Chapter 27 Solutions

27.1 $I = \dfrac{\Delta Q}{\Delta t}$ $\Delta Q = I\,\Delta t = (30.0 \times 10^{-6}\text{ A})(40.0\text{ s}) = 1.20 \times 10^{-3}\text{ C}$

$$N = \frac{Q}{e} = \frac{1.20 \times 10^{-3}\text{ C}}{1.60 \times 10^{-19}\text{ C/electron}} = \boxed{7.50 \times 10^{15}\text{ electrons}}$$

***27.2** The atomic weight of silver = 107.9, and the volume V is

$V = \text{(area)(thickness)} = (700 \times 10^{-4}\text{ m}^2)(0.133 \times 10^{-3}\text{ m}) = 9.31 \times 10^{-6}\text{ m}^3$

The mass of silver deposited is $m_{\text{Ag}} = \rho V = \left(10.5 \times 10^3\text{ kg/m}^3\right)\left(9.31 \times 10^{-6}\text{ m}^3\right) = 9.78 \times 10^{-2}\text{ kg}.$

and the number of silver atoms deposited is

$$N = (9.78 \times 10^{-2}\text{ kg})\frac{6.02 \times 10^{26}\text{ atoms}}{107.9\text{ kg}} = 5.45 \times 10^{23}$$

$$I = \frac{V}{R} = \frac{12.0\text{ V}}{1.80\ \Omega} = 6.67\text{ A} = 6.67\text{ C/s}$$

$$\Delta t = \frac{\Delta Q}{I} = \frac{Ne}{I} = \frac{(5.45 \times 10^{23})(1.60 \times 10^{-19}\text{ C})}{6.67\text{ C/s}} = 1.31 \times 10^4\text{ s} = \boxed{3.64\text{ h}}$$

27.3 $Q(t) = \int_0^t I\,dt = I_0\tau(1 - e^{-t/\tau})$

(a) $Q(\tau) = I_0\tau(1 - e^{-1}) = \boxed{(0.632)I_0\tau}$

(b) $Q(10\tau) = I_0\tau(1 - e^{-10}) = \boxed{(0.99995)I_0\tau}$

(c) $Q(\infty) = I_0\tau(1 - e^{-\infty}) = \boxed{I_0\tau}$

27.4 (a) Using $\dfrac{k_e e^2}{r^2} = \dfrac{mv^2}{r}$, we get: $v = \sqrt{\dfrac{k_e e^2}{mr}} = \boxed{2.19 \times 10^6\text{ m/s}}$.

(b) The time for the electron to revolve around the proton once is:

$$t = \frac{2\pi r}{v} = \frac{2\pi(5.29 \times 10^{-11}\text{ m})}{(2.19 \times 10^6\text{ m/s})} = 1.52 \times 10^{-16}\text{ s}$$

The total charge flow in this time is 1.60×10^{-19} C, so the current is

$$I = \frac{1.60 \times 10^{-19}\ \text{C}}{1.52 \times 10^{-16}\ \text{s}} = 1.05 \times 10^{-3}\ \text{A} = \boxed{1.05\ \text{mA}}$$

27.5 $\omega = \dfrac{2\pi}{T}$ where T is the period.

$$I = \frac{q}{T} = \frac{q\omega}{2\pi} = \frac{(8.00\times10^{-9}\text{ C})(100\pi\text{ rad / s})}{2\pi} = 4.00\times10^{-7}\text{ A} = \boxed{400\text{ nA}}$$

27.6 The period of revolution for the sphere is $T = \dfrac{2\pi}{\omega}$, and the average current represented by this revolving charge is $I = \dfrac{q}{T} = \boxed{\dfrac{q\omega}{2\pi}}$.

27.7 $q = 4t^3 + 5t + 6$ $\quad A = (2.00\text{ cm}^2)\left(\dfrac{1.00\text{ m}}{100\text{ cm}}\right)^2 = 2.00\times10^{-4}\text{ m}^2$

(a) $I(1.00\text{ s}) = \left.\dfrac{dq}{dt}\right|_{t=1.00\text{ s}} = \left.\left(12t^2 + 5\right)\right|_{t=1.00\text{ s}} = \boxed{17.0\text{ A}}$

(b) $J = \dfrac{I}{A} = \dfrac{17.0\text{ A}}{2.00\times10^{-4}\text{ m}^2} = \boxed{85.0\text{ kA / m}^2}$

27.8 $I = \dfrac{dq}{dt}$

$$q = \int dq = \int I\,dt = \int_0^{1/240\text{ s}} (100\text{ A})\sin(120\pi t/\text{s})\,dt$$

$$q = \frac{-100\text{ C}}{120\pi}\left[\cos(\pi/2) - \cos 0\right] = \frac{+100\text{ C}}{120\pi} = \boxed{0.265\text{ C}}$$

27.9 (a) $J = \dfrac{I}{A} = \dfrac{5.00\text{ A}}{\pi(4.00\times10^{-3}\text{ m})^2} = \boxed{99.5\text{ kA/m}^2}$

(b) $J_2 = \dfrac{1}{4}J_1$; $\quad \dfrac{I}{A_2} = \dfrac{1}{4}\dfrac{I}{A_1}$

$A_1 = \dfrac{1}{4}A_2$ so $\quad \pi(4.00\times10^{-3})^2 = \dfrac{1}{4}\pi r_2^2$

$r_2 = 2(4.00\times10^{-3}) = 8.00\times10^{-3}\text{ m} = \boxed{8.00\text{ mm}}$

27.10 (a) The speed of each deuteron is given by $K = \frac{1}{2}mv^2$

$(2.00 \times 10^6)(1.60 \times 10^{-19}\text{ J}) = \frac{1}{2}(2 \times 1.67 \times 10^{-27}\text{ kg})\, v^2$ and $v = 1.38 \times 10^7\text{ m/s}$

The time between deuterons passing a stationary point is t in $I = q/t$

$10.0 \times 10^{-6}\text{ C/s} = 1.60 \times 10^{-19}\text{ C}/t$ or $t = 1.60 \times 10^{-14}\text{ s}$

So the distance between them is $vt = \left(1.38 \times 10^7\text{ m/s}\right)\left(1.60 \times 10^{-14}\text{ s}\right) = \boxed{2.21 \times 10^{-7}\text{ m}}$

(b) One nucleus will put its nearest neighbor at potential

$$V = \frac{k_e q}{r} = \frac{(8.99 \times 10^9\text{ N}\cdot\text{m}^2/\text{C}^2)(1.60 \times 10^{-19}\text{ C})}{2.21 \times 10^{-7}\text{ m}} = \boxed{6.49 \times 10^{-3}\text{ V}}$$

This is very small compared to the 2 MV accelerating potential, so repulsion within the beam is a small effect.

27.11 (a) $J = \frac{I}{A} = \frac{8.00 \times 10^{-6}\text{ A}}{\pi\left(1.00 \times 10^{-3}\text{ m}\right)^2} = \boxed{2.55\text{ A/m}^2}$

(b) From $J = nev_d$, we have $n = \frac{J}{ev_d} = \frac{2.55\text{ A/m}^2}{\left(1.60 \times 10^{-19}\text{ C}\right)\left(3.00 \times 10^8\text{ m/s}\right)} = \boxed{5.31 \times 10^{10}\text{ m}^{-3}}$

(c) From $I = \Delta Q/\Delta t$, we have $\Delta t = \frac{\Delta Q}{I} = \frac{N_A e}{I} = \frac{\left(6.02 \times 10^{23}\right)\left(1.60 \times 10^{-19}\text{ C}\right)}{8.00 \times 10^{-6}\text{ A}} = \boxed{1.20 \times 10^{10}\text{ s}}$

(This is about 381 years!)

