pi R^2}\right)\left(\pi r^2\right), \quad \text{or} \quad B = \frac{\mu_0 I r}{2\pi R^2}$$

The magnetic energy per unit length within the wire is then

$$\frac{U}{\ell} = \int_0^R \frac{B^2}{2\mu_0}(2\pi r\, dr) = \frac{\mu_0 I^2}{4\pi R^4}\int_0^R r^3\, dr = \frac{\mu_0 I^2}{4\pi R^4}\left(\frac{R^4}{4}\right) = \boxed{\frac{\mu_0 I^2}{16\pi}}$$

This is independent of the radius of the wire.

32.68 The primary circuit (containing the battery and solenoid) is an RL circuit with $R = 14.0\ \Omega$, and



$$L = \frac{\mu_0 N^2 A}{l} = \frac{(4\pi\times10^{-7})(12\ 500)^2(1.00\times10^{-4})}{0.0700} = 0.280\ \text{H}$$

(a) The time for the current to reach 63.2% of the maximum value is the time constant of the circuit:

$$\tau = \frac{L}{R} = \frac{0.280\ \text{H}}{14.0\ \Omega} = 0.0200\ \text{s} = \boxed{20.0\ \text{ms}}$$

(b) The solenoid's average back emf is $\left|\bar{\mathcal{E}}_L\right| = L\left(\frac{\Delta I}{\Delta t}\right) = L\left(\frac{I_f - 0}{\Delta t}\right)$

where $I_f = 0.632 I_{\max} = 0.632\left(\frac{\Delta V}{R}\right) = 0.632\left(\frac{60.0\ \text{V}}{14.0\ \Omega}\right) = 2.71\ \text{A}$

Thus, $\left|\bar{\mathcal{E}}_L\right| = (0.280\ \text{H})\left(\frac{2.71\ \text{A}}{0.0200\ \text{s}}\right) = \boxed{37.9\ \text{V}}$

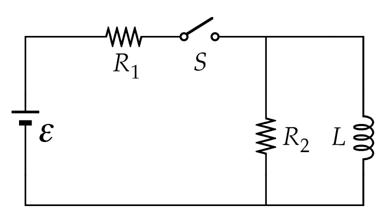
(c) The average rate of change of flux through each turn of the overwrapped concentric coil is the same as that through a turn on the solenoid:

$$\frac{\Delta\Phi_B}{\Delta t} = \frac{\mu_0 n(\Delta I)A}{\Delta t} = \frac{(4\pi\times10^{-7}\ \text{T}\cdot\text{m/A})(12\,500/0.0700\ \text{m})(2.71\ \text{A})(1.00\times10^{-4}\ \text{m}^2)}{0.0200\ \text{s}} = \boxed{3.04\ \text{mV}}$$

(d) The magnitude of the average induced emf in the coil is $\left|\bar{\mathcal{E}}_L\right| = N(\Delta\Phi_B/\Delta t)$ and magnitude of the average induced current is

$$I = \frac{\left|\bar{\mathcal{E}}_L\right|}{R} = \frac{N}{R}\left(\frac{\Delta\Phi_B}{\Delta t}\right) = \frac{820}{24.0\ \Omega}(3.04\times10^{-3}\ \text{V}) = 0.104\ \text{A} = \boxed{104\ \text{mA}}$$

32.69 Left-hand loop: $\mathcal{E} - (I + I_2)R_1 - I_2 R_2 = 0$



Outside loop: $\mathcal{E} - (I + I_2)R_1 - L\frac{dI}{dt} = 0$

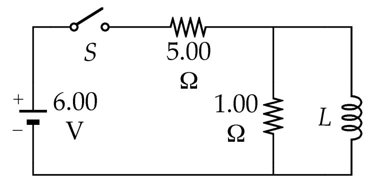
Eliminating I_2 gives $\mathcal{E}' - IR' - L\frac{dI}{dt} = 0$

This is of the same form as Equation 32.6, so its solution is of the same form as Equation 32.7: $I(t) = \frac{\mathcal{E}'}{R'}(1 - e^{-R't/L})$

