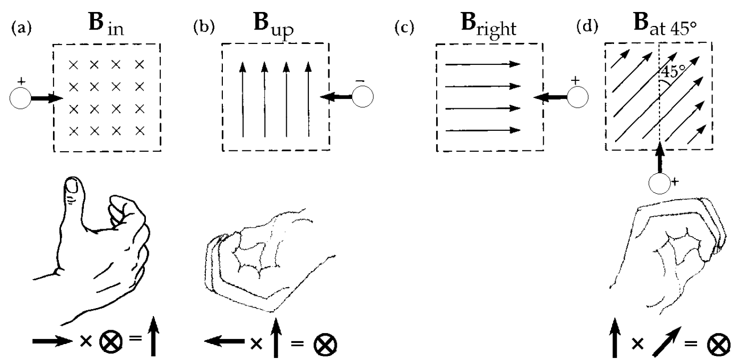
Chapter 29 Solutions

29.1 (a) up

(b) out of the page, since the charge is negative.

(c) no deflection

(d) into the page



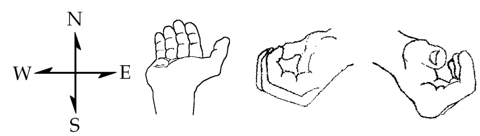
29.2 At the equator, the Earth's magnetic field is horizontally north. Because an electron has negative charge, $\mathbf{F} = q\mathbf{v} \times \mathbf{B}$ is opposite in direction to $\mathbf{v} \times \mathbf{B}$. Figures are drawn looking down.

(a) Down × North = East, so the force is directed $\boxed{\text{West}}$

(b) North × North = sin 0° = 0: $\boxed{\text{Zero deflection}}$

(c) West × North = Down, so the force is directed $\boxed{\text{Up}}$

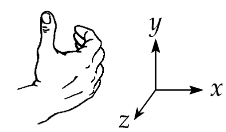
(d) Southeast × North = Up, so the force is $\boxed{\text{Down}}$



(a) (c) (d)

29.3 $\mathbf{F}_B = q\mathbf{v} \times \mathbf{B}$; $\left|\mathbf{F}_B\right|(-\mathbf{j}) = -e\left|\mathbf{v}\right|\mathbf{i} \times \mathbf{B}$

Therefore, $B = \left|\mathbf{B}\right|(-\mathbf{k})$ which indicates the $\boxed{\text{negative } z \text{ direction}}$



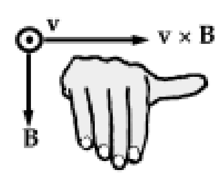
***29.4** (a) $F_B = qvB \sin\theta = (1.60 \times 10^{-19}\ \text{C})(3.00 \times 10^{6}\ \text{m/s})(3.00 \times 10^{-1}\ \text{T}) \sin 37.0°$

$$F_B = \boxed{8.67 \times 10^{-14}\ \text{N}}$$

(b) $$a = \frac{F}{m} = \frac{8.67 \times 10^{-14}\ \text{N}}{1.67 \times 10^{-27}\ \text{kg}} = \boxed{5.19 \times 10^{13}\ \text{m/s}^2}$$

29.5 $F = ma = (1.67 \times 10^{-27}\ \text{kg})(2.00 \times 10^{13}\ \text{m/s}^2) = 3.34 \times 10^{-14}\ \text{N} = qvB \sin 90°$

$$B = \frac{F}{qv} = \frac{3.34 \times 10^{-14}\ \text{N}}{(1.60 \times 10^{-19}\ \text{C})(1.00 \times 10^{7}\ \text{m/s})} = \boxed{2.09 \times 10^{-2}\ \text{T}}$$



The right-hand rule shows that B must be in the $-y$ direction to yield a force in the $+x$ direction when v is in the z direction.

***29.6** First find the speed of the electron: $\Delta K = \frac{1}{2} mv^2 = e(\Delta V) = \Delta U$

$$v = \sqrt{\frac{2e(\Delta V)}{m}} = \sqrt{\frac{2\left(1.60\times10^{-19}\text{ C}\right)(2400\text{ J / C})}{\left(9.11\times10^{-31}\text{ kg}\right)}} = 2.90\times10^{7}\text{ m / s}$$

(a) $F_{B,\max} = qvB = (1.60\times10^{-19}\text{ C})(2.90\times10^{7}\text{ m/s})(1.70\text{ T}) = \boxed{7.90\times10^{-12}\text{ N}}$

(b) $F_{B,\min} = \boxed{0}$ occurs when v is either parallel to or anti-parallel to B

29.7 Gravitational force: $F_g = mg = (9.11\times10^{-31}\text{ kg})(9.80\text{ m/s}^2) = \boxed{8.93\times10^{-30}\text{ N down}}$

Electric force: $F_e = qE = (-1.60\times10^{-19}\text{ C})100\text{ N/C down} = \boxed{1.60\times10^{-17}\text{ N up}}$

Magnetic force: $\mathbf{F}_B = q\mathbf{v}\times\mathbf{B} = \left(-1.60\times10^{-19}\text{ C}\right)\left(6.00\times10^{6}\ \frac{\text{m}}{\text{s}}\ \mathbf{E}\right)\times\left(50.0\times10^{-6}\ \frac{\text{N}\cdot\text{s}}{\text{C}\cdot\text{m}}\ \mathbf{N}\right)$

$\mathbf{F}_B = -\,4.80\times10^{-17}\text{ N up} = \boxed{4.80\times10^{-17}\text{ N down}}$

29.8 We suppose the magnetic force is small compared to gravity. Then its horizontal velocity component stays nearly constant. We call it $v\,\mathbf{i}$.

From $v_y^2 = v_{yi}^2 + 2a_y(y - y_i)$, the vertical component at impact is $-\sqrt{2gh}\,\mathbf{j}$. Then,

$$\mathbf{F}_B = q\mathbf{v}\times\mathbf{B} = Q\left(v\mathbf{i} - \sqrt{2gh}\,\mathbf{j}\right)\times B\mathbf{k} = QvB(-\mathbf{j}) - Q\sqrt{2gh}\,B\mathbf{i}$$

$\mathbf{F}_B = QvB$ vertical $+\ Q\sqrt{2gh}\,B$ horizontal

$\mathbf{F}_B = 5.00\times10^{-6}\text{ C}(20.0\text{ m/s})(0.0100\text{ T})\ \mathbf{j} + 5.00\times10^{-6}\text{ C}\sqrt{2(9.80\text{ m/s}^2)(20.0\text{ m})}\ (0.0100\text{ T})\ \mathbf{i}$

$\mathbf{F}_B = \boxed{(1.00\times10^{-6}\text{ N})\text{ vertical} + (0.990\times10^{-6}\text{ N})\text{ horizontal}}$

29.9 $F_B = qvB\sin\theta$ so $8.20\times10^{-13}\text{ N} = (1.60\times10^{-19}\text{ C})(4.00\times10^{6}\text{ m/s})(1.70\text{ T})\sin\theta$

$\sin\theta = 0.754$ and $\theta = \sin^{-1}(0.754) = \boxed{48.9^\circ\text{ or }131^\circ}$

29.10 $q\mathbf{E} = \left(-1.60\times10^{-19}\text{ C}\right)\left(20.0\text{ N / C}\right)\mathbf{k} = \left(-3.20\times10^{-18}\text{ N}\right)\mathbf{k}$

$\Sigma\mathbf{F} = q\mathbf{E} + q\mathbf{v}\times\mathbf{B} = m\mathbf{a}$

$\left(-3.20\times10^{-18}\text{ N}\right)\mathbf{k} - 1.60\times10^{-19}\text{ C}(1.20\times10^{4}\text{ m / s }\mathbf{i})\times\mathbf{B} = (9.11\times10^{-31})(2.00\times10^{12}\text{ m / s}^2)\mathbf{k}$

$-(3.20\times10^{-18}\text{ N})\mathbf{k} - (1.92\times10^{-15}\text{ C}\cdot\text{m / s})\mathbf{i}\times\mathbf{B} = (1.82\times10^{-18}\text{ N})\mathbf{k}$

$(1.92\times10^{-15}\text{ C}\cdot\text{m / s})\mathbf{i}\times\mathbf{B} = -(5.02\times10^{-18}\text{ N})\mathbf{k}$

The magnetic field $\boxed{\text{may have any } x\text{-component}}$. $B_z = \boxed{0}$ and $B_y = \boxed{-2.62\text{ mT}}$

29.11 $\mathbf{F}_B = q\mathbf{v}\times\mathbf{B}$

$$\mathbf{v}\times\mathbf{B} = \begin{vmatrix}\mathbf{i} & \mathbf{j} & \mathbf{k}\\ +2 & -4 & +1\\ +1 & +2 & -3\end{vmatrix} = (12-2)\mathbf{i} + (1+6)\mathbf{j} + (4+4)\mathbf{k} = 10\mathbf{i} + 7\mathbf{j} + 8\mathbf{k}$$

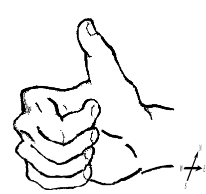
$|\mathbf{v}\times\mathbf{B}| = \sqrt{10^2 + 7^2 + 8^2} = 14.6\text{ T}\cdot\text{m / s}$