***27.12** We use $I = nqAv_d$ where n is the number of charge carriers per unit volume, and is identical to the number of atoms per unit volume). We assume a contribution of 1 free electron per atom in the relationship above. For aluminum, which has a molecular weight of 27, we know that Avogadro's number of atoms, N_A, has a mass of 27.0 g. Thus, the mass per atom is

$$\frac{27.0\text{ g}}{N_A} = \frac{27.0\text{ g}}{6.02 \times 10^{23}} = 4.49 \times 10^{-23}\text{ g/atom}$$

Thus, $n = \frac{\text{density of aluminum}}{\text{mass per atom}} = \frac{2.70\text{ g/cm}^3}{4.49 \times 10^{-23}\text{ g/atom}}$

$$n = 6.02 \times 10^{22}\ \frac{\text{atoms}}{\text{cm}^3} = 6.02 \times 10^{28}\ \frac{\text{atoms}}{\text{m}^3}$$

Therefore, $v_d = \frac{I}{nqA} = \frac{5.00\text{ A}}{(6.02 \times 10^{28}\text{ m}^{-3})(1.60 \times 10^{-19}\text{ C})(4.00 \times 10^{-6}\text{ m}^2)} = 1.30 \times 10^{-4}\text{ m/s}$

or, $\boxed{v_d = 0.130\text{ mm/s}}$

***27.13** $I = \frac{\Delta V}{R} = \frac{120\text{ V}}{240\ \Omega} = 0.500\text{ A} = \boxed{500\text{ mA}}$

27.14 (a) Applying its definition, we find the resistance of the rod,

$$R = \frac{\Delta V}{I} = \frac{15.0\text{ V}}{4.00 \times 10^{-3}\text{ A}} = 3750\ \Omega = \boxed{3.75\text{ k}\Omega}$$

(b) The length of the rod is determined from Equation 27.11: $R = \rho\ell / A$. Solving for ℓ and substituting numerical values for R, A, and the values of ρ given for carbon in Table 27.1, we obtain

$$\ell = \frac{RA}{\rho} = \frac{(3.75 \times 10^3\ \Omega)(5.00 \times 10^{-6}\text{ m}^2)}{(3.50 \times 10^{-5}\ \Omega \cdot \text{m})} = \boxed{536\text{ m}}$$

27.15 $\Delta V = IR$ and $R = \frac{\rho l}{A}$: $A = 0.600\text{ mm}^2\left(\frac{1.00\text{ m}}{1000\text{ mm}}\right)^2 = 6.00 \times 10^{-7}\text{ m}^2$

$\Delta V = \frac{I\rho l}{A}$: $I = \frac{\Delta V A}{\rho l} = \frac{(0.900\text{ V})(6.00 \times 10^{-7}\text{ m}^2)}{(5.60 \times 10^{-8}\ \Omega \cdot \text{m})(1.50\text{ m})}$

$I = \boxed{6.43\text{ A}}$

27.16 $J = \frac{I}{\pi r^2} = \sigma E = \frac{3.00\text{ A}}{\pi(0.0120\text{ m})^2} = \sigma(120\text{ N/C})$

$\sigma = 55.3(\Omega \cdot \text{m})^{-1}$ $\rho = \frac{1}{\sigma} = \boxed{0.0181\ \Omega \cdot \text{m}}$

27.17 (a) Given $M = \rho_d V = \rho_d A l$ where $\rho_d \equiv$ mass density, we obtain: $A = \frac{M}{\rho_d l}$

Taking $\rho_r \equiv$ resistivity, $R = \frac{\rho_r l}{A} = \frac{\rho_r l}{\left(\frac{M}{\rho_d l}\right)} = \frac{\rho_r \rho_d l^2}{M}$

Thus, $l = \sqrt{\frac{MR}{\rho_r \rho_d}} = \sqrt{\frac{(1.00 \times 10^{-3})(0.500)}{(1.70 \times 10^{-8})(8.92 \times 10^3)}} = \boxed{1.82\text{ m}}$

(b) $V = \dfrac{M}{\rho_d}$, or $\pi r^2 \ell = \dfrac{M}{\rho_d}$

Thus, $r = \sqrt{\dfrac{M}{\pi \rho_d \ell}} = \sqrt{\dfrac{1.00 \times 10^{-3}}{\pi (8.92 \times 10^3)(1.82)}} = 1.40 \times 10^{-4}\ \text{m}$

The diameter is twice this distance: $\boxed{\text{diameter} = 280\ \mu\text{m}}$

***27.18** (a) Suppose the rubber is 10 cm long and 1 mm in diameter.

$$R = \frac{\rho \ell}{A} = \frac{4\rho \ell}{\pi d^2} \sim \frac{4\left(10^{13}\ \Omega \cdot \text{m}\right)\left(10^{-1}\ \text{m}\right)}{\pi \left(10^{-3}\ \text{m}\right)^2} = \boxed{\sim 10^{18}\ \Omega}$$

(b) $R = \dfrac{4\rho \ell}{\pi d^2} \sim \dfrac{4(1.7 \times 10^{-8}\ \Omega \cdot \text{m})(10^{-3}\ \text{m})}{\pi (2 \times 10^{-2}\ \text{m})^2}\ \boxed{\sim 10^{-7}\ \Omega}$

(c) $I = \dfrac{\Delta V}{R} \sim \dfrac{10^2\ \text{V}}{10^{18}\ \Omega}\ \boxed{\sim 10^{-16}\ \text{A}}$

$I \sim \dfrac{10^2\ \text{V}}{10^{-7}\ \Omega}\ \boxed{\sim 10^{9}\ \text{A}}$

27.19 The distance between opposite faces of the cube is $\ell = \left(\dfrac{90.0\ \text{g}}{10.5\ \text{g/cm}^3}\right)^{1/3} = 2.05\ \text{cm}$

(a) $R = \dfrac{\rho \ell}{A} = \dfrac{\rho \ell}{\ell^2} = \dfrac{\rho}{\ell} = \dfrac{1.59 \times 10^{-8}\ \Omega \cdot \text{m}}{2.05 \times 10^{-2}\ \text{m}} = 7.77 \times 10^{-7}\ \Omega = \boxed{777\ \text{n}\Omega}$

(b) $I = \dfrac{\Delta V}{R} = \dfrac{1.00 \times 10^{-5}\ \text{V}}{7.77 \times 10^{-7}\ \Omega} = 12.9\ \text{A}$

$$n = \frac{10.5\ \text{g/cm}^3}{107.87\ \text{g/mol}}\left(6.02 \times 10^{23}\ \frac{\text{electrons}}{\text{mol}}\right)$$

$$n = \left(5.86 \times 10^{22}\ \frac{\text{electrons}}{\text{cm}^3}\right)\left(\frac{1.00 \times 10^6\ \text{cm}^3}{1.00\ \text{m}^3}\right) = 5.86 \times 10^{28}/\text{m}^3$$

$I = nqvA$ and $v = \dfrac{I}{nqA} = \dfrac{12.9\ \text{C/s}}{\left(5.86 \times 10^{28}/\text{m}^3\right)\left(1.60 \times 10^{-19}\ \text{C}\right)(0.0205\ \text{m})^2} = \boxed{3.28\ \mu\text{m/s}}$

27.20 Originally, $R = \frac{\rho \ell}{A}$

Finally, $R_f = \frac{\rho(\ell/3)}{3A} = \frac{\rho \ell}{9A} = \boxed{\frac{R}{9}}$

