Chapter 26 Solutions

***26.1** (a) $Q = C(\Delta V) = (4.00 \times 10^{-6}\text{ F})(12.0\text{ V}) = 4.80 \times 10^{-5}\text{ C} = \boxed{48.0\ \mu\text{C}}$

(b) $Q = C(\Delta V) = (4.00 \times 10^{-6}\text{ F})(1.50\text{ V}) = 6.00 \times 10^{-6}\text{ C} = \boxed{6.00\ \mu\text{C}}$

26.2 (a) $C = \dfrac{Q}{\Delta V} = \dfrac{10.0 \times 10^{-6}\text{ C}}{10.0\text{ V}} = 1.00 \times 10^{-6}\text{ F} = \boxed{1.00\ \mu\text{F}}$

(b) $\Delta V = \dfrac{Q}{C} = \dfrac{100 \times 10^{-6}\text{ C}}{1.00 \times 10^{-6}\text{ F}} = \boxed{100\text{ V}}$

26.3 $E = \dfrac{k_e q}{r^2}$: $\quad q = \dfrac{(4.90 \times 10^4\text{ N / C})(0.210\text{ m})^2}{8.99 \times 10^9\text{ N} \cdot \text{m}^2\text{ / C}^2} = 0.240\ \mu\text{C}$

(a) $\sigma = \dfrac{q}{A} = \dfrac{0.240 \times 10^{-6}}{4\pi(0.120)^2} = \boxed{1.33\ \mu\text{C / m}^2}$

(b) $C = 4\pi \epsilon_0 r = 4\pi(8.85 \times 10^{-12})(0.120) = \boxed{13.3\text{ pF}}$

26.4 (a) $C = 4\pi \epsilon_0 R$

$R = \dfrac{C}{4\pi \epsilon_0} = k_e C = (8.99 \times 10^9\text{ N} \cdot \text{m}^2/\text{C}^2)(1.00 \times 10^{-12}\text{ F}) = \boxed{8.99\text{ mm}}$

(b) $C = 4\pi \epsilon_0 R = \dfrac{4\pi(8.85 \times 10^{-12}\text{ C}^2)(2.00 \times 10^{-3}\text{ m})}{\text{N} \cdot \text{m}^2} = \boxed{0.222\text{ pF}}$

(c) $Q = CV = (2.22 \times 10^{-13}\text{ F})(100\text{ V}) = \boxed{2.22 \times 10^{-11}\text{ C}}$

26.5 (a) $\dfrac{Q_1}{Q_2} = \dfrac{R_1}{R_2}$

$Q_1 + Q_2 = \left(1 + \dfrac{R_1}{R_2}\right)Q_2 = 3.50 Q_2 = 7.00\ \mu\text{C}$

$\boxed{Q_2 = 2.00\ \mu\text{C}}$ $\quad \boxed{Q_1 = 5.00\ \mu\text{C}}$

(b) $V_1 = V_2 = \dfrac{Q_1}{C_1} = \dfrac{Q_2}{C_2} = \dfrac{5.00\ \mu\text{C}}{\left(8.99 \times 10^9\text{ m/F}\right)^{-1}(0.500\text{ m})} = 8.99 \times 10^4\text{ V} = \boxed{89.9\text{ kV}}$

***26.6** $C = \frac{\kappa \epsilon_0 A}{d} = \frac{(1.00)(8.85 \times 10^{-12}\ \text{C}^2)(1.00 \times 10^3\ \text{m})^2}{\text{N} \cdot \text{m}^2(800\ \text{m})} = \boxed{11.1\ \text{nF}}$

The potential between ground and cloud is

$\Delta V = Ed = (3.00 \times 10^6\ \text{N/C})(800\ \text{m}) = 2.40 \times 10^9\ \text{V}$

$Q = C(\Delta V) = (11.1 \times 10^{-9}\ \text{C/V})(2.40 \times 10^9\ \text{V}) = \boxed{26.6\ \text{C}}$

26.7 (a) $\Delta V = Ed$

$E = \frac{20.0\ \text{V}}{1.80 \times 10^{-3}\ \text{m}} = \boxed{11.1\ \text{kV/m}}$