$|\mathbf{F}_B| = q|\mathbf{v}\times\mathbf{B}| = \left(1.60\times10^{-19}\text{ C}\right)\left(14.6\text{ T}\cdot\text{m/s}\right) = \boxed{2.34\times10^{-18}\text{ N}}$

29.12 $$\mathbf{F}_B = q\mathbf{v}\times\mathbf{B} = \left(-1.60\times10^{-19}\right)\begin{vmatrix}\mathbf{i} & \mathbf{j} & \mathbf{k}\\ 0 & 3.70\times10^{5} & 0\\ 1.40 & 2.10 & 0\end{vmatrix}$$

$$\mathbf{F}_B = \left(-1.60\times10^{-19}\text{ C}\right)\left[(0-0)\mathbf{i} + (0-0)\mathbf{j} + \left(0-(1.40\text{ T})\left(3.70\times10^{5}\text{ m/s}\right)\right)\mathbf{k}\right] = \boxed{\left(8.29\times10^{-14}\ \mathbf{k}\right)\text{N}}$$

29.13 $F_B = ILB\sin\theta$ with $F_B = F_g = mg$



$mg = ILB\sin\theta$ so $\frac{m}{L}g = IB\sin\theta$

$I = 2.00\text{ A}$ and $\frac{m}{L} = \left(0.500\text{ g / cm}\right)\left(\frac{100\text{ cm / m}}{1000\text{ g / kg}}\right) = 5.00\times10^{-2}\text{ kg / m}$

Thus $(5.00\times10^{-2})(9.80) = (2.00)B\sin 90.0°$

$B = \boxed{0.245\text{ Tesla}}$ with the direction given by right-hand rule: $\boxed{\text{eastward}}$

Goal Solution

A wire having a mass per unit length of 0.500 g/cm carries a 2.00-A current horizontally to the south. What are the direction and magnitude of the minimum magnetic field needed to lift this wire vertically upward?

G: Since $\mathbf{I} = 2.00$ A south, **B** must be to the east to make **F** upward according to the right-hand rule for currents in a magnetic field.

The magnitude of **B** should be significantly greater than the earth's magnetic field (~ 50 μT), since we do not typically see wires levitating when current flows through them.

O: The force on a current-carrying wire in a magnetic field is $\mathbf{F}_B = I\mathbf{l}\times\mathbf{B}$, from which we can find **B**.

A: With I to the south and **B** to the east, the force on the wire is simply $F_B = IlB\sin 90°$, which must oppose the weight of the wire, mg. So,

$$B = \frac{F_B}{Il} = \frac{mg}{Il} = \frac{g}{I}\left(\frac{m}{l}\right) = \left(\frac{9.80\text{ m / s}^2}{2.00\text{ A}}\right)\left(0.500\ \frac{\text{g}}{\text{cm}}\right)\left(\frac{10^2\text{ cm / m}}{10^3\text{ g / kg}}\right) = 0.245\text{ T}$$

L: The required magnetic field is about 5000 times stronger than the earth's magnetic field. Thus it was reasonable to ignore the earth's magnetic field in this problem. In other situations the earth's field can have a significant effect.

29.14 $\mathbf{F}_B = I\mathbf{L}\times\mathbf{B} = (2.40\text{ A})(0.750\text{ m})\mathbf{i}\times(1.60\text{ T})\mathbf{k} = \boxed{(-2.88\,\mathbf{j})\text{ N}}$

29.15 (a) $F_B = ILB\sin\theta = (5.00\text{ A})(2.80\text{ m})(0.390\text{ T})\sin 60.0° = \boxed{4.73\text{ N}}$

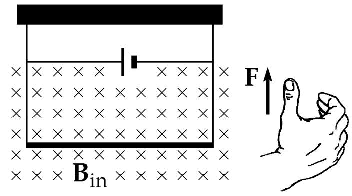
(b) $F_B = (5.00\text{ A})(2.80\text{ m})(0.390\text{ T})\sin 90.0° = \boxed{5.46\text{ N}}$

(c) $F_B = (5.00\text{ A})(2.80\text{ m})(0.390\text{ T})\sin 120° = \boxed{4.73\text{ N}}$

29.16 $$\frac{|\mathbf{F}_B|}{L} = \frac{mg}{L} = \frac{I|\mathbf{L}\times\mathbf{B}|}{L}$$

$$I = \frac{mg}{BL} = \frac{(0.0400\text{ kg / m})(9.80\text{ m / s}^2)}{3.60\text{ T}} = \boxed{0.109\text{ A}}$$

The direction of I in the bar is $\boxed{\text{to the right}}$.



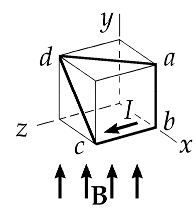
29.17 The magnetic and gravitational forces must balance. Therefore, it is necessary to have $F_B = BIL = mg$, or $I = (mg/BL) = (\lambda g/B)$ [λ is the mass per unit length of the wire].

Thus, $I = \dfrac{(1.00 \times 10^{-3}\ \text{kg/m})(9.80\ \text{m/s}^2)}{(5.00 \times 10^{-5}\ \text{T})} = \boxed{196\ \text{A}}$ (if $B = 50.0\ \mu\text{T}$)

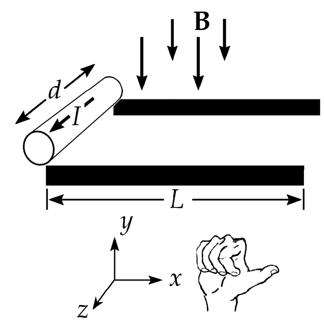
The required direction of the current is $\boxed{\text{eastward}}$, since East × North = Up.

29.18 For each segment, $I = 5.00$ A and $\mathbf{B} = 0.0200\ \text{N/A}\cdot\text{m}\ \mathbf{j}$

Segment	$\mathbf{L}$	$\mathbf{F}_B = I(\mathbf{L} \times \mathbf{B})$
ab	$-0.400\ \text{m}\ \mathbf{j}$	$\boxed{0}$
bc	$0.400\ \text{m}\ \mathbf{k}$	$\boxed{(40.0\ \text{mN})(-\mathbf{i})}$
cd	$-0.400\ \text{m}\ \mathbf{i} + 0.400\ \text{m}\ \mathbf{j}$	$\boxed{(40.0\ \text{mN})(-\mathbf{k})}$
da	$0.400\ \text{m}\ \mathbf{i} - 0.400\ \text{m}\ \mathbf{k}$	$\boxed{(40.0\ \text{mN})(\mathbf{k} + \mathbf{i})}$



29.19 The rod feels force $\mathbf{F}_B = I(\mathbf{d} \times \mathbf{B}) = Id(\mathbf{k}) \times B(-\mathbf{j}) = IdB(\mathbf{i})$



The work-energy theorem is $(K_{\text{trans}} + K_{\text{rot}})_i + \Delta E = (K_{\text{trans}} + K_{\text{rot}})_f$

$$0 + 0 + Fs\cos\theta = \tfrac{1}{2}mv^2 + \tfrac{1}{2}I\omega^2$$

$$IdBL\cos 0^\circ = \tfrac{1}{2}mv^2 + \tfrac{1}{2}\left(\tfrac{1}{2}mR^2\right)\left(\frac{v}{R}\right)^2 \quad \text{and} \quad IdBL = \tfrac{3}{4}mv^2$$

$$v = \sqrt{\frac{4IdBL}{3m}} = \sqrt{\frac{4(48.0\ \text{A})(0.120\ \text{m})(0.240\ \text{T})(0.450\ \text{m})}{3(0.720\ \text{kg})}} = \boxed{1.07\ \text{m/s}}$$

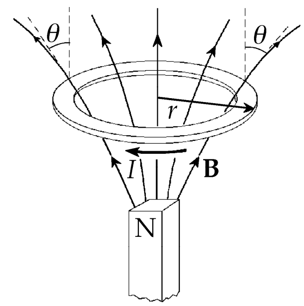
29.20 The rod feels force $\mathbf{F}_B = I(\mathbf{d} \times \mathbf{B}) = Id(\mathbf{k}) \times B(-\mathbf{j}) = IdB(\mathbf{i})$

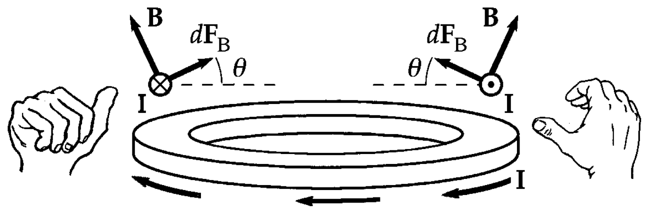
The work-energy theorem is $(K_{\text{trans}} + K_{\text{rot}})_i + \Delta E = (K_{\text{trans}} + K_{\text{rot}})_f$

$$0 + 0 + Fs\cos\theta = \tfrac{1}{2}mv^2 + \tfrac{1}{2}I\omega^2$$

$$IdBL\cos 0^\circ = \tfrac{1}{2}mv^2 + \tfrac{1}{2}\left(\tfrac{1}{2}mR^2\right)\left(\frac{v}{R}\right)^2 \quad \text{and} \quad v = \boxed{\sqrt{\frac{4IdBL}{3m}}}$$

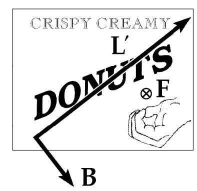
29.21 The magnetic force on each bit of ring is $I\,d\mathbf{s} \times \mathbf{B} = I\,ds\,B$ radially inward and upward, at angle θ above the radial line. The radially inward components tend to squeeze the ring but all cancel out as forces. The upward components $I\,ds\,B\sin\theta$ all add to $\boxed{I\,2\pi rB\sin\theta \text{ up}}$.