But $R' = R_1R_2/(R_1 + R_2)$ and $\mathcal{E}' = R_2\mathcal{E}/(R_1 + R_2)$, so $\frac{\mathcal{E}'}{R'} = \frac{\mathcal{E}R_2/(R_1+R_2)}{R_1R_2/(R_1+R_2)} = \frac{\mathcal{E}}{R_1}$

Thus $I(t) = \frac{\mathcal{E}}{R_1}(1 - e^{-R't/L})$

32.70 When switch is closed, steady current $I_0 = 1.20$ A. When the switch is opened after being closed a long time, the current in the right loop is



$$I = I_0 e^{-R_2 t/L}$$

so $e^{Rt/L} = \frac{I_0}{I}$ and $\frac{Rt}{L} = \ln\left(\frac{I_0}{I}\right)$

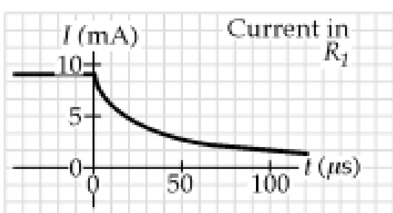
Therefore, $L = \frac{R_2 t}{\ln(I_0/I)} = \frac{(1.00\ \Omega)(0.150\ \text{s})}{\ln(1.20\ \text{A}/0.250\ \text{A})} = 0.0956\ \text{H} = \boxed{95.6\ \text{mH}}$

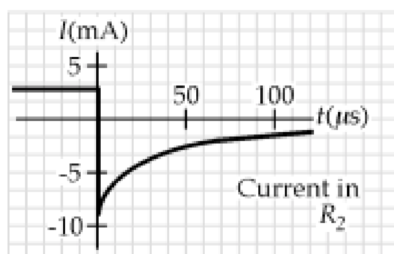
32.71 (a) While steady-state conditions exist, a 9.00 mA flows clockwise around the right loop of the circuit. Immediately after the switch is opened, a 9.00 mA current will flow around the outer loop of the circuit. Applying Kirchhoff's loop rule to this loop gives:

$$+\mathcal{E}_0 - \left[(2.00 + 6.00)\times 10^3\ \Omega\right]\left(9.00\times 10^{-3}\ \text{A}\right) = 0$$

$$+\mathcal{E}_0 = \boxed{72.0\ \text{V with end } b \text{ at the higher potential}}$$

(b)





(c) After the switch is opened, the current around the outer loop decays as

$I = I_{max} e^{-Rt/L}$ with $I_{max} = 9.00$ mA, $R = 8.00$ kΩ, and $L = 0.400$ H

Thus, when the current has reached a value $I = 2.00$ mA, the elapsed time is:

$$t = \left(\frac{L}{R}\right)\ln\left(\frac{I_{max}}{I}\right) = \left(\frac{0.400\ \text{H}}{8.00\times 10^3\ \Omega}\right)\ln\left(\frac{9.00}{2.00}\right) = 7.52\times 10^{-5}\ \text{s} = \boxed{75.2\ \mu\text{s}}$$

32.72 (a) The instant after the switch is closed, the situation is as shown in the circuit diagram of Figure (a). The requested quantities are:

$$\boxed{I_L = 0, \quad I_C = \mathcal{E}_0/R, \quad I_R = \mathcal{E}_0/R}$$

$$\boxed{\Delta V_L = \mathcal{E}_0, \quad \Delta V_C = 0, \quad \Delta V_R = \mathcal{E}_0}$$

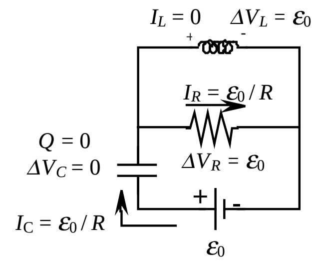


Figure (a)

(b) After the switch has been closed a long time, the steady-state conditions shown in Figure (b) will exist. The currents and voltages are:

$$\boxed{I_L = 0, \quad I_C = 0, \quad I_R = 0}$$

$$\boxed{\Delta V_L = 0, \quad \Delta V_C = \mathcal{E}_0, \quad \Delta V_R = 0}$$

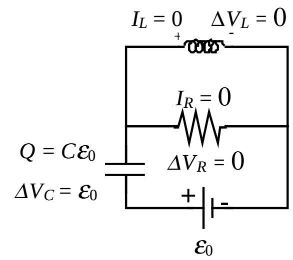


Figure (b)