27.21 The total volume of material present does not change, only its shape. Thus,

$A_f \ell_f = A_f(1.25\ell_i) = A_i \ell_i$ giving $A_f = A_i/1.25$

The final resistance is then: $R_f = \frac{\rho \ell_f}{A_f} = \frac{\rho(1.25\ell_i)}{A_i/1.25} = 1.56\left(\frac{\rho \ell_i}{A_i}\right) = \boxed{1.56R}$

27.22 $\frac{\rho_{Al}\ell}{\pi(r_{Al})^2} = \frac{\rho_{Cu}\ell}{\pi(r_{Cu})^2}$

$\frac{r_{\text{Al}}}{r_{\text{Cu}}} = \sqrt{\frac{\rho_{\text{Al}}}{\rho_{\text{Cu}}}} = \sqrt{\frac{2.82\times10^{-8}}{1.70\times10^{-8}}} = \boxed{1.29}$

27.23 $J = \sigma E$ so $\sigma = \frac{J}{E} = \frac{6.00\times10^{-13}\text{ A/m}^2}{100\text{ V/m}} = \boxed{6.00\times10^{-15}\,(\Omega\cdot\text{m})^{-1}}$

27.24 $R = \frac{\rho_1 \ell_1}{A_1} + \frac{\rho_2 \ell_2}{A_2} = (\rho_1 \ell_1 + \rho_2 \ell_2)/d^2$

$$R = \frac{(4.00\times10^{-3}\ \Omega\cdot\text{m})(0.250\text{ m}) + (6.00\times10^{-3}\ \Omega\cdot\text{m})(0.400\text{ m})}{(3.00\times10^{-3}\text{ m})^2} = \boxed{378\ \Omega}$$

27.25 $\rho = \frac{m}{nq^2\tau}$ so $\tau = \frac{m}{\rho n q^2} = \frac{9.11\times10^{-31}}{(1.70\times10^{-8})(8.49\times10^{28})(1.60\times10^{19})^2} = 2.47\times10^{-14}\text{ s}$

$v_d = \frac{qE}{m}\tau$ so $7.84\times10^{-4} = \frac{(1.60\times10^{-19})E(2.47\times10^{-14})}{9.11\times10^{-31}}$

Therefore $\boxed{E = 0.181\text{ V/m}}$

Goal Solution

If the drift velocity of free electrons in a copper wire is 7.84×10^{-4} m / s, what is the electric field in the conductor?

G: For electrostatic cases, we learned that the electric field inside a conductor is always zero. On the other hand, if there is a current, a non-zero electric field must be maintained by a battery or other source to make the charges flow. Therefore, we might expect the electric field to be small, but definitely **not** zero.

O: The drift velocity of the electrons can be used to find the current density, which can be used with Ohm's law to find the electric field inside the conductor.

A: We first need the electron density in copper, which from Example 27.1 is $n = 8.49\times10^{28}$ e$^-$ / m^3. The current density in this wire is then

$$J = nqv_d = (8.49\times10^{28}\ \text{e}^- / \text{m}^3)(1.60\times10^{-19}\ \text{C} / \text{e}^-)(7.84\times10^{-4}\ \text{m} / \text{s}) = 1.06\times10^7\ \text{A} / \text{m}^2$$

Ohm's law can be stated as $J = \sigma E = E/\rho$ where $\rho = 1.7\times10^{-8}\ \Omega\cdot\text{m}$ for copper, so then

$$E = \rho J = (1.70\times10^{-8}\ \Omega\cdot\text{m})(1.06\times10^7\ \text{A} / \text{m}^2) = 0.181\ \text{V} / \text{m}$$

L: This electric field is certainly smaller than typical static values outside charged objects. The direction of the electric field should be along the length of the conductor, otherwise the electrons would be forced to leave the wire! The reality is that excess charges arrange themselves on the surface of the wire to create an electric field that "steers" the free electrons to flow along the length of the wire from low to high potential (opposite the direction of a positive test charge). It is also interesting to note that when the electric field is being established it travels at the speed of light; but the drift velocity of the electrons is literally at a "snail's pace"!

27.26 (a) n is $\boxed{\text{unaffected}}$

(b) $|J| = \frac{I}{A} \propto I$ so it $\boxed{\text{doubles}}$

(c) $J = nev_d$ so v_d $\boxed{\text{doubles}}$

(d) $\tau = \frac{m\sigma}{nq^2}$ is $\boxed{\text{unchanged}}$ as long as σ does not change due to heating.

27.27 From Equation 27.17,

$$\tau = \frac{m_e}{nq^2\rho} = \frac{9.11\times10^{-31}}{\left(8.49\times10^{28}\right)\left(1.60\times10^{-19}\right)^2\left(1.70\times10^{-8}\right)} = 2.47\times10^{-14}\ \text{s}$$

$$l = v\tau = \left(8.60\times10^5\ \text{m} / \text{s}\right)\left(2.47\times10^{-14}\ \text{s}\right) = 2.12\times10^{-8}\ \text{m} = \boxed{21.2\ \text{nm}}$$

27.28 At the low temperature T_C we write $R_C = \dfrac{\Delta V}{I_C} = R_0\left[1+\alpha(T_C - T_0)\right]$ where $T_0 = 20.0$°C

At the high temperature T_h, $R_h = \dfrac{\Delta V}{I_h} = \dfrac{\Delta V}{1\text{ A}} = R_0\left[1+\alpha(T_h - T_0)\right]$

Then $$\frac{(\Delta V)/(1.00\text{ A})}{(\Delta V)/I_C} = \frac{1 + (3.90\times 10^{-3})(38.0)}{1 + (3.90\times 10^{-3})(-108)}$$

and $I_C = (1.00\text{ A})(1.15/0.579) = \boxed{1.98\text{ A}}$

***27.29** $R = R_0\left[1+\alpha(\Delta T)\right]$ gives $140\ \Omega = (19.0\ \Omega)\left[1+(4.50\times 10^{-3}/°\text{C})\Delta T\right]$

Solving, $\Delta T = 1.42\times 10^{3}\ °\text{C} = T - 20.0\ °\text{C}$

And, the final temperature is $\boxed{T = 1.44\times 10^{3}\ °\text{C}}$

27.30 $R = R_c + R_n = R_c\left[1+\alpha_c(T-T_0)\right] + R_n\left[1+\alpha_n(T-T_0)\right]$

$0 = R_c\alpha_c\,(T - T_0) + R_n\alpha_n\,(T - T_0)$ so $R_c = -R_n\dfrac{\alpha_n}{\alpha_c}$

$$R = -R_n\frac{\alpha_n}{\alpha_c} + R_n$$

$R_n = R(1 - \alpha_n/\alpha_c)^{-1}$ $\quad R_c = R(1 - \alpha_c/\alpha_n)^{-1}$

$$R_n = 10.0\text{ k}\Omega\left[1 - \frac{(0.400\times 10^{-3}/\text{C°})}{(-0.500\times 10^{-3}/\text{C°})}\right]^{-1}$$

$\boxed{R_n = 5.56\text{ k}\Omega}$ and $\boxed{R_c = 4.44\text{ k}\Omega}$

27.31 (a) $\rho = \rho_0\left[1+\alpha(T-T_0)\right] = (2.82\times 10^{-8}\ \Omega\cdot\text{m})\left[1+3.90\times 10^{-3}(30.0°)\right] = \boxed{3.15\times 10^{-8}\ \Omega\cdot\text{m}}$

(b) $J = \dfrac{E}{\rho} = \dfrac{0.200\text{ V/m}}{3.15\times 10^{-8}\ \Omega\cdot\text{m}} = \boxed{6.35\times 10^{6}\text{ A/m}^2}$