***29.22** Take the x-axis east, the y-axis up, and the z-axis south. The field is

$$\mathbf{B} = (52.0\ \mu\text{T})\cos 60.0^\circ\,(-\mathbf{k}) + (52.0\ \mu\text{T})\sin 60.0^\circ\,(-\mathbf{j})$$



The current then has equivalent length: $\mathbf{L}' = 1.40\ \text{m}(-\mathbf{k}) + 0.850\ \text{m}(\mathbf{j})$

$$\mathbf{F}_B = I\mathbf{L}'\times\mathbf{B} = (0.0350\ \text{A})(0.850\mathbf{j} - 1.40\mathbf{k})\text{m} \times (-45.0\mathbf{j} - 26.0\mathbf{k})10^{-6}\ \text{T}$$

$$\mathbf{F}_B = 3.50\times10^{-8}\ \text{N}(-22.1\mathbf{i} - 63.0\mathbf{i}) = 2.98\times10^{-6}\ \text{N}(-\mathbf{i}) = \boxed{2.98\ \mu\text{N west}}$$

29.23 (a) $2\pi r = 2.00$ m so $r = 0.318$ m

$$\mu = IA = (17.0\times10^{-3}\ \text{A})\left[\pi(0.318)^2\ \text{m}^2\right] = \boxed{5.41\ \text{mA}\cdot\text{m}^2}$$

(b) $\tau = \boldsymbol{\mu}\times\mathbf{B}$ so $\tau = (5.41\times10^{-3}\ \text{A}\cdot\text{m}^2)(0.800\ \text{T}) = \boxed{4.33\ \text{mN}\cdot\text{m}}$

***29.24** $\tau = \mu B\sin\theta$ so $4.60\times10^{-3}\ \text{N}\cdot\text{m} = \mu(0.250)\sin 90.0^\circ$

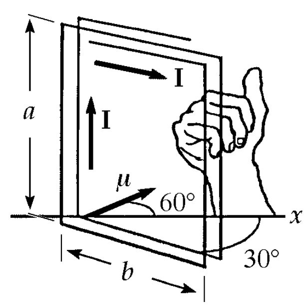
$$\mu = 1.84\times10^{-2}\ \text{A}\cdot\text{m}^2 = \boxed{18.4\ \text{mA}\cdot\text{m}^2}$$

29.25 $\tau = NBAI\sin\theta$

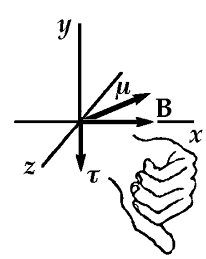
$$\tau = 100(0.800\ \text{T})(0.400\times0.300\ \text{m}^2)(1.20\ \text{A})\sin 60^\circ$$

$$\tau = \boxed{9.98\ \text{N}\cdot\text{m}}$$

Note that θ is the angle between the magnetic moment and the **B** field. The loop will rotate so as to align the magnetic moment with the **B** field. Looking down along the y-axis, the loop will rotate in a $\boxed{\text{clockwise}}$ direction.



(a)



(b)

29.26 (a) Let θ represent the unknown angle; L, the total length of the wire; and d, the length of one side of the square coil. Then, use the right-hand rule to find

$$\mu = NAI = \left(\frac{L}{4d}\right)d^2 I \quad \text{at angle } \theta \text{ with the horizontal.}$$

At equilibrium, $\Sigma\tau = (\boldsymbol{\mu} \times \mathbf{B}) - (\mathbf{r} \times m\mathbf{g}) = 0$

$$\left(\frac{ILBd}{4}\right)\sin(90.0° - \theta) - \left(\frac{mgd}{2}\right)\sin\theta = 0 \quad \text{and} \quad \left(\frac{mgd}{2}\right)\sin\theta = \left(\frac{ILBd}{4}\right)\cos\theta$$

$$\theta = \tan^{-1}\left(\frac{ILB}{2mg}\right) = \tan^{-1}\left(\frac{(3.40\text{ A})(4.00\text{ m})(0.0100\text{ T})}{2(0.100\text{ kg})(9.80\text{ m/s}^2)}\right) = \boxed{3.97°}$$

(b) $$\tau_m = \left(\frac{ILBd}{4}\right)\cos\theta = \frac{1}{4}(3.40\text{ A})(4.00\text{ m})(0.0100\text{ T})(0.100\text{ m})\cos 3.97° = \boxed{3.39\text{ mN}\cdot\text{m}}$$

29.27 From $\tau = \boldsymbol{\mu} \times \mathbf{B} = I\mathbf{A} \times \mathbf{B}$, the magnitude of the torque is $IAB \sin 90.0°$

(a) Each side of the triangle is 40.0 cm / 3.

Its altitude is $\sqrt{13.3^2 - 6.67^2}$ cm = 11.5 cm and its area is

$$A = \frac{1}{2}(11.5\text{ cm})(13.3\text{ cm}) = 7.70 \times 10^{-3}\text{ m}^2$$

Then $\tau = (20.0\text{ A})(7.70 \times 10^{-3}\text{ m}^2)(0.520\text{ N}\cdot\text{s/C}\cdot\text{m}) = \boxed{80.1\text{ mN}\cdot\text{m}}$

(b) Each side of the square is 10.0 cm and its area is 100 cm^2 = 10^{-2} m^2.

$$\tau = (20.0\text{ A})(10^{-2}\text{ m}^2)(0.520\text{ T}) = \boxed{0.104\text{ N}\cdot\text{m}}$$

(c) $r = 0.400\text{ m}/2\pi = 0.0637\text{ m}$

$$A = \pi r^2 = 1.27 \times 10^{-2}\text{ m}^2$$

$$\tau = (20.0\text{ A})(1.27 \times 10^{-2}\text{ m}^2)(0.520) = \boxed{0.132\text{ N}\cdot\text{m}}$$

(d) The circular loop experiences the largest torque.

***29.28** Choose $U = 0$ when the dipole moment is at $\theta = 90.0°$ to the field. The field exerts torque of magnitude $\mu B \sin\theta$ on the dipole, tending to turn the dipole moment in the direction of decreasing θ. Its energy is given by

$$U - 0 = \int_{90.0°}^{\theta} \mu B \sin\theta\, d\theta = \mu B(-\cos\theta)\Big|_{90.0°}^{\theta} = -\mu B\cos\theta + 0 \qquad \text{or} \qquad \boxed{U = -\boldsymbol{\mu}\cdot\mathbf{B}}$$

***29.29** (a) The field exerts torque on the needle tending to align it with the field, so the minimum energy orientation of the needle is:

$\boxed{\text{pointing north at 48.0° below the horizontal}}$

where its energy is $U_{\min} = -\mu B\cos 0° = -(9.70\times10^{-3}\ \text{A}\cdot\text{m}^2)(55.0\times10^{-6}\ \text{T}) = -5.34\times10^{-7}\ \text{J}$

It has maximum energy when pointing in the opposite direction,

$\boxed{\text{south at 48.0° above the horizontal}}$

where its energy is $U_{\max} = -\mu B\cos 180° = +(9.70\times10^{-3}\ \text{A}\cdot\text{m}^2)(55.0\times10^{-6}\ \text{T}) = +5.34\times10^{-7}\ \text{J}$

(b) $U_{\min} + W = U_{\max}$: $W = U_{\max} - U_{\min} = +5.34\times10^{-7}\ \text{J} - (-5.34\times10^{-7}\ \text{J}) = \boxed{1.07\ \mu\text{J}}$

29.30 (a) $\tau = \boldsymbol{\mu}\times\mathbf{B}$, so $\tau = |\boldsymbol{\mu}\times\mathbf{B}| = \mu B\sin\theta = NIAB\sin\theta$

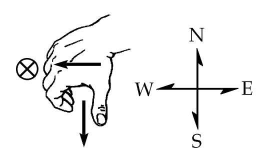
$\tau_{\max} = NIAB\sin 90.0° = 1(5.00\ \text{A})\left[\pi(0.0500\ \text{m})^2\right](3.00\times10^{-3}\ \text{T}) = \boxed{118\ \mu\text{N}\cdot\text{m}}$

(b) $U = -\boldsymbol{\mu}\cdot\mathbf{B}$, so $-\mu B \le U \le +\mu B$

Since $\mu B = (NIA)B = 1(5.00\ \text{A})\left[\pi(0.0500\ \text{m})^2\right](3.00\times10^{-3}\ \text{T}) = 118\ \mu\text{J}$,

the range of the potential energy is: $\boxed{-118\ \mu\text{J} \le U \le +118\ \mu\text{J}}$