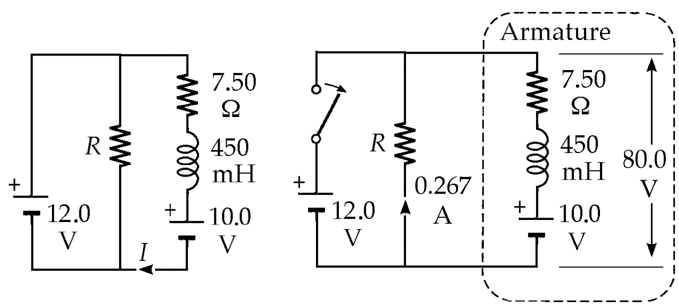
32.73 When the switch is closed, as shown in Figure (a), the current in the inductor is I:

$12.0 - 7.50I - 10.0 = 0 \rightarrow I = 0.267$ A

When the switch is opened, the initial current in the inductor remains at 0.267 A.

$IR = \Delta V$: $(0.267 \text{ A})R \le 80.0$ V

$$\boxed{R \le 300\ \Omega}$$



(a) **(b)**

Goal Solution

To prevent damage from arcing in an electric motor, a discharge resistor is sometimes placed in parallel with the armature. If the motor is suddenly unplugged while running, this resistor limits the voltage that appears across the armature coils. Consider a 12.0-V dc motor with an armature that has a resistance of 7.50 Ω and an inductance of 450 mH. Assume that the back emf in the armature coils is 10.0 V when the motor is running at normal speed. (The equivalent circuit for the armature is shown in Figure P32.73.) Calculate the maximum resistance R that limits the voltage across the armature to 80.0 V when the motor is unplugged.

G: We should expect R to be significantly greater than the resistance of the armature coil, for otherwise a large portion of the source current would be diverted through R and much of the total power would be wasted on heating this discharge resistor.

O: When the motor is unplugged, the 10-V back emf will still exist for a short while because the motor's inertia will tend to keep it spinning. Now the circuit is reduced to a simple series loop with an emf, inductor, and two resistors. The current that was flowing through the armature coil must now flow through the discharge resistor, which will create a voltage across R that we wish to limit to 80 V. As time passes, the current will be reduced by the opposing back emf, and as the motor slows down, the back emf will be reduced to zero, and the current will stop.

A: The steady-state coil current when the switch is closed is found from applying Kirchhoff's loop rule to the outer loop:

$$+12.0\text{ V} - I(7.50\ \Omega) - 10.0\text{ V} = 0$$

so $$I = \frac{2.00\text{ V}}{7.50\ \Omega} = 0.267\text{ A}$$

We then require that $$\Delta V_R = 80.0\text{ V} = (0.267\text{ A})R$$

so $$R = \frac{\Delta V_R}{I} = \frac{80.0\text{ V}}{0.267\text{ A}} = 300\ \Omega$$

L: As we expected, this discharge resistance is considerably greater than the coil's resistance. Note that while the motor is running, the discharge resistor turns $\mathcal{P} = (12\text{ V})^2/300\ \Omega = 0.48\text{ W}$ of power into heat (or wastes 0.48 W). The source delivers power at the rate of about $\mathcal{P} = IV = [0.267\text{ A} + (12\text{ V} / 300\ \Omega)](12\text{ V}) = 3.68\text{ W}$, so the discharge resistor wastes about 13% of the total power. For a sense of perspective, this 4-W motor could lift a 40-N weight at a rate of 0.1 m/s.

32.74 (a) $$L_1 = \frac{\mu_0 N_1^2 A}{\ell_1} = \frac{(4\pi\times10^{-7}\text{ T}\cdot\text{m/A})(1000)^2(1.00\times10^{-4}\text{ m}^2)}{0.500\text{ m}} = 2.51\times10^{-4}\text{ H} = \boxed{251\ \mu\text{H}}$$

(b) $$M = \frac{N_2\Phi_2}{I_1} = \frac{N_2\Phi_1}{I_1} = \frac{N_2BA}{I_1} = \frac{N_2[\mu_0(N_1/\ell_1)I_1]A}{I_1} = \frac{\mu_0 N_1 N_2 A}{\ell_1}$$

$$M = \frac{(4\pi\times10^{-7}\text{ T}\cdot\text{m/A})(1000)(100)(1.00\times10^{-4}\text{ m}^2)}{0.500\text{ m}} = 2.51\times10^{-5}\text{ H} = \boxed{25.1\ \mu\text{H}}$$