(c) $I = JA = \dfrac{\pi d^2}{4}J = \dfrac{\pi(1.00\times 10^{-4}\text{ m})^2}{4}(6.35\times 10^{6}\text{ A/m}^2) = \boxed{49.9\text{ mA}}$

(d) $n = \dfrac{6.02\times 10^{23}\text{ electrons}}{\left(\dfrac{26.98\text{ g}}{2.70\times 10^{6}\text{ g/m}^3}\right)} = 6.02\times 10^{28}\text{ electrons/m}^3$

$$v_d = \frac{J}{ne} = \frac{(6.35\times 10^{6}\text{ A/m}^2)}{(6.02\times 10^{28}\text{ electrons/m}^3)(1.60\times 10^{-19}\text{ C})} = \boxed{659\ \mu\text{m/s}}$$

(e) $\Delta V = E\,\ell = (0.200\text{ V/m})(2.00\text{ m}) = \boxed{0.400\text{ V}}$

***27.32** For aluminum, $\alpha_E = 3.90 \times 10^{-3}/°\text{C}$ (Table 27.1) $\alpha = 24.0 \times 10^{-6}/°\text{C}$ (Table 19.2)

$$R = \frac{\rho l}{A} = \frac{\rho_0(1+\alpha_E \Delta T)(1+\alpha \Delta T)}{A(1+\alpha \Delta T)^2} = R_0 \frac{(1 + \alpha_E\, \Delta T)}{(1 + \alpha\, \Delta T)} = (1.234\ \Omega) \frac{(1.39)}{(1.0024)} = \boxed{1.71\ \Omega}$$

27.33 $R = R_0[1 + \alpha\, \Delta T]$

$R - R_0 = R_0 \alpha\, \Delta T$

$$\frac{R - R_0}{R_0} = \alpha\, \Delta T = (5.00 \times 10^{-3})25.0 = \boxed{0.125}$$

27.34 Assuming linear change of resistance with temperature, $R = R_0(1 + \alpha\, \Delta T)$

$$R_{77\,K} = (1.00\ \Omega)\left[1 + \left(3.92 \times 10^{-3}\right)(-216°\text{ C})\right] = \boxed{0.153\ \Omega}$$

27.35 $\rho = \rho_0(1 + \alpha\, \Delta T)$ or $\Delta T_W = \dfrac{1}{\alpha_W}\left(\dfrac{\rho_W}{\rho_{0W}} - 1\right)$

Require that $\rho_W = 4\rho_{0_{\text{Cu}}}$ so that $\Delta T_W = \left(\dfrac{1}{4.50 \times 10^{-3}\ /°\text{C}}\right)\left(\dfrac{4\left(1.70 \times 10^{-8}\right)}{5.60 \times 10^{-8}} - 1\right) = 47.6\ °\text{C}$

Therefore, $T_W = 47.6\ °\text{C} + T_0 = \boxed{67.6°\text{C}}$

27.36

$$\alpha = \frac{1}{R_0}\left(\frac{\Delta R}{\Delta T}\right) = \left(\frac{1}{R_0}\right)\frac{2R_0 - R_0}{T - T_0} = \frac{1}{T - T_0}$$

so, $T = \left(\dfrac{1}{\alpha}\right) + T_0$ and $T = \left(\dfrac{1}{0.400 \times 10^{-3}\ \text{C}°^{-1}}\right) + 20.0\ °\text{C}$ so $T = \boxed{2.52 \times 10^3\ °\text{C}}$

***27.37**

$$I = \frac{\mathcal{P}}{\Delta V} = \frac{600\ \text{W}}{120\ \text{V}} = \boxed{5.00\ \text{A}}$$

and $R = \dfrac{\Delta V}{I} = \dfrac{120\ \text{V}}{5.00\ \text{A}} = \boxed{24.0\ \Omega}$

27.38 $\mathcal{P} = 0.800(1500\text{ hp})(746\text{ W/hp}) = 8.95 \times 10^5\text{ W}$

$\mathcal{P} = I(\Delta V)$

$8.95 \times 10^5 = I(2000)$

$\boxed{I = 448\text{ A}}$

27.39 The heat that must be added to the water is

$Q = mc\,\Delta T = (1.50\text{ kg})(4186\text{ J/kg°C})(40.0\text{°C}) = 2.51 \times 10^5\text{ J}$

Thus, the power supplied by the heater is

$$\mathcal{P} = \frac{W}{t} = \frac{Q}{t} = \frac{2.51 \times 10^5\text{ J}}{600\text{ s}} = 419\text{ W}$$

and the resistance is $R = \dfrac{(\Delta V)^2}{\mathcal{P}} = \dfrac{(110\text{ V})^2}{419\text{ W}} = \boxed{28.9\ \Omega}$

27.40 The heat that must be added to the water is $Q = mc(T_2 - T_1)$

Thus, the power supplied by the heat is $\mathcal{P} = \dfrac{W}{\Delta t} = \dfrac{Q}{\Delta t} = \dfrac{mc(T_2 - T_1)}{t}$

and the resistance is $R = \dfrac{(\Delta V)^2}{\mathcal{P}} = \boxed{\dfrac{(\Delta V)^2 t}{mc(T_2 - T_1)}}$

27.41 $$\frac{\mathcal{P}}{\mathcal{P}_0} = \frac{(\Delta V)^2 / R}{(\Delta V_0)^2 / R} = \left(\frac{\Delta V}{\Delta V_0}\right)^2 = \left(\frac{140}{120}\right)^2 = 1.361$$

$$\Delta\% = \left(\frac{\mathcal{P} - \mathcal{P}_0}{\mathcal{P}_0}\right)(100\%) = \left(\frac{\mathcal{P}}{\mathcal{P}_0} - 1\right)(100\%) = (1.361 - 1)100 = \boxed{36.1\%}$$

Goal Solution

Suppose that a voltage surge produces 140 V for a moment. By what percentage does the power output of a 120-V, 100-W light bulb increase? (Assume that its resistance does not change.)

G: The voltage increases by about 20%, but since $\mathcal{P} = (\Delta V)^2 / R$, the power will increase as the square of the voltage:

$$\frac{\mathcal{P}_f}{\mathcal{P}_i} = \frac{(\Delta V_f)^2 / R}{(\Delta V_i)^2 / R} = \frac{(140\text{ V})^2}{(120\text{ V})^2} = 1.361 \text{ or a } 36.1\% \text{ increase.}$$

O: We have already found an answer to this problem by reasoning in terms of ratios, but we can also calculate the power explicitly for the bulb and compare with the original power by using Ohm's law and the equation for electrical power. To find the power, we must first find the resistance of the bulb, which should remain relatively constant during the power surge (we can check the validity of this assumption later).

A: From $\mathcal{P} = (\Delta V)^2 / R$, we find that $R = \frac{(\Delta V_i)^2}{\mathcal{P}} = \frac{(120\text{ V})^2}{100\text{ W}} = 144\ \Omega$

The final current is, $I_f = \frac{\Delta V_f}{R} = \frac{140\text{ V}}{144\ \Omega} = 0.972\text{ A}$

The power during the surge is $\mathcal{P}_f = \frac{(\Delta V_f)^2}{R} = \frac{(140\text{ V})^2}{144\ \Omega} = 136\text{ V}$

So the percentage increase is $\frac{136\text{ W} - 100\text{ W}}{100\text{ W}} = 0.361 = 36.1\%$

L: Our result tells us that this 100-W light bulb momentarily acts like a 136-W light bulb, which explains why it would suddenly get brighter. Some electronic devices (like computers) are sensitive to voltage surges like this, which is the reason that **surge protectors** are recommended to protect these devices from being damaged.