29.31 (a) $B = 50.0\times10^{-6}$ T; $v = 6.20\times10^{6}$ m/s



Direction is given by the right-hand-rule: $\boxed{\text{southward}}$

$F_B = qvB\sin\theta$

$F_B = (1.60\times10^{-19}\ \text{C})(6.20\times10^{6}\ \text{m/s})(50.0\times10^{-6}\ \text{T})\sin 90.0° = \boxed{4.96\times10^{-17}\ \text{N}}$

(b) $F = \dfrac{mv^2}{r}$ so $r = \dfrac{mv^2}{F} = \dfrac{(1.67\times10^{-27}\ \text{kg})(6.20\times10^{6}\ \text{m/s})^2}{4.96\times10^{-17}\ \text{N}} = \boxed{1.29\ \text{km}}$

29.32 (a) $\frac{1}{2}mv^2 = q(\Delta V)$ $\quad \frac{1}{2}(3.20\times10^{-26}\ \text{kg})v^2 = (1.60\times10^{-19}\ \text{C})(833\ \text{V})$ $\quad v = 91.3$ km/s

The magnetic force provides the centripetal force: $qvB\sin\theta = \dfrac{mv^2}{r}$

$$r = \frac{mv}{qB\sin 90.0°} = \frac{(3.20\times 10^{-26}\text{ kg})(9.13\times 10^{4}\text{ m/s})}{(1.60\times 10^{-19}\text{ C})(0.920\text{ N}\cdot\text{s/C}\cdot\text{m})} = \boxed{1.98\text{ cm}}$$

29.33 For each electron, $|q|vB \sin 90.0° = \frac{mv^2}{r}$ and $v = \frac{eBr}{m}$

The electrons have no internal structure to absorb energy, so the collision must be perfectly elastic:

$$K = \tfrac{1}{2}mv_{1i}^2 + 0 = \tfrac{1}{2}mv_{1f}^2 + \tfrac{1}{2}mv_{2f}^2$$

$$K = \tfrac{1}{2}m\left(\frac{e^2B^2R_1^2}{m^2}\right) + \tfrac{1}{2}m\left(\frac{e^2B^2R_2^2}{m^2}\right) = \frac{e^2B^2}{2m}\left(R_1^2 + R_2^2\right)$$

$$K = \frac{e(1.60\times10^{-19}\text{ C})(0.0440\text{ N}\cdot\text{s / C}\cdot\text{m})^2}{2(9.11\times10^{-31}\text{ kg})}\left[(0.0100\text{ m})^2 + (0.0240\text{ m})^2\right] = \boxed{115\text{ keV}}$$

29.34 We begin with $qvB = \frac{mv^2}{R}$, so $v = \frac{qRB}{m}$

The time to complete one revolution is $T = \frac{2\pi R}{v} = \frac{2\pi R}{\left(\frac{qRB}{m}\right)} = \frac{2\pi m}{qB}$

Solving for B, $B = \frac{2\pi m}{qT} = \boxed{6.56\times10^{-2}\text{ T}}$

29.35 $q(\Delta V) = \tfrac{1}{2}mv^2$ or $v = \sqrt{\frac{2q(\Delta V)}{m}}$

Also, $qvB = \frac{mv^2}{r}$ so $r = \frac{mv}{qB} = \frac{m}{qB}\sqrt{\frac{2q(\Delta V)}{m}} = \sqrt{\frac{2m(\Delta V)}{qB^2}}$

Therefore, $r_p^2 = \frac{2m_p(\Delta V)}{eB^2}$

$$r_d^2 = \frac{2m_d(\Delta V)}{q_dB^2} = \frac{2(2m_p)(\Delta V)}{eB^2} = 2\left(\frac{2m_p(\Delta V)}{eB^2}\right) = 2r_p^2$$

and $r_\alpha{}^2 = \frac{2m_\alpha(\Delta V)}{q_\alpha B^2} = \frac{2(4m_p)(\Delta V)}{(2e)B^2} = 2\left(\frac{2m_p(\Delta V)}{eB^2}\right) = 2r_p{}^2$

The conclusion is: $\boxed{r_\alpha = r_d = \sqrt{2}\,r_p}$

Goal Solution

29.35 A proton (charge $+e$, mass m_p), a deuteron (charge $+e$, mass $2m_p$), and an alpha particle, (charge $+2e$, mass $4m_p$) are accelerated through a common potential difference ΔV. The particles enter a uniform magnetic field **B** with a velocity in a direction perpendicular to **B**. The proton moves in a circular path of radius r_p. Determine the values of the radii of the circular orbits for the deuteron r_d and the alpha particle r_α in terms of r_p.

G: In general, particles with greater speed, more mass, and less charge will have larger radii as they move in a circular path due to a constant magnetic force. Since the effects of mass and charge have opposite influences on the path radius, it is somewhat difficult to predict which particle will have the larger radius. However, since the mass and charge ratios of the three particles are all similar in magnitude within a factor of four, we should expect that the radii also fall within a similar range.

O: The radius of each particle's path can be found by applying Newton's second law, where the force causing the centripetal acceleration is the magnetic force: $\mathbf{F} = q\mathbf{v} \times \mathbf{B}$. The speed of the particles can be found from the kinetic energy resulting from the change in electric potential given.

A: An electric field changes the speed of each particle according to $(K+U)_i = (K+U)_f$. Therefore, assuming that the particles start from rest, we can write $q\Delta V = \frac{1}{2}mv^2$.

The magnetic field changes their direction as described by $\Sigma\mathbf{F} = m\mathbf{a}$: $qvB\sin 90° = \dfrac{mv^2}{r}$

thus $$r = \frac{mv}{qB} = \frac{m}{qB}\sqrt{\frac{2q\Delta V}{m}} = \frac{1}{B}\sqrt{\frac{2m\Delta V}{q}}$$

For the protons, $$r_p = \frac{1}{B}\sqrt{\frac{2m_p\Delta V}{e}}$$

For the deuterons, $$r_d = \frac{1}{B}\sqrt{\frac{2(2m_p)\Delta V}{e}} = \sqrt{2}r_p$$

For the alpha particles, $$r_\alpha = \frac{1}{B}\sqrt{\frac{2(4m_p)\Delta V}{2e}} = \sqrt{2}r_p$$

L: Somewhat surprisingly, the radii of the deuterons and alpha particles are the same and are only 41% greater than for the protons.

29.36 (a) We begin with $qvB = \dfrac{mv^2}{R}$, or $qRB = mv$. But, $L = mvR = qR^2B$.

Therefore, $$R = \sqrt{\frac{L}{qB}} = \sqrt{\frac{4.00 \times 10^{-25}\ \text{J}\cdot\text{s}}{(1.60 \times 10^{-19}\ \text{C})(1.00 \times 10^{-3}\ \text{T})}} = 0.0500\ \text{m} = \boxed{5.00\ \text{cm}}$$

(b) Thus, $$v = \frac{L}{mR} = \frac{4.00 \times 10^{-25}\ \text{J}\cdot\text{s}}{(9.11 \times 10^{-31}\ \text{kg})(0.0500\ \text{m})} = \boxed{8.78 \times 10^6\ \text{m/s}}$$

29.37 $\omega = \dfrac{qB}{m} = \dfrac{(1.60\times10^{-19}\text{ C})(5.20\text{ T})}{1.67\times10^{-27}\text{ kg}} = \boxed{4.98\times10^{8}\text{ rad/s}}$

29.38 $\frac{1}{2}mv^2 = q(\Delta V)$ so $v = \sqrt{\dfrac{2q(\Delta V)}{m}}$

$r = \dfrac{mv}{qB}$ so $r = \dfrac{m\sqrt{2q(\Delta V)/m}}{qB}$

$r^2 = \dfrac{m}{q}\cdot\dfrac{2(\Delta V)}{B^2}$ and $(r')^2 = \dfrac{m'}{q'}\cdot\dfrac{2(\Delta V)}{B^2}$

$m = \dfrac{qB^2r^2}{2(\Delta V)}$ and $(m') = \dfrac{(q')B^2(r')^2}{2(\Delta V)}$ so $\dfrac{m'}{m} = \dfrac{q'}{q}\cdot\dfrac{(r')^2}{r^2} = \left(\dfrac{2e}{e}\right)\left(\dfrac{2R}{R}\right)^2 = \boxed{8}$

29.39 $E = \frac{1}{2}mv^2 = e(\Delta V)$ and $evB\sin 90° = mv^2/R$

$B = \dfrac{mv}{eR} = \dfrac{m}{eR}\sqrt{\dfrac{2e(\Delta V)}{m}} = \dfrac{1}{R}\sqrt{\dfrac{2m(\Delta V)}{e}}$

$B = \dfrac{1}{5.80\times10^{10}\text{ m}}\sqrt{\dfrac{2(1.67\times10^{-27}\text{ kg})(10.0\times10^{6}\text{ V})}{1.60\times10^{-19}\text{ C}}} = \boxed{7.88\times10^{-12}\text{ T}}$

29.40 $r = \dfrac{mv}{qB}$ so $m = \dfrac{rqB}{v} = \dfrac{(7.94\times10^{-3}\text{ m})(1.60\times10^{-19}\text{ C})(1.80\text{ T})}{4.60\times10^{5}\text{ m/s}}$

$m = 4.97\times10^{-27}\text{ kg}\left(\dfrac{1\text{ u}}{1.66\times10^{-27}\text{ kg}}\right) = \boxed{2.99\text{ u}}$

The particle is singly ionized: either a tritium ion, $\boxed{{}^{3}_{1}\text{H}^{+}}$, or a helium ion, $\boxed{{}^{3}_{2}\text{He}^{+}}$.