(c) $$\mathcal{E}_1 = -M\frac{dI_2}{dt}, \quad \text{or} \quad I_1R_1 = -M\frac{dI_2}{dt} \quad \text{and} \quad I_1 = \frac{dQ_1}{dt} = -\frac{M}{R_1}\frac{dI_2}{dt}$$

$$Q_1 = -\frac{M}{R_1}\int_0^{t_f} dI_2 = -\frac{M}{R_1}(I_{2f} - I_{2i}) = -\frac{M}{R_1}(0 - I_{2i}) = \frac{MI_{2i}}{R_1}$$

$$Q_1 = \frac{(2.51\times10^{-5}\text{ H})(1.00\text{ A})}{1000\ \Omega} = 2.51\times10^{-8}\text{ C} = \boxed{25.1\text{ nC}}$$

32.75 (a) It has a magnetic field, and it stores energy, so $L = \frac{2U}{I^2}$ is non-zero.

(b) Every field line goes through the rectangle between the conductors.

(c) $\Phi = LI$ so $L = \frac{\Phi}{I} = \frac{1}{I}\int_{y=a}^{w-a} B\,da$

$$L = \frac{1}{I}\int_a^{w-a} x\,dy\left(\frac{\mu_0 I}{2\pi y} + \frac{\mu_0 I}{2\pi(w-y)}\right) = \frac{2}{I}\int \frac{\mu_0 Ix}{2\pi y}dy = \frac{2\mu_0 x}{2\pi}\ln y\Big|_a^{w-a}$$

Thus $L = \frac{\mu_0 x}{\pi}\ln\left(\frac{w-a}{a}\right)$

32.76 For an RL circuit, $I(t) = I_{\max}e^{-\frac{R}{L}t}$: $\frac{I(t)}{I_{\max}} = 1 - 10^{-9} = e^{-\frac{R}{L}t} \cong 1 - \frac{R}{L}t$

$\frac{R}{L}t = 10^{-9}$ so $R_{\max} = \frac{(3.14\times10^{-8})(10^{-9})}{(2.50\text{ yr})(3.16\times10^{7}\text{ s / yr})} = \boxed{3.97\times10^{-25}\ \Omega}$

(If the ring were of purest copper, of diameter 1 cm, and cross-sectional area 1 mm^2, its resistance would be at least $10^{-6}\ \Omega$).

32.77 (a) $U_B = \frac{1}{2}LI^2 = \frac{1}{2}(50.0\text{ H})(50.0\times10^3\text{ A})^2 = \boxed{6.25\times10^{10}\text{ J}}$

(b) Two adjacent turns are parallel wires carrying current in the same direction. Since the loops have such large radius, a one-meter section can be regarded as straight.

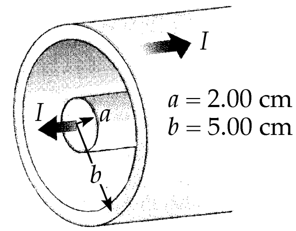
Then one wire creates a field of $B = \frac{\mu_0 I}{2\pi r}$

This causes a force on the next wire of $F = I\ell B\sin\theta$

giving $F = I\ell\frac{\mu_0 I}{2\pi r}\sin 90° = \frac{\mu_0 \ell I^2}{2\pi r}$

Solving for the force, $F = (4\pi\times10^{-7}\text{ N / A}^2)\frac{(1.00\text{ m})(50.0\times10^3\text{ A})^2}{(2\pi)(0.250\text{ m})} = \boxed{2000\text{ N}}$

32.78 $\mathcal{P} = I(\Delta V)$



$$I = \frac{\mathcal{P}}{\Delta V} = \frac{1.00 \times 10^9 \text{ W}}{200 \times 10^3 \text{ V}} = 5.00 \times 10^3 \text{ A}$$

From Ampere's law, $B(2\pi r) = \mu_0 I_{\text{enclosed}}$ or $B = \dfrac{\mu_0 I_{\text{enclosed}}}{2\pi r}$