In solving this problem, we assumed that the resistance of the bulb did not change during the voltage surge, but we should check this assumption. Let us assume that the filament is made of tungsten and that its resistance will change linearly with temperature according to equation 27.21. Let us further assume that the increased voltage lasts for a time long enough so that the filament comes to a new equilibrium temperature. The temperature change can be estimated from the power surge according to Stefan's law (equation 20.18), assuming that all the power loss is due to radiation. By this law, $T \propto \sqrt[4]{\mathcal{P}}$ so that a 36% change in power should correspond to only about a 8% increase in temperature. A typical operating temperature of a white light bulb is about 3000 °C, so $\Delta T \approx 0.08(3273\ °\text{C}) = 260\ °\text{C}$. Then the increased resistance would be roughly

$$R = R_0\left(1 + \alpha(T - T_0)\right) = (144\ \Omega)\left(1 + 4.5 \times 10^{-3}(260)\right) \cong 310\ \Omega$$

It appears that the resistance could change double from 144 Ω. On the other hand, if the voltage surge lasts only a very short time, the 136 W we calculated originally accurately describes the conversion of electrical into internal energy in the filament.

27.42 $\mathcal{P} = I(\Delta V) = \dfrac{(\Delta V)^2}{R} = 500\ \text{W}$

$$R = \frac{(110\ \text{V})^2}{(500\ \text{W})} = 24.2\ \Omega$$

(a) $R = \dfrac{\rho}{A}\ell$ so $\ell = \dfrac{RA}{\rho} = \dfrac{(24.2\ \Omega)\pi(2.50\times10^{-4}\ \text{m})^2}{1.50\times10^{-6}\ \Omega\cdot\text{m}} = \boxed{3.17\ \text{m}}$

(b) $R = R_0[1 + \alpha\,\Delta T] = 24.2\ \Omega\left[1+(0.400\times10^{-3})(1180)\right] = 35.6\ \Omega$

$$\mathcal{P} = \frac{(\Delta V)^2}{R} = \frac{(110)^2}{35.6} = \boxed{340\ \text{W}}$$

27.43 $R = \dfrac{\rho\ell}{A} = \dfrac{(1.50\times10^{-6}\ \Omega\cdot\text{m})25.0\ \text{m}}{\pi(0.200\times10^{-3}\ \text{m})^2} = 298\ \Omega$

$$\Delta V = IR = (0.500\ \text{A})(298\ \Omega) = 149\ \text{V}$$

(a) $E = \dfrac{\Delta V}{\ell} = \dfrac{149\ \text{V}}{25.0\ \text{m}} = \boxed{5.97\ \text{V/m}}$

(b) $\mathcal{P} = (\Delta V)I = (149\ \text{V})(0.500\ \text{A}) = \boxed{74.6\ \text{W}}$

(c) $R = R_0\left[1+\alpha(T-T_0)\right] = 298\ \Omega\left[1+(0.400\times10^{-3}\ /\ \text{C}°)320\ \text{C}°\right] = 337\ \Omega$

$$I = \frac{\Delta V}{R} = \frac{(149\ \text{V})}{(337\ \Omega)} = 0.443\ \text{A}$$

$$\mathcal{P} = (\Delta V)I = (149\ \text{V})(0.443\ \text{A}) = \boxed{66.1\ \text{W}}$$

27.44 (a) $\Delta U = q(\Delta V) = It(\Delta V) = (55.0\ \text{A}\cdot\text{h})(12.0\ \text{V})\left(\dfrac{1\ \text{C}}{1\ \text{A}\cdot\text{s}}\right)\left(\dfrac{1\ \text{J}}{1\ \text{V}\cdot\text{C}}\right)\left(\dfrac{1\ \text{W}\cdot\text{s}}{1\ \text{J}}\right) = 660\ \text{W}\cdot\text{h} = \boxed{0.660\ \text{kWh}}$

(b) $\text{Cost} = 0.660\ \text{kWh}\left(\dfrac{\$0.0600}{1\ \text{kWh}}\right) = \boxed{3.96¢}$

27.45 $\mathcal{P} = I(\Delta V)$ $\Delta V = IR$

$$\mathcal{P} = \frac{(\Delta V)^2}{R} = \frac{(10.0)^2}{120} = \boxed{0.833\ \text{W}}$$

27.46 The total clock power is $\left(270\times10^{6}\text{ clocks}\right)\left(2.50\ \dfrac{\text{J/s}}{\text{clock}}\right)\left(\dfrac{3600\text{ s}}{1\text{ h}}\right)=2.43\times10^{12}\text{ J/h}$

From $e=\dfrac{W_{\text{out}}}{Q_{\text{in}}}$, the power input to the generating plants must be:

$$\frac{Q_{\text{in}}}{t}=\frac{W_{\text{out}}/t}{e}=\frac{2.43\times10^{12}\text{ J/h}}{0.250}=9.72\times10^{12}\text{ J/h}$$

and the rate of coal consumption is

$$\text{Rate}=\left(9.72\times10^{12}\text{ J/h}\right)\left(\frac{1.00\text{ kg coal}}{33.0\times10^{6}\text{ J}}\right)=2.95\times10^{5}\ \frac{\text{kg coal}}{\text{h}}=\boxed{295\ \frac{\text{metric ton}}{\text{h}}}$$

27.47 $\mathcal{P}=I(\Delta V)=(1.70\text{ A})(110\text{ V})=187\text{ W}$

Energy used in a 24-hour day = (0.187 kW)(24.0 h) = 4.49 kWh

$\therefore$ cost = 4.49 kWh $\left(\dfrac{\$0.0600}{\text{kWh}}\right)$ = \$0.269 = $\boxed{26.9¢}$

27.48 $\mathcal{P}=I(\Delta V)=(2.00\text{ A})(120\text{ V})=240\text{ W}$

$\Delta U=(0.500\text{ kg})(4186\text{ J/kg°C})(77.0\text{°C})=161\text{ kJ}$

$$t=\frac{\Delta U}{\mathcal{P}}=\frac{1.61\times10^{5}\text{ J}}{240\text{ W}}=\boxed{672\text{ s}}$$

27.49 At operating temperature,

(a) $\mathcal{P}=I(\Delta V)=(1.53\text{ A})(120\text{ V})=\boxed{184\text{ W}}$

(b) Use the change in resistance to find the final operating temperature of the toaster.

$R=R_0(1+\alpha\,\Delta T)$

$$\frac{120}{1.53}=\frac{120}{1.80}\left[1+(0.400\times10^{-3})\Delta T\right]$$

$\Delta T=441\text{°C}$

$T=20.0\text{°C}+441\text{°C}=\boxed{461\text{°C}}$

Goal Solution

A certain toaster has a heating element made of Nichrome resistance wire. When the toaster is first connected to a 120-V source of potential difference (and the wire is at a temperature of 20.0 °C), the initial current is 1.80 A. However, the current begins to decrease as the resistive element warms up. When the toaster has reached its final operating temperature, the current has dropped to 1.53 A. (a) Find the power the toaster consumes when it is at its operating temperature. (b) What is the final temperature of the heating element?

G: Most toasters are rated at about 1000 W (usually stamped on the bottom of the unit), so we might expect this one to have a similar power rating. The temperature of the heating element should be hot enough to toast bread but low enough that the nickel-chromium alloy element does not melt. (The melting point of nickel is 1455 °C, and chromium melts at 1907 °C.)

O: The power can be calculated directly by multiplying the current and the voltage. The temperature can be found from the linear conductivity equation for Nichrome, with $\alpha = 0.4 \times 10^{-3}\ °C^{-1}$ from Table 27.1.