29.41 $F_B = F_e$ so $qvB = qE$ where $v = \sqrt{2K/m}$. K is kinetic energy of the electrons.

$E = vB = \sqrt{\dfrac{2K}{m}}B = \left(\dfrac{2(750)(1.60\times10^{-19})}{9.11\times10^{-31}}\right)^{1/2}(0.0150) = \boxed{244\text{ kV/m}}$

29.42 $K = \frac{1}{2} mv^2 = q(\Delta V)$ so $v = \sqrt{\frac{2q(\Delta V)}{m}}$

$$|\mathbf{F}_B| = |q\mathbf{v} \times \mathbf{B}| = \frac{mv^2}{r} \quad r = \frac{mv}{qB} = \frac{m}{q}\frac{\sqrt{2q(\Delta V)/m}}{B} = \frac{1}{B}\sqrt{\frac{2m(\Delta V)}{q}}$$

(a) $r_{238} = \sqrt{\frac{2(238 \times 1.66 \times 10^{-27})2000}{1.60 \times 10^{-19}}}\left(\frac{1}{1.20}\right) = 8.28 \times 10^{-2} \text{ m} = \boxed{8.28 \text{ cm}}$

(b) $r_{235} = \boxed{8.23 \text{ cm}}$

$$\frac{r_{238}}{r_{235}} = \sqrt{\frac{m_{238}}{m_{235}}} = \sqrt{\frac{238.05}{235.04}} = 1.0064$$

The ratios of the orbit radius for different ions are independent of ΔV and B.

29.43 In the velocity selector: $v = \frac{E}{B} = \frac{2500 \text{ V/m}}{0.0350 \text{ T}} = 7.14 \times 10^4 \text{ m/s}$

In the deflection chamber: $r = \frac{mv}{qB} = \frac{(2.18 \times 10^{-26} \text{ kg})(7.14 \times 10^4 \text{ m/s})}{(1.60 \times 10^{-19} \text{ C})(0.0350 \text{ T})} = \boxed{0.278 \text{ m}}$

29.44 $K = \frac{1}{2} mv^2$: $(34.0 \times 10^6 \text{ eV})(1.60 \times 10^{-19} \text{ J/eV}) = \frac{1}{2}(1.67 \times 10^{-27} \text{ kg})v^2$

$v = 8.07 \times 10^7 \text{ m/s}$ $r = \frac{mv}{qB} = \frac{(1.67 \times 10^{-27} \text{ kg})(8.07 \times 10^7 \text{ m/s})}{(1.60 \times 10^{-19} \text{ C})(5.20 \text{ T})} = \boxed{0.162 \text{ m}}$

29.45 (a) $F_B = qvB = \frac{mv^2}{R}$

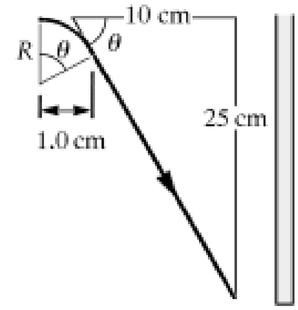
$$\omega = \frac{v}{R} = \frac{qBR}{mR} = \frac{qB}{m} = \frac{(1.60 \times 10^{-19} \text{ C})(0.450 \text{ T})}{1.67 \times 10^{-27} \text{ kg}} = \boxed{4.31 \times 10^7 \text{ rad/s}}$$

(b) $v = \frac{qBR}{m} = \frac{(1.60 \times 10^{-19} \text{ C})(0.450 \text{ T})(1.20 \text{ m})}{1.67 \times 10^{-27} \text{ kg}} = \boxed{5.17 \times 10^7 \text{ m/s}}$

29.46 $F_B = qvB = \frac{mv^2}{r}$

$$B = \frac{mv}{qr} = \frac{4.80 \times 10^{-16} \text{ kg} \cdot \text{m/s}}{(1.60 \times 10^{-19} \text{ C})(1000 \text{ m})} = \boxed{3.00 \text{ T}}$$

29.47 $\theta = \tan^{-1}\dfrac{25.0}{10.0} = 68.2°$ and $R = \dfrac{1.00 \text{ cm}}{\sin 68.2°} = 1.08 \text{ cm}$



Ignoring relativistic correction, the kinetic energy of the electrons is

$\frac{1}{2}mv^2 = q(\Delta V)$ so $v = \sqrt{\dfrac{2q(\Delta V)}{m}} = 1.33 \times 10^8 \text{ m/s}$

From the centripetal force $\dfrac{mv^2}{R} = qvB$, we find the magnetic field

$$B = \frac{mv}{|q|R} = \frac{(9.11\times10^{-31}\text{ kg})(1.33\times10^{8}\text{ m/s})}{(1.60\times10^{-19}\text{ C})(1.08\times10^{-2}\text{ m})} = \boxed{70.1\text{ mT}}$$

29.48 (a) $R_H \equiv \dfrac{1}{nq}$ so $n = \dfrac{1}{qR_H} = \dfrac{1}{(1.60\times10^{-19}\text{ C})(0.840\times10^{-10}\text{ m}^3/\text{C})} = \boxed{7.44\times10^{28}\text{ m}^{-3}}$

(b) $\Delta V_H = \dfrac{IB}{nqt}$

$$B = \frac{nqt(\Delta V_H)}{I} = \frac{(7.44\times10^{28}\text{ m}^{-3})(1.60\times10^{-19}\text{ C})(0.200\times10^{-3}\text{ m})(15.0\times10^{-6}\text{ V})}{20.0\text{ A}} = \boxed{1.79\text{ T}}$$

29.49 $$\frac{1}{nq} = \frac{t(\Delta V_H)}{IB} = \frac{(35.0\times10^{-6}\text{ V})(0.400\times10^{-2}\text{ m})}{(21.0\text{ A})(1.80\text{ T})} = \boxed{3.70\times10^{-9}\text{ m}^3/\text{C}}$$

29.50 Since $\Delta V_H = \dfrac{IB}{nqt}$, and given that $I = 50.0$ A, $B = 1.30$ T, and $t = 0.330$ mm, the number of charge carriers per unit volume is

$$n = \frac{IB}{e(\Delta V_H)t} = \boxed{1.28\times10^{29}\text{ m}^{-3}}$$

The number density of atoms we compute from the density:

$$n_0 = \frac{8.92\text{ g}}{\text{cm}^3}\left(\frac{1\text{ mole}}{63.5\text{ g}}\right)\left(\frac{6.02\times10^{23}\text{ atoms}}{\text{mole}}\right)\left(\frac{10^6\text{ cm}^3}{1\text{ m}^3}\right) = 8.46\times10^{28}\text{ atom/m}^3$$

So the number of conduction electrons per atom is

$$\frac{n}{n_0} = \frac{1.28\times10^{29}}{8.46\times10^{28}} = \boxed{1.52}$$

29.51 $$B = \frac{nqt(\Delta V_H)}{I} = \frac{\left(8.48\times10^{28}\ \text{m}^{-3}\right)\left(1.60\times10^{-19}\ \text{C}\right)\left(5.00\times10^{-3}\ \text{m}\right)\left(5.10\times10^{-12}\ \text{V}\right)}{8.00\ \text{A}}$$

$$B = 4.32\times10^{-5}\ \text{T} = \boxed{43.2\ \mu\text{T}}$$

Goal Solution

In an experiment designed to measure the Earth's magnetic field using the Hall effect, a copper bar 0.500 cm thick is positioned along an east-west direction. If a current of 8.00 A in the conductor results in a Hall voltage of 5.10 pV, what is the magnitude of the Earth's magnetic field? (Assume that $n = 8.48\times10^{28}$ electrons/m^3 and that the plane of the bar is rotated to be perpendicular to the direction of **B**.)

G: The Earth's magnetic field is about 50 μT (see Table 29.1), so we should expect a result of that order of magnitude.

O: The magnetic field can be found from the Hall effect voltage:

$$\Delta V_H = \frac{IB}{nqt} \quad \text{or} \quad B = \frac{nqt\Delta V_H}{I}$$

A: From the Hall voltage,

$$B = \frac{\left(8.48\times10^{28}\ \text{e}^-/\text{m}^3\right)\left(1.60\times10^{-19}\ \text{C}/\text{e}^-\right)(0.00500\ \text{m})\left(5.10\times10^{-12}\ \text{V}\right)}{8.00\ \text{A}} = 4.32\times10^{-5}\ \text{T} = 43.2\ \mu\text{T}$$

L: The calculated magnetic field is slightly less than we expected but is reasonable considering that the Earth's local magnetic field varies in both magnitude and direction.