(b) $E = \frac{\sigma}{\epsilon_0}$

$\sigma = (1.11 \times 10^4\ \text{N/C})(8.85 \times 10^{-12}\ \text{C}^2/\text{N} \cdot \text{m}^2) = \boxed{98.3\ \text{nC/m}^2}$

(c) $C = \frac{\epsilon_0 A}{d} = \frac{\left(8.85 \times 10^{-12}\ \text{C}^2/\text{N} \cdot \text{m}^2\right)\left(7.60\ \text{cm}^2\right)\left(1.00\ \text{m}/100\ \text{cm}\right)^2}{1.80 \times 10^{-3}\ \text{m}} = \boxed{3.74\ \text{pF}}$

(d) $\Delta V = \frac{Q}{C}$ $\quad Q = (20.0\ \text{V})(3.74 \times 10^{-12}\ \text{F}) = \boxed{74.7\ \text{pC}}$

26.8 $C = \frac{\kappa \epsilon_0 A}{d} = 60.0 \times 10^{-15}\ \text{F}$

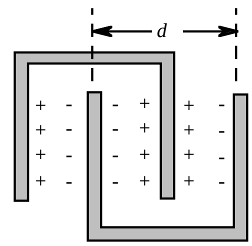
$d = \frac{\kappa \epsilon_0 A}{C} = \frac{(1)\left(8.85 \times 10^{-12}\right)\left(21.0 \times 10^{-12}\right)}{60.0 \times 10^{-15}}$

$d = 3.10 \times 10^{-9}\ \text{m} = \boxed{3.10\ \text{nm}}$

26.9 $Q = \frac{\epsilon_0 A}{d}(\Delta V)$ $\quad \frac{Q}{A} = \sigma = \frac{\epsilon_0(\Delta V)}{d}$

$d = \frac{\epsilon_0(\Delta V)}{\sigma} = \frac{\left(8.85 \times 10^{-12}\ \text{C}^2/\text{N} \cdot \text{m}^2\right)(150\ \text{V})}{\left(30.0 \times 10^{-9}\ \text{C/cm}^2\right)\left(1.00 \times 10^4\ \text{cm}^2/\text{m}^2\right)} = \boxed{4.42\ \mu\text{m}}$

26.10 With $\theta = \pi$, the plates are out of mesh and the overlap area is zero. With $\theta = 0$, the overlap area is that of a semi-circle, $\pi R^2/2$. By proportion, the effective area of a single sheet of charge is $(\pi - \theta)R^2/2$.



When there are two plates in each comb, the number of adjoining sheets of positive and negative charge is 3, as shown in the sketch. When there are N plates on each comb, the number of parallel capacitors is $2N - 1$ and the total capacitance is

$$C = (2N-1)\frac{\epsilon_0 A_{\text{effective}}}{\text{distance}} = \frac{(2N-1)\epsilon_0(\pi-\theta)R^2/2}{d/2} = \boxed{\frac{(2N-1)\,\epsilon_0\,(\pi-\theta)R^2}{d}}$$

26.11 (a) $C = \dfrac{l}{2k_e \ln\left(\dfrac{b}{a}\right)} = \dfrac{50.0}{2(8.99\times10^9)\ln\left(\dfrac{7.27}{2.58}\right)} = \boxed{2.68\text{ nF}}$

(b) Method 1: $\Delta V = 2k_e\,\lambda\,\ln\left(\dfrac{b}{a}\right)$

$$\lambda = q\ /\ l = \frac{8.10\times10^{-6}\text{ C}}{50.0\text{ m}} = 1.62\times10^{-7}\text{ C/m}$$

$$\Delta V = 2(8.99\times10^9)(1.62\times10^{-7})\ln\left(\frac{7.27}{2.58}\right) = \boxed{3.02\text{ kV}}$$

Method 2: $\Delta V = \dfrac{Q}{C} = \dfrac{8.10\times10^{-6}}{2.68\times10^{-9}} = \boxed{3.02\text{ kV}}$