A: (a) $\mathcal{P} = (\Delta V)I = (120\ \text{V})(1.53\ \text{A}) = 184\ \text{W}$

(b) The resistance at 20.0 °C is $R_0 = \frac{\Delta V}{I} = \frac{120\ \text{V}}{1.80\ \text{A}} = 66.7\ \Omega$

At operating temperature, $R = \frac{120\ \text{V}}{1.53\ \text{A}} = 78.4\ \Omega$

Neglecting thermal expansion, $R = \frac{\rho l}{A} = \frac{\rho_0\left(1 + \alpha(T - T_0)\right)l}{A} = R_0\left(1 + \alpha(T - T_0)\right)$

$$T = T_0 + \frac{R/R_0 - 1}{\alpha} = 20.0\ °C + \frac{78.4\ \Omega/66.7\ \Omega - 1}{0.4 \times 10^{-3}\ °C^{-1}} = 461\ °C$$

L: Although this toaster appears to use significantly less power than most, the temperature seems high enough to toast a piece of bread in a reasonable amount of time. In fact, the temperature of a typical 1000-W toaster would only be slightly higher because Stefan's radiation law (Eq. 20.18) tells us that (assuming all power is lost through radiation) $T \propto \sqrt[4]{\mathcal{P}}$, so that the temperature might be about 700 °C. In either case, the operating temperature is well below the melting point of the heating element.

27.50 $\mathcal{P} = (10.0\ \text{W/ft}^2)(10.0\ \text{ft})(15.0\ \text{ft}) = 1.50\ \text{kW}$

$\text{Energy} = \mathcal{P}\,t = (1.50\ \text{kW})(24.0\ \text{h}) = 36.0\ \text{kWh}$

$\text{Cost} = (36.0\ \text{kWh})(\$0.0800/\text{kWh}) = \boxed{\$2.88}$

***27.51** Consider a 400-W blow dryer used for ten minutes daily for a year. The energy converted is

$$\mathcal{P}\,t = (400\ \text{J/s})(600\ \text{s/d})(365\ \text{d}) \cong 9 \times 10^7\ \text{J}\left(\frac{1\ \text{kWh}}{3.6 \times 10^6\ \text{J}}\right) \cong 20\ \text{kWh}$$

We suppose that electrical energy costs on the order of ten cents per kilowatt-hour. Then the cost of using the dryer for a year is on the order of

$\text{Cost} \cong (20\ \text{kWh})(\$0.100/\text{kWh}) = \$2 \quad \boxed{\sim\$1}$

***27.52** (a) $I = \frac{\Delta V}{R}$ so $\mathcal{P} = (\Delta V)I = \frac{(\Delta V)^2}{R}$

$R = \frac{(\Delta V)^2}{\mathcal{P}} = \frac{(120\ \text{V})^2}{25.0\ \text{W}} = \boxed{576\ \Omega}$ and $R = \frac{(\Delta V)^2}{\mathcal{P}} = \frac{(120\ \text{V})^2}{100\ \text{W}} = \boxed{144\ \Omega}$

(b) $I = \frac{\mathcal{P}}{\Delta V} = \frac{25.0\ \text{W}}{120\ \text{V}} = 0.208\ \text{A} = \frac{Q}{t} = \frac{1.00\ \text{C}}{t}$

$t = \frac{1.00\ \text{C}}{0.208\ \text{A}} = \boxed{4.80\ \text{s}}$ The charge has $\boxed{\text{lower potential energy}}$.

(c) $\mathcal{P} = 25.0\ \text{W} = \frac{\Delta U}{t} = \frac{1.00\ \text{J}}{t}$

$t = \frac{1.00\ \text{J}}{25.0\ \text{W}} = \boxed{0.0400\ \text{s}}$ The energy $\boxed{\text{changes from electrical to heat and light}}$.

(d) $\Delta U = \mathcal{P}t = (25.0\ \text{J/s})(86400\ \text{s/d})(30.0\ \text{d}) = 64.8 \times 10^6\ \text{J}$

The energy company sells $\boxed{\text{energy}}$.

$\text{Cost} = 64.8 \times 10^6\ \text{J}\left(\frac{\$0.0700}{\text{kWh}}\right)\left(\frac{\text{k}}{1000}\right)\left(\frac{\text{W} \cdot \text{s}}{\text{J}}\right)\left(\frac{\text{h}}{3600\ \text{s}}\right) = \boxed{\$1.26}$

$\text{Cost per joule} = \frac{\$0.0700}{\text{kWh}}\left(\frac{\text{kWh}}{3.60 \times 10^6\ \text{J}}\right) = \boxed{\$1.94 \times 10^{-8}/\text{J}}$

***27.53** We find the drift velocity from $I = nqv_d A = nqv_d \pi r^2$

$$v_d = \frac{I}{nq\pi r^2} = \frac{1000\ \text{A}}{8.00 \times 10^{28}\ \text{m}^{-3}(1.60 \times 10^{-19}\ \text{C})\pi(10^{-2}\ \text{m})^2} = 2.49 \times 10^{-4}\ \text{m/s}$$

$$v = \frac{x}{t} \qquad t = \frac{x}{v} = \frac{200 \times 10^3\ \text{m}}{2.49 \times 10^{-4}\ \text{m/s}} = 8.04 \times 10^8\ \text{s} = \boxed{25.5\ \text{yr}}$$

***27.54** The resistance of one wire is $\left(\frac{0.500\ \Omega}{\text{mi}}\right)(100\ \text{mi}) = 50.0\ \Omega$

The whole wire is at nominal 700 kV away from ground potential, but the potential difference between its two ends is

$IR = (1000\ \text{A})(50.0\ \Omega) = 50.0\ \text{kV}$

Then it radiates as heat power $\mathcal{P} = (\Delta V)I = (50.0 \times 10^3\ \text{V})(1000\ \text{A}) = \boxed{50.0\ \text{MW}}$

27.55 We begin with the differential equation $\alpha = \frac{1}{\rho}\frac{d\rho}{dT}$

(a) Separating variables, $\int_{\rho_0}^{\rho} \frac{d\rho}{\rho} = \int_{T_0}^{T} \alpha\, dT$

$\ln\left(\frac{\rho}{\rho_0}\right) = \alpha(T - T_0)$ and $\boxed{\rho = \rho_0 e^{\alpha(T - T_0)}}$

(b) From the series expansion $e^x \cong 1 + x$, $(x << 1)$,

$\boxed{\rho \cong \rho_0[1 + \alpha(T - T_0)]}$

***27.56** Consider a 1.00-m length of cable. The potential difference between its ends is

$$\Delta V = \frac{\mathcal{P}}{I} = \frac{2.00\text{ W}}{300\text{ A}} = 6.67\text{ mV}$$

The resistance is $R = \frac{\Delta V}{I} = \frac{6.67 \times 10^{-3}\text{ V}}{300\text{ A}} = 22.2\ \mu\Omega$

Then $R = \frac{\rho \ell}{A} = \frac{\rho \ell}{\pi r^2}$ gives $r = \sqrt{\frac{\rho \ell}{\pi R}} = \sqrt{\frac{(1.70 \times 10^{-8}\ \Omega \cdot \text{m})(1.00\text{ m})}{\pi(22.2 \times 10^{-6}\ \Omega)}} = \boxed{1.56\text{ cm}}$

27.57 $\rho = \frac{RA}{\ell} = \frac{(\Delta V)}{I}\frac{A}{\ell}$

ℓ (m)	$R(\Omega)$	$\rho(\Omega \cdot \text{m})$
0.540	10.4	1.41×10^{-6}
1.028	21.1	1.50×10^{-6}
1.543	31.8	1.50×10^{-6}