29.52 (a) $\Delta V_H = \dfrac{IB}{nqt}$ so $\dfrac{nqt}{I} = \dfrac{B}{\Delta V_H} = \dfrac{0.0800\ \text{T}}{0.700\times10^{-6}\ \text{V}} = 1.14\times10^5\ \dfrac{\text{T}}{\text{V}}$

Then, the unknown field is $B = \left(\dfrac{nqt}{I}\right)(\Delta V_H)$

$$B = \left(1.14\times10^5\ \text{T/V}\right)\left(0.330\times10^{-6}\ \text{V}\right) = 0.0377\ \text{T} = \boxed{37.7\ \text{mT}}$$

(b) $\dfrac{nqt}{I} = 1.14\times10^5\ \dfrac{\text{T}}{\text{V}}$ so $n = \left(1.14\times10^5\ \dfrac{\text{T}}{\text{V}}\right)\dfrac{I}{qt}$

$$n = \left(1.14\times10^5\ \frac{\text{T}}{\text{V}}\right)\frac{0.120\ \text{A}}{\left(1.60\times10^{-19}\ \text{C}\right)\left(2.00\times10^{-3}\ \text{m}\right)} = \boxed{4.29\times10^{25}\ \text{m}^{-3}}$$

29.53 $|q| vB \sin 90° = \dfrac{mv^2}{r}$ $\quad \therefore \ \omega = \dfrac{v}{r} = \dfrac{eB}{m} = \dfrac{\theta}{t}$

(a) The time it takes the electron to complete π radians is

$$t = \frac{\theta}{\omega} = \frac{\theta m}{eB} = \frac{(\pi \text{ rad})(9.11 \times 10^{-31} \text{ kg})}{(1.60 \times 10^{-19} \text{ C})(0.100 \text{ N} \cdot \text{s/C} \cdot \text{m})} = \boxed{1.79 \times 10^{-10} \text{ s}}$$

(b) Since $v = \dfrac{|q| Br}{m}$,

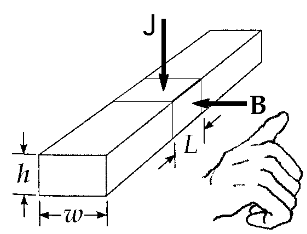
$$K_e = \tfrac{1}{2} mv^2 = \frac{q^2 B^2 r^2}{2m} = \frac{e(1.60 \times 10^{-19} \text{ C})(0.100 \text{ N} \cdot \text{s / Cm})^2 (2.00 \times 10^{-2} \text{ m})^2}{2(9.11 \times 10^{-31} \text{ kg})} = \boxed{351 \text{ keV}}$$

29.54 $\sum F_y = 0$: $\quad +n - mg = 0$

$\sum F_x = 0$: $\quad -\mu_k n + IBd \sin 90.0° = 0$

$$B = \frac{\mu_k m g}{Id} = \frac{0.100(0.200 \text{ kg})(9.80 \text{ m/s}^2)}{(10.0 \text{ A})(0.500 \text{ m})} = \boxed{39.2 \text{ mT}}$$

29.55 (a) The $\boxed{\text{electric current experiences a magnetic force}}$.



$I(\mathbf{h} \times \mathbf{B})$ in the direction of $\mathbf{L}$.

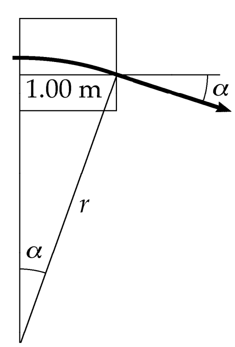
(b) The sodium, consisting of ions and electrons, flows along the pipe transporting no net charge. But inside the section of length L, electrons drift upward to constitute downward electric current $\mathbf{J} \times (\text{area}) = \mathbf{J}Lw$.

The current then feels a magnetic force $\quad I|\mathbf{h} \times \mathbf{B}| = JLwhB \sin 90°$

This force along the pipe axis will make the fluid move, exerting pressure

$$\frac{F}{\text{area}} = \frac{JLwhB}{hw} = \boxed{JLB}$$

29.56 The magnetic force on each proton, $\quad \mathbf{F}_B = q\mathbf{v} \times \mathbf{B} = qvB \sin 90°$



downward perpendicular to velocity, supplies centripetal force, guiding it into a circular path of radius r, with

$$qvB = \frac{mv^2}{r} \quad \text{and} \quad r = \frac{mv}{qB}$$

We compute this radius by first finding the proton's speed: $\quad K = \frac{1}{2} mv^2$

$$v = \sqrt{\frac{2K}{m}} = \sqrt{\frac{2(5.00 \times 10^6 \text{ eV})(1.60 \times 10^{-19} \text{ J / eV})}{1.67 \times 10^{-27} \text{ kg}}} = 3.10 \times 10^7 \text{ m / s}$$

Now, $r = \frac{mv}{qB} = \frac{(1.67 \times 10^{-27}\text{ kg})(3.10 \times 10^{7}\text{ m/s})(\text{C}\cdot\text{m})}{(1.60 \times 10^{-19}\text{ C})(0.0500\text{ N}\cdot\text{s})} = 6.46\text{ m}$

(b) From the figure, observe that $\sin \propto = \frac{1.00\text{ m}}{r} = \frac{1\text{ m}}{6.46\text{ m}}$ $\boxed{\alpha = 8.90°}$

(a) The magnitude of the proton momentum stays constant, and its final y component is

$-(1.67 \times 10^{-27}\text{ kg})(3.10 \times 10^{7}\text{ m/s})\sin(8.90°) = \boxed{-8.00 \times 10^{-21}\text{ kg}\cdot\text{m/s}}$

***29.57** (a) If $\mathbf{B} = B_x\,\mathbf{i} + B_y\,\mathbf{j} + B_z\,\mathbf{k}$, $\mathbf{F}_B = q\mathbf{v}\times\mathbf{B} = e(v_i\,\mathbf{i})\times(B_x\,\mathbf{i} + B_y\,\mathbf{j} + B_z\,\mathbf{k}) = 0 + ev_iB_y\,\mathbf{k} - ev_iB_z\,\mathbf{j}$

Since the force actually experienced is $\mathbf{F}_B = F_i\,\mathbf{j}$, observe that

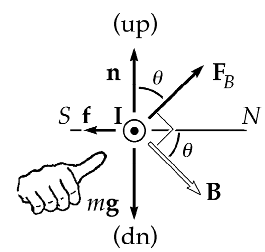
$\boxed{B_x \text{ could have any value}}$, $\boxed{B_y = 0}$, and $\boxed{B_z = -F_i/ev_i}$

(b) If $\mathbf{v} = -v_i\,\mathbf{i}$, then $\mathbf{F}_B = q\mathbf{v}\times\mathbf{B} = e(-v_i\,\mathbf{i})\times(B_x\,\mathbf{i} + 0\,\mathbf{j} - F_i/ev_i\,\mathbf{k}) = \boxed{-F_i\,\mathbf{j}}$

(c) If $q = -e$ and $\mathbf{v} = v_i\,\mathbf{i}$, then $\mathbf{F}_B = q\mathbf{v}\times\mathbf{B} = -e(v_i\,\mathbf{i})\times(B_x\,\mathbf{i} + 0\,\mathbf{j} - F_i/ev_i\,\mathbf{k}) = \boxed{-F_i\,\mathbf{j}}$

Reversing either the velocity or the sign of the charge reverses the force.

29.58 A key to solving this problem is that reducing the normal force will reduce the friction force: $F_B = BIL$ or $B = F_B/IL$



When the wire is just able to move, $\Sigma F_y = n + F_B\cos\theta - mg = 0$

so $n = mg - F_B\cos\theta$

and $f = \mu(mg - F_B\cos\theta)$

Also, $\Sigma F_x = F_B\sin\theta - f = 0$

so $F_B\sin\theta = f$: $F_B\sin\theta = \mu(mg - F_B\cos\theta)$ and $F_B = \frac{\mu mg}{\sin\theta + \mu\cos\theta}$

We minimize B by minimizing F_B: $\frac{dF_B}{d\theta} = (\mu mg)\frac{\cos\theta - \mu\sin\theta}{(\sin\theta + \mu\cos\theta)^2} = 0 \Rightarrow \mu\sin\theta = \cos\theta$

Thus, $\theta = \tan^{-1}\left(\frac{1}{\mu}\right) = \tan^{-1}(5.00) = 78.7°$ for the smallest field, and

$$B = \frac{F_B}{IL} = \left(\frac{\mu g}{I}\right)\frac{(m/L)}{\sin\theta + \mu\cos\theta}$$

$$B_{\min} = \left[\frac{(0.200)(9.80\text{ m/s}^2)}{1.50\text{ A}}\right]\frac{0.100\text{ kg/m}}{\sin 78.7° + (0.200)\cos 78.7°} = 0.128\text{ T}$$

$\boxed{B_{\min} = 0.128\text{ T pointing north at an angle of }78.7°\text{ below the horizontal}}$