(a) At $r = a = 0.0200$ m, $I_{\text{enclosed}} = 5.00 \times 10^3$ A and

$$B = \frac{(4\pi \times 10^{-7} \text{ T}\cdot\text{m/A})(5.00 \times 10^3 \text{ A})}{2\pi(0.0200 \text{ m})} = 0.0500 \text{ T} = \boxed{50.0 \text{ mT}}$$

(b) At $r = b = 0.0500$ m, $I_{\text{enclosed}} = I = 5.00 \times 10^3$ A and

$$B = \frac{(4\pi \times 10^{-7} \text{ T}\cdot\text{m/A})(5.00 \times 10^3 \text{ A})}{2\pi(0.0500 \text{ m})} = 0.0200 \text{ T} = \boxed{20.0 \text{ mT}}$$

(c) $$U = \int u\,dV = \int_{r=a}^{r=b} \frac{[B(r)]^2 (2\pi r \ell\, dr)}{2\mu_0} = \frac{\mu_0 I^2 \ell}{4\pi}\int_a^b \frac{dr}{r} = \frac{\mu_0 I^2 \ell}{4\pi}\ln\left(\frac{b}{a}\right)$$

$$U = \frac{(4\pi \times 10^{-7} \text{ T}\cdot\text{m/A})(5.00 \times 10^3 \text{ A})^2 (1000 \times 10^3 \text{ m})}{4\pi}\ln\left(\frac{5.00 \text{ cm}}{2.00 \text{ cm}}\right) = 2.29 \times 10^6 \text{ J} = \boxed{2.29 \text{ MJ}}$$

(d) The magnetic field created by the inner conductor exerts a force of repulsion on the current in the outer sheath. The strength of this field, from part (b), is 20.0 mT. Consider a small rectangular section of the outer cylinder of length ℓ and width w. It carries a current of

$$(5.00 \times 10^3 \text{ A})\left(\frac{w}{2\pi(0.0500 \text{ m})}\right)$$

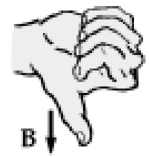
and experiences an outward force $$F = I\ell B \sin\theta = \frac{(5.00 \times 10^3 \text{ A})w}{2\pi(0.0500 \text{ m})}\ell(20.0 \times 10^{-3} \text{ T})\sin 90.0°$$

The pressure on it is $$P = \frac{F}{A} = \frac{F}{w\ell} = \frac{(5.00 \times 10^3 \text{ A})(20.0 \times 10^{-3} \text{ T})}{2\pi(0.0500 \text{ m})} = \boxed{318 \text{ Pa}}$$

***32.79** (a) $B = \frac{\mu_0 NI}{l} = \frac{(4\pi \times 10^{-7}\ \text{T} \cdot \text{m/A})(1400)(2.00\ \text{A})}{1.20\ \text{m}} = \boxed{2.93 \times 10^{-3}\ \text{T (upward)}}$

(b) $u = \frac{B^2}{2\mu_0} = \frac{(2.93 \times 10^{-3}\ \text{T})^2}{2(4\pi \times 10^{-7}\ \text{T} \cdot \text{m/A})} = 3.42\ \frac{\text{J}}{\text{m}^3}\left(\frac{1\ \text{N} \cdot \text{m}}{1\ \text{J}}\right) = 3.42\ \frac{\text{N}}{\text{m}^2} = \boxed{3.42\ \text{Pa}}$

(c) To produce a downward magnetic field, the surface of the super conductor must carry a $\boxed{\text{clockwise}}$ current.



(d) The vertical component of the field of the solenoid exerts an inward force on the superconductor. The total horizontal force is zero. Over the top end of the solenoid, its field diverges and has a radially outward horizontal component. This component exerts upward force on the clockwise superconductor current. The total force on the core is $\boxed{\text{upward}}$. You can think of it as a force of repulsion between the solenoid with its north end pointing up, and the core, with its north end pointing down.

(e) $F = PA = (3.42\ \text{Pa})\left[\pi(1.10 \times 10^{-2}\ \text{m})^2\right] = \boxed{1.30 \times 10^{-3}\ \text{N}}$

Note that we have not proven that energy density is pressure. In fact, it is not in some cases; see problem 12 in Chapter 21.