$\bar{\rho} = \boxed{1.47 \times 10^{-6}\ \Omega \cdot \text{m}}$ (in agreement with tabulated value)

$\rho = \frac{RA}{\ell} = \boxed{1.50 \times 10^{-6}\ \Omega \cdot \text{m}}$ (Table 27.1)

27.58 2 wires $\rightarrow \ell = 100$ m

$$R = \frac{0.108\ \Omega}{300\ \text{m}}(100\ \text{m}) = 0.0360\ \Omega$$

(a) $(\Delta V)_{\text{home}} = (\Delta V)_{\text{line}} - IR = 120 - (110)(0.0360) = \boxed{116\ \text{V}}$

(b) $\mathcal{P} = I(\Delta V) = (110\ \text{A})(116\ \text{V}) = \boxed{12.8\ \text{kW}}$

(c) $\mathcal{P}_{\text{wires}} = I^2R = (110\ \text{A})^2(0.0360\ \Omega) = \boxed{436\ \text{W}}$

***27.59** (a) $\mathbf{E} = -\frac{dV}{dx}\mathbf{i} = -\frac{(0-4.00)\text{V}}{(0.500-0)\ \text{m}} = \boxed{8.00\mathbf{i}\ \text{V/m}}$

(b) $R = \frac{\rho \ell}{A} = \frac{(4.00\times10^{-8}\ \Omega\cdot\text{m})(0.500\ \text{m})}{\pi(1.00\times10^{-4}\ \text{m})^2} = \boxed{0.637\ \Omega}$

(c) $I = \frac{\Delta V}{R} = \frac{4.00\ \text{V}}{0.637\ \Omega} = \boxed{6.28\ \text{A}}$

(d) $\mathbf{J} = \frac{I}{A}\mathbf{i} = \frac{6.28\ \text{A}}{\pi(1.00\times10^{-4}\ \text{m})^2} = 2.00\times10^8\ \mathbf{i}\ \text{A/m}^2 = \boxed{200\mathbf{i}\ \text{MA/m}^2}$

(e) $\rho\mathbf{J} = (4.00\times10^{-8}\ \Omega\cdot\text{m})(2.00\times10^8\ \mathbf{i}\ \text{A/m}^2) = 8.00\mathbf{i}\ \text{V/m} = \mathbf{E}$

***27.60** (a) $\mathbf{E} = -\frac{dV(x)}{dx}\ \mathbf{i} = \boxed{\frac{V}{L}\mathbf{i}}$

(b) $R = \frac{\rho \ell}{A} = \boxed{\frac{4\rho L}{\pi d^2}}$

(c) $I = \frac{\Delta V}{R} = \boxed{\frac{V\pi d^2}{4\rho L}}$

(d) $\mathbf{J} = \frac{I}{A}\ \mathbf{i} = \boxed{\frac{V}{\rho L}\mathbf{i}}$

(e) $\rho\,\mathbf{J} = \frac{V}{L}\ \mathbf{i} = \boxed{\mathbf{E}}$

27.61 $R = R_0[1 + \alpha(T - T_0)]$ so $T = T_0 + \frac{1}{\alpha}\left[\frac{R}{R_0} - 1\right] = T_0 + \frac{1}{\alpha}\left[\frac{I_0}{I} - 1\right]$

In this case, $I = \frac{I_0}{10}$, so $T = T_0 + \frac{1}{\alpha}(9) = 20° + \frac{9}{0.00450/°\text{C}} = \boxed{2020\ °\text{C}}$

27.62 $R = \frac{\Delta V}{I} = \frac{12.0}{I} = \frac{6.00}{(I - 3.00)}$ thus $12.0I - 36.0 = 6.00I$ and $I = 6.00\text{ A}$

Therefore, $R = \frac{12.0\text{ V}}{6.00\text{ A}} = \boxed{2.00\ \Omega}$

27.63 (a) $\mathcal{P} = I(\Delta V)$ so $I = \frac{\mathcal{P}}{\Delta V} = \frac{8.00 \times 10^3\text{ W}}{12.0\text{ V}} = \boxed{667\text{ A}}$

(b) $t = \frac{\Delta U}{\mathcal{P}} = \frac{2.00 \times 10^7\text{ J}}{8.00 \times 10^3\text{ W}} = 2.50 \times 10^3\text{ s}$ and $d = vt = (20.0\text{ m / s})(2.50 \times 10^3\text{ s}) = \boxed{50.0\text{ km}}$

27.64 (a) We begin with $R = \frac{\rho l}{A} = \frac{\rho_0[1 + \alpha(T - T_0)]\, l_0[1 + \alpha'(T - T_0)]}{A_0[1 + 2\alpha'(T - T_0)]}$,

which reduces to $R = \boxed{\frac{R_0[1 + \alpha(T - T_0)]\,[1 + \alpha'(T - T_0)]}{[1 + 2\alpha'(T - T_0)]}}$

(b) For copper: $\rho_0 = 1.70 \times 10^{-8}\ \Omega \cdot \text{m}$, $\alpha = 3.90 \times 10^{-3}\ °\text{C}^{-1}$, and $\alpha' = 17.0 \times 10^{-6}\ °\text{C}^{-1}$

$$R_0 = \frac{\rho_0 l_0}{A_0} = \frac{(1.70 \times 10^{-8})(2.00)}{\pi(0.100 \times 10^{-3})^2} = \boxed{1.08\ \Omega}$$

The simple formula for R gives:

$$R = (1.08\ \Omega)\left[1 + (3.90 \times 10^{-3}\ °\text{C}^{-1})(100°\text{C} - 20.0°\text{C})\right] = \boxed{1.420\ \Omega}$$

while the more complicated formula gives:

$$R = \frac{(1.08\ \Omega)\left[1 + (3.90 \times 10^{-3}\ °\text{C}^{-1})(80.0°\text{C})\right]\left[1 + (17.0 \times 10^{-6}\ °\text{C}^{-1})(80.0°\text{C})\right]}{\left[1 + 2(17.0 \times 10^{-6}\ °\text{C}^{-1})(80.0°\text{C})\right]} = \boxed{1.418\ \Omega}$$

27.65 Let α be the temperature coefficient at 20.0°C, and α' be the temperature coefficient at 0 °C. Then $\rho = \rho_0[1+\alpha(T-20.0°\text{C})]$, and $\rho = \rho'[1+\alpha'(T-0°\text{C})]$ must both give the correct resistivity at any temperature *T*. That is, we must have:

$$\rho_0[1+\alpha(T-20.0°\text{C})] = \rho'[1+\alpha'(T-0°\text{C})] \quad (1)$$

Setting $T = 0$ in equation (1) yields: $\rho' = \rho_0[1-\alpha(20.0°\text{C})]$,

and setting $T = 20.0$°C in equation (1) gives: $\rho_0 = \rho'[1+\alpha'(20.0°\text{C})]$

Put ρ' from the first of these results into the second to obtain:

$$\rho_0 = \rho_0[1-\alpha(20.0°\text{C})][1+\alpha'(20.0°\text{C})]$$

Therefore $1+\alpha'(20.0°\text{C}) = \dfrac{1}{1-\alpha(20.0°\text{C})}$

which simplifies to $\alpha' = \dfrac{\alpha}{[1-\alpha(20.0\ °\text{C})]}$

From this, the temperature coefficient, based on a reference temperature of 0°C, may be computed for any material. For example, using this, Table 27.1 becomes at 0°C :