29.59 (a) The net force is the Lorentz force given by $\mathbf{F} = q\mathbf{E} + q\mathbf{v}\times\mathbf{B} = q(\mathbf{E} + \mathbf{v}\times\mathbf{B})$

$$\mathbf{F} = \left(3.20\times10^{-19}\right)\left[\left(4\,\mathbf{i} - 1\mathbf{j} - 2\,\mathbf{k}\right) + \left(2\,\mathbf{i} + 3\,\mathbf{j} - 1\mathbf{k}\right)\times\left(2\,\mathbf{i} + 4\,\mathbf{j} + 1\mathbf{k}\right)\right]\text{ N}$$

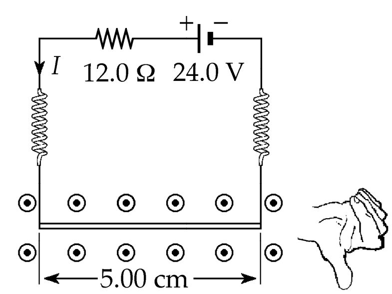
Carrying out the indicated operations, we find: $\mathbf{F} = \boxed{\left(3.52\,\mathbf{i} - 1.60\,\mathbf{j}\right)\times10^{-18}\text{ N}}$

(b) $\theta = \cos^{-1}\left(\frac{F_x}{F}\right) = \cos^{-1}\left(\frac{3.52}{\sqrt{(3.52)^2 + (1.60)^2}}\right) = \boxed{24.4°}$

29.60 $r = \frac{mv}{qB} = \frac{(1.67\times10^{-27})(1.50\times10^{8})}{(1.60\times10^{-19})(5.00\times10^{-5})}\text{ m} = \boxed{3.13\times10^{4}\text{ m}} = 31.3\text{ km}$

No, $\boxed{\text{the proton will not hit the Earth}}$.

29.61 Let Δx_1 be the elongation due to the weight of the wire and let Δx_2 be the additional elongation of the springs when the magnetic field is turned on. Then $F_{\text{magnetic}} = 2k\,\Delta x_2$ where k is the force constant of the spring and can be determined from $k = mg/2\Delta x_1$. (The factor 2 is included in the two previous equations since there are 2 springs in parallel.) Combining these two equations, we find



$$F_{\text{magnetic}} = 2\left(\frac{mg}{2\Delta x_1}\right)\Delta x_2 = \frac{mg\,\Delta x_2}{\Delta x_1}; \quad \text{but} \quad \left|\mathbf{F}_B\right| = I\left|\mathbf{L}\times\mathbf{B}\right| = ILB$$

Therefore, where $I = \frac{24.0\text{ V}}{12.0\ \Omega} = 2.00\text{ A}$, $B = \frac{mg\,\Delta x_2}{IL\,\Delta x_1} = \frac{(0.0100)(9.80)(3.00\times10^{-3})}{(2.00)(0.0500)(5.00\times10^{-3})} = \boxed{0.588\text{ T}}$

***29.62** Suppose the input power is $120\text{ W} = (120\text{ V})I$: $\boxed{I \sim 1\text{ A} = 10^{0}\text{ A}}$

Suppose $\omega = 2000\,\frac{\text{rev}}{\text{min}}\left(\frac{1\text{ min}}{60\text{ s}}\right)\left(\frac{2\pi\text{ rad}}{1\text{ rev}}\right) \sim 200\,\frac{\text{rad}}{\text{s}}$

and the output power is $20\text{ W} = \tau\omega = \tau\left(200\,\frac{\text{rad}}{\text{s}}\right)$ $\boxed{\tau \sim 10^{-1}\text{ N}\cdot\text{m}}$

Suppose the area is about $(3\text{ cm})\times(4\text{ cm})$, or $\boxed{A \sim 10^{-3}\text{ m}^2}$

From Table 29.1, suppose that the field is $\boxed{B \sim 10^{-1}\text{ T}}$

Then, the number of turns in the coil may be found from $\tau \cong NIAB$:

$0.1\text{ N}\cdot\text{m} \sim N\left(1\,\frac{\text{C}}{\text{s}}\right)\left(10^{-3}\text{ m}^2\right)\left(10^{-1}\,\frac{\text{N}\cdot\text{s}}{\text{Cm}}\right)$ giving $\boxed{N \sim 10^{3}}$

29.63 Call the length of the rod L and the tension in each wire alone $T/2$. Then, at equilibrium:

$$\Sigma F_x = T\sin\theta - ILB\sin 90.0^0 = 0 \quad \text{or} \quad T\sin\theta = ILB$$
$$\Sigma F_y = T\cos\theta - mg = 0, \quad \text{or} \quad T\cos\theta = mg$$

Therefore, $\tan\theta = \dfrac{ILB}{mg} = \dfrac{IB}{(m/L)g}$ or $B = \dfrac{(m/L)g}{I}\tan\theta$

$$B = \frac{(0.0100 \text{ kg/m})(9.80 \text{ m/s}^2)}{5.00 \text{ A}}\tan(45.0^\circ) = \boxed{19.6 \text{ mT}}$$

29.64 Call the length of the rod L and the tension in each wire alone $T/2$. Then, at equilibrium:

$$\Sigma F_x = T\sin\theta - ILB\sin 90.0^0 = 0 \quad \text{or} \quad T\sin\theta = ILB$$
$$\Sigma F_y = T\cos\theta - mg = 0, \quad \text{or} \quad T\cos\theta = mg$$

$$\tan\theta = \frac{ILB}{mg} = \frac{IB}{(m/L)g} \quad \text{or} \quad B = \frac{(m/L)g}{I}\tan\theta = \boxed{\frac{\mu g}{I}\tan\theta}$$

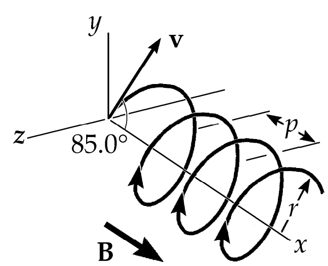
29.65 $\Sigma F = ma$ or $qvB\sin 90.0° = \dfrac{mv^2}{r}$

$\therefore$ the angular frequency for each ion is $\dfrac{v}{r} = \omega = \dfrac{qB}{m} = 2\pi f$ and

$$\Delta f = f_{12} - f_{14} = \frac{qB}{2\pi}\left(\frac{1}{m_{12}} - \frac{1}{m_{14}}\right) = \frac{(1.60\times10^{-19}\text{ C})(2.40\text{ T})}{2\pi(1.66\times10^{-27}\text{ kg / u})}\left(\frac{1}{12.0\text{ u}} - \frac{1}{14.0\text{ u}}\right)$$

$$\Delta f = f_{12} - f_{14} = 4.38\times10^5\text{ s}^{-1} = \boxed{438\text{ kHz}}$$

29.66 Let v_x and $v_\perp$ be the components of the velocity of the positron parallel to and perpendicular to the direction of the magnetic field.



(a) The pitch of trajectory is the distance moved along x by the positron during each period, T (see Equation 29.15).

$$p = v_x T = (v\cos 85.0°)\left(\frac{2\pi m}{Bq}\right)$$

$$p = \frac{(5.00\times10^6)(\cos 85.0°)(2\pi)(9.11\times10^{-31})}{(0.150)(1.60\times10^{-19})} = \boxed{1.04\times10^{-4}\text{ m}}$$

(b) From Equation 29.13, $r = \dfrac{mv_\perp}{Bq} = \dfrac{mv\sin 85.0°}{Bq}$

$$r = \frac{(9.11\times10^{-31})(5.00\times10^6)(\sin 85.0°)}{(0.150)(1.60\times10^{-19})} = \boxed{1.89\times10^{-4}\text{ m}}$$

29.67 $|\tau| = IAB$ where the effective current due to the orbiting electrons is $I = \dfrac{\Delta q}{\Delta t} = \dfrac{q}{T}$

and the period of the motion is $T = \dfrac{2\pi R}{v}$

The electron's speed in its orbit is found by requiring $\dfrac{k_e q^2}{R^2} = \dfrac{mv^2}{R}$ or $v = q\sqrt{\dfrac{k_e}{mR}}$

Substituting this expression for v into the equation for T, we find $T = 2\pi\sqrt{\dfrac{mR^3}{q^2 k_e}}$

$$T = 2\pi\sqrt{\frac{(9.11\times10^{-31})(5.29\times10^{-11})^3}{(1.60\times10^{-19})^2(8.99\times10^{9})}} = 1.52\times10^{-16}\text{ s}$$

Therefore, $$|\tau| = \left(\frac{q}{T}\right)AB = \frac{1.60\times10^{-19}}{1.52\times10^{-16}}\pi(5.29\times10^{-11})^2(0.400) = \boxed{3.70\times10^{-24}\text{ N}\cdot\text{m}}$$

Goal Solution

Consider an electron orbiting a proton and maintained in a fixed circular path of radius $R = 5.29\times10^{-11}$ m by the Coulomb force. Treating the orbiting charge as a current loop, calculate the resulting torque when the system is in a magnetic field of 0.400 T directed perpendicular to the magnetic moment of the electron.