Material	Temp Coefficients at 0°C
Silver	$4.1\times10^{-3}/°\text{C}$
Copper	$4.2\times10^{-3}/°\text{C}$
Gold	$3.6\times10^{-3}/°\text{C}$
Aluminum	$4.2\times10^{-3}/°\text{C}$
Tungsten	$4.9\times10^{-3}/°\text{C}$
Iron	$5.6\times10^{-3}/°\text{C}$
Platinum	$4.25\times10^{-3}/°\text{C}$
Lead	$4.2\times10^{-3}/°\text{C}$
Nichrome	$0.4\times10^{-3}/°\text{C}$
Carbon	$-0.5\times10^{-3}/°\text{C}$
Germanium	$-24\times10^{-3}/°\text{C}$
Silicon	$-30\times10^{-3}/°\text{C}$

27.66 (a) $R = \dfrac{\rho \ell}{A} = \boxed{\dfrac{\rho L}{\pi(r_b^2 - r_a^2)}}$

(b) $R = \dfrac{(3.50\times10^5\ \Omega\cdot\text{m})(0.0400\ \text{m})}{\pi[(0.0120\ \text{m})^2 - (0.00500\ \text{m})^2]} = 3.74\times10^7\ \Omega = \boxed{37.4\ \text{M}\Omega}$

(c) $dR = \dfrac{\rho\, d\ell}{A} = \dfrac{\rho\, dr}{(2\pi r)L} = \left(\dfrac{\rho}{2\pi L}\right)\dfrac{dr}{r}$, so $R = \dfrac{\rho}{2\pi L}\displaystyle\int_{r_a}^{r_b}\dfrac{dr}{r} = \boxed{\dfrac{\rho}{2\pi L}\ln\left(\dfrac{r_b}{r_a}\right)}$

(d) $R = \dfrac{(3.50\times10^5\ \Omega\cdot\text{m})}{2\pi(0.0400\ \text{m})}\ln\left(\dfrac{1.20}{0.500}\right) = 1.22\times10^6\ \Omega = \boxed{1.22\ \text{M}\Omega}$

27.67 Each speaker receives 60.0 W of power. Using $\mathcal{P} = I^2 R$, we then have

$$I = \sqrt{\frac{\mathcal{P}}{R}} = \sqrt{\frac{60.0\text{ W}}{4.00\ \Omega}} = 3.87\text{ A}$$

The system is $\boxed{\text{not adequately protected}}$ since the $\boxed{\text{fuse should be set to melt at 3.87 A, or less}}$.

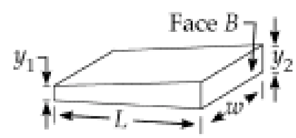
27.68 $\Delta V = -E \cdot \ell$ or $dV = -E \cdot dx$

$\Delta V = -IR = -E \cdot \ell$

$$I = \frac{dq}{dt} = \frac{E \cdot \ell}{R} = \frac{A}{\rho \ell} E \cdot \ell = \frac{A}{\rho} E = -\sigma A \frac{dV}{dx} = \boxed{\sigma A \left| \frac{dV}{dx} \right|}$$

Current flows in the direction of decreasing voltage. Energy flows as heat in the direction of decreasing temperature.

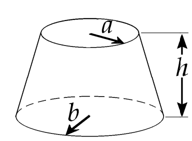
27.69 $R = \int \frac{\rho\, dx}{A} = \int \frac{\rho\, dx}{wy}$ where $y = y_1 + \frac{y_2 - y_1}{L} x$



$$R = \frac{\rho}{w} \int_0^L \frac{dx}{y_1 + \frac{y_2 - y_1}{L} x} = \frac{\rho L}{w(y_2 - y_1)} \ln\left[y_1 + \frac{y_2 - y_1}{L} x \right]\Bigg|_0^L$$

$$R = \boxed{\frac{\rho L}{w(y_2 - y_1)} \ln\left(\frac{y_2}{y_1}\right)}$$

27.70 From the geometry of the longitudinal section of the resistor shown in the figure, we see that



$$\frac{(b - r)}{y} = \frac{(b - a)}{h}$$

From this, the radius at a distance y from the base is $r = (a - b)\frac{y}{h} + b$

For a disk-shaped element of volume $dR = \frac{\rho\, dy}{\pi r^2}$: $\quad R = \frac{\rho}{\pi} \int_0^h \frac{dy}{\left[(a - b)(y / h) + b\right]^2}$.

Using the integral formula $\int \frac{du}{(au + b)^2} = -\frac{1}{a(au + b)}$, $\quad \boxed{R = \frac{\rho}{\pi} \frac{h}{ab}}$

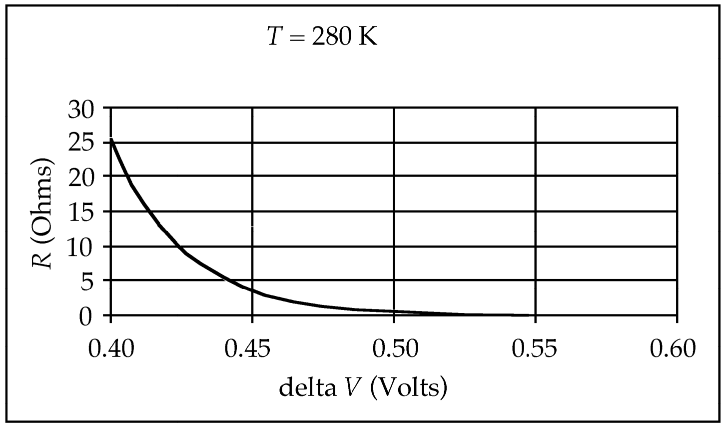
27.71 $I = I_0\left[\exp\left(e\Delta V / k_B T\right) - 1\right]$ and $R = \dfrac{\Delta V}{I}$

with $I_0 = 1.00 \times 10^{-9}$ A, $e = 1.60 \times 10^{-19}$ C, and $k_B = 1.38 \times 10^{-23}$ J/K.

The following includes a partial table of calculated values and a graph for each of the specified temperatures.

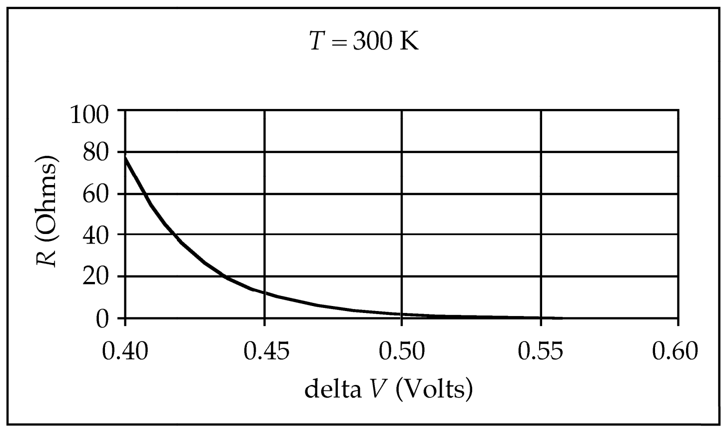
(i) For $T = 280$ K:

ΔV (V)	I (A)	R (Ω)
0.400	0.0156	25.6
0.440	0.0818	5.38
0.480	0.429	1.12
0.520	2.25	0.232
0.560	11.8	0.0476
0.600	61.6	0.0097



(ii) For $T = 300$ K:

ΔV (V)	I (A)	R (Ω)
0.400	0.005	77.3
0.440	0.024	18.1
0.480	0.114	4.22
0.520	0.534	0.973
0.560	2.51	0.223
0.600	11.8	0.051



(iii) For $T = 320$ K:

ΔV (V)	I (A)	R (Ω)
0.400	0.0020	203
0.440	0.0084	52.5
0.480	0.0357	13.4
0.520	0.152	3.42
0.560	0.648	0.864
0.600	2.76	0.217