G: Since the mass of the electron is very small (~10^{-30} kg), we should expect that the torque on the orbiting charge will be very small as well, perhaps ~10^{-30} N·m.

O: The torque on a current loop that is perpendicular to a magnetic field can be found from $|\tau| = IAB\sin\theta$. The magnetic field is given, $\theta = 90°$, the area of the loop can be found from the radius of the circular path, and the current can be found from the centripetal acceleration that results from the Coulomb force that attracts the electron to proton.

A: The area of the loop is $A = \pi r^2 = \pi\left(5.29\times10^{-11}\text{ m}\right)^2 = 8.79\times10^{-21}\text{ m}^2$.

If v is the speed of the electron, then the period of its circular motion will be $T = 2\pi R/v$, and the effective current due to the orbiting electron is $I = \Delta Q/\Delta t = e/T$. Applying Newton's second law with the Coulomb force acting as the central force gives

$$\Sigma F = \frac{k_e q^2}{R^2} = \frac{mv^2}{R} \quad\text{so that}\quad v = q\sqrt{\frac{k_e}{mR}} \quad\text{and}\quad T = 2\pi\sqrt{\frac{mR^3}{q^2 k_e}}$$

$$T = 2\pi\sqrt{\frac{(9.10\times10^{-31}\text{ kg})(5.29\times10^{-11}\text{ m})^3}{\left(1.60\times10^{-19}\text{ C}\right)^2\left(8.99\times10^{9}\text{ N}\cdot\text{m}^2/\text{C}^2\right)}} = 1.52\times10^{-16}\text{ s}$$

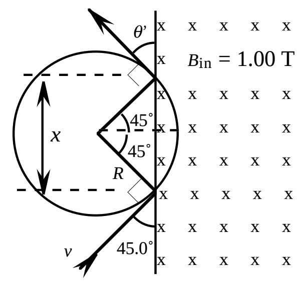
The torque is $|\tau| = \left(\dfrac{q}{T}\right)AB$: $|\tau| = \dfrac{1.60\times10^{19}\text{ C}}{1.52\times10^{-16}\text{ s}}(\pi)(5.29\times10^{-11}\text{ m})^2(0.400\text{ T}) = 3.70\times10^{-24}\text{ N}\cdot\text{m}$

L: The torque is certainly small, but a million times larger than we guessed. This torque will cause the atom to precess with a frequency proportional to the applied magnetic field. A similar process on the nuclear, rather than the atomic, level leads to nuclear magnetic resonance (NMR), which is used for magnetic resonance imaging (MRI) scans employed for medical diagnostic testing (see Section 44.2).

29.68 Use the equation for cyclotron frequency $\omega = \frac{qB}{m}$ or $m = \frac{qB}{\omega} = \frac{qB}{2\pi f}$

$$m = \frac{(1.60\times10^{-19}\ \text{C})(5.00\times10^{-2}\ \text{T})}{(2\pi)(5.00\ \text{rev}\ /\ 1.50\times10^{-3}\ \text{s})} = \boxed{3.82\times10^{-25}\ \text{kg}}$$

29.69 (a) $K = \frac{1}{2}mv^2 = 6.00\ \text{MeV} = \left(6.00\times10^{6}\ \text{eV}\right)\left(1.60\times10^{-19}\ \frac{\text{J}}{\text{eV}}\right)$



$K = 9.60\times10^{-13}$ J

$$v = \sqrt{\frac{2\left(9.60\times10^{-13}\ \text{J}\right)}{1.67\times10^{-27}\ \text{kg}}} = 3.39\times10^{7}\ \text{m s}$$

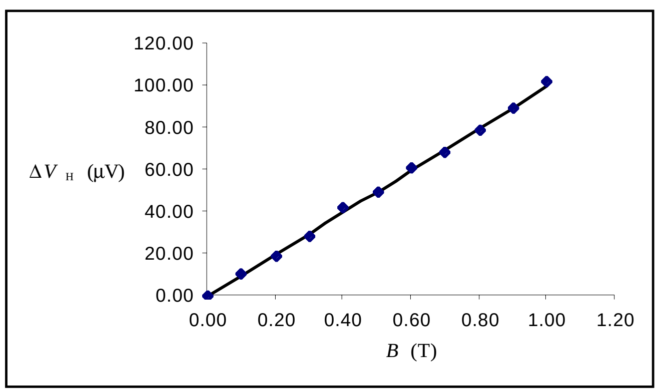
$$F_B = qvB = \frac{mv^2}{R} \quad \text{so} \quad R = \frac{mv}{qB} = \frac{\left(1.67\times10^{-27}\ \text{kg}\right)\left(3.39\times10^{7}\ \text{m/s}\right)}{\left(1.60\times10^{-19}\ \text{C}\right)\left(1.00\ \text{T}\right)} = 0.354\ \text{m}$$

Then, from the diagram, $x = 2R\sin 45.0^\circ = 2(0.354\ \text{m})\sin 45.0^\circ = \boxed{0.501\ \text{m}}$

(b) From the diagram, observe that $\theta' = \boxed{45.0^\circ}$.

29.70 (a) See graph to the right. The Hall voltage is directly proportional to the magnetic field. A least-square fit to the data gives the equation of the best fitting line as:

$$\boxed{\Delta V_H = \left(1.00\times10^{-4}\ \text{V/T}\right)B}$$



(b) Comparing the equation of the line which fits the data best to

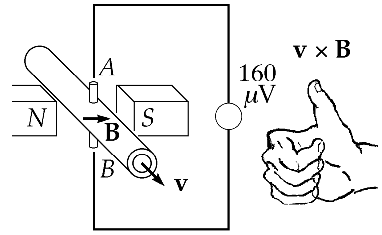
$$\Delta V_H = \left(\frac{I}{nqt}\right)B$$

observe that: $\frac{I}{nqt} = 1.00\times10^{-4}\ \text{V/T}$, or $t = \frac{I}{nq\left(1.00\times10^{-4}\ \text{V/T}\right)}$

Then, if $I = 0.200$ A, $q = 1.60\times10^{-19}$ C, and $n = 1.00\times10^{26}\ \text{m}^{-3}$, the thickness of the sample is

$$t = \frac{0.200\ \text{A}}{\left(1.00\times10^{26}\ \text{m}^{-3}\right)\left(1.60\times10^{-19}\ \text{C}\right)\left(1.00\times10^{-4}\ \text{V/T}\right)} = 1.25\times10^{-4}\ \text{m} = \boxed{0.125\ \text{mm}}$$

***29.71** (a) The magnetic force acting on ions in the blood stream will deflect positive charges toward point *A* and negative charges toward point *B*. This separation of charges produces an electric field directed from *A* toward *B*. At equilibrium, the electric force caused by this field must balance the magnetic force,

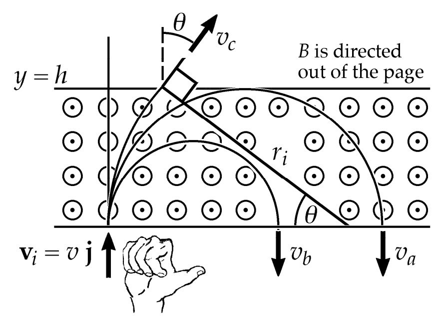


so $$qvB = qE = q\left(\frac{\Delta V}{d}\right)$$

or $$v = \frac{\Delta V}{Bd} = \frac{160\times10^{-6}\text{ V}}{(0.040\,0\text{ T})(3.00\times10^{-3}\text{ m})} = \boxed{1.33\text{ m/s}}$$

(b) $\boxed{\text{No}}$. Negative ions moving in the direction of v would be deflected toward point *B*, giving *A* a higher potential than *B*. Positive ions moving in the direction of v would be deflected toward *A*, again giving *A* a higher potential than *B*. Therefore, the sign of the potential difference does not depend on whether the ions in the blood are positively or negatively charged.

***29.72** When in the field, the particles follow a circular path according to $qvB = mv^2/r$, so the radius of the path is: $r = mv/qB$



(a) When $r = h = \frac{mv}{qB}$, that is, when $\boxed{v = \frac{qBh}{m}}$, the particle will cross the band of field. It will move in a full semicircle of radius h, leaving the field at $(2h, 0, 0)$ with velocity $\boxed{\mathbf{v}_f = -v\mathbf{j}}$.

(b) When $v < \frac{qBh}{m}$, the particle will move in a smaller $\boxed{\text{semicircle}}$ of radius $r = \frac{mv}{qB} < h$. It will leave the field at $(2r, 0, 0)$ with velocity $\boxed{\mathbf{v}_f = -v\mathbf{j}}$.

(c) When $v > \frac{qBh}{m}$, the particle moves in a $\boxed{\text{circular arc}}$ of radius $r = \frac{mv}{qB} > h$, centered at $(r, 0, 0)$. The arc subtends an angle given by $\theta = \sin^{-1}(h/r)$. It will leave the field at the point with coordinates $[r(1-\cos\theta), h, 0]$ with velocity $\boxed{\mathbf{v}_f = v\sin\theta\,\mathbf{i} + v\cos\theta\,\mathbf{j}}$.