26.12 Let the radii be b and a with $b = 2a$. Put charge Q on the inner conductor and $-Q$ on the outer. Electric field exists only in the volume between them. The potential of the inner sphere is $V_a = k_eQ/a$; that of the outer is $V_b = k_eQ/b$. Then

$$V_a - V_b = \frac{k_e Q}{a} - \frac{k_e Q}{b} = \frac{Q}{4\pi\epsilon_0}\left(\frac{b-a}{ab}\right) \quad \text{and} \quad C = \frac{Q}{V_a - V_b} = \frac{4\pi\epsilon_0 ab}{b-a}$$

Here $C = \dfrac{4\pi\epsilon_0\, 2a^2}{a} = 8\pi\epsilon_0\, a \qquad a = \dfrac{C}{8\pi\epsilon_0}$

The intervening volume is $\text{Volume} = \frac{4}{3}\pi b^3 - \frac{4}{3}\pi a^3 = 7\left(\frac{4}{3}\pi a^3\right) = 7\left(\frac{4}{3}\pi\right)\dfrac{C^3}{8^3\pi^3\epsilon_0^{\,3}} = \dfrac{7C^3}{384\pi^2\epsilon_0^{\,3}}$

$$\text{Volume} = \frac{7(20.0\times10^{-6}\text{ C}^2/\text{N}\cdot\text{m})^3}{384\pi^2(8.85\times10^{-12}\text{ C}^2/\text{N}\cdot\text{m}^2)^3} = \boxed{2.13\times10^{16}\text{ m}^3}$$

The outer sphere is 360 km in diameter.

26.13 $\Sigma F_y = 0$: $T\cos\theta - mg = 0$ $\Sigma F_x = 0$: $T\sin\theta - Eq = 0$

Dividing, $\tan\theta = \dfrac{Eq}{mg}$, so $E = \dfrac{mg}{q}\tan\theta$

$$\Delta V = Ed = \frac{mgd\tan\theta}{q} = \frac{(350\times10^{-6}\text{ kg})(9.80\text{ m/s}^2)(4.00\times10^{-2}\text{ m})\tan 15.0°}{30.0\times10^{-9}\text{ C}} = \boxed{1.23\text{ kV}}$$

26.14 $\Sigma F_y = 0$: $T\cos\theta - mg = 0$ $\Sigma F_x = 0$: $T\sin\theta - Eq = 0$

Dividing, $\tan\theta = \dfrac{Eq}{mg}$, so $E = \dfrac{mg}{q}\tan\theta$ and $\Delta V = Ed = \boxed{\dfrac{mgd\tan\theta}{q}}$

26.15 (a) $C = \dfrac{ab}{k_e(b-a)} = \dfrac{(0.0700)(0.140)}{(8.99\times10^9)(0.140-0.0700)} = \boxed{15.6\text{ pF}}$

(b) $C = \dfrac{Q}{\Delta V}$ $\Delta V = \dfrac{Q}{C} = \dfrac{4.00\times10^{-6}\text{ C}}{15.6\times10^{-12}\text{ F}} = \boxed{256\text{ kV}}$

Goal Solution

An air-filled spherical capacitor is constructed with inner and outer shell radii of 7.00 and 14.0 cm, respectively. (a) Calculate the capacitance of the device. (b) What potential difference between the spheres results in a charge of 4.00 μC on the capacitor?

G: Since the separation between the inner and outer shells is much larger than a typical electronic capacitor with $d \sim 0.1$ mm and capacitance in the microfarad range, we might expect the capacitance of this spherical configuration to be on the order of picofarads, (based on a factor of about 700 times larger spacing between the conductors). The potential difference should be sufficiently low to prevent sparking through the air that separates the shells.

O: The capacitance can be found from the equation for spherical shells, and the voltage can be found from $Q = C\Delta V$.

A: (a) For a spherical capacitor with inner radius a and outer radius b,

$$C = \frac{ab}{k(b-a)} = \frac{(0.0700\text{ m})(0.140\text{ m})}{\left(8.99\times10^9\text{ N}\cdot\text{m}^2/\text{C}^2\right)(0.140-0.0700)\text{ m}} = 1.56\times10^{-11}\text{ F} = 15.6\text{ pF}$$

(b) $$\Delta V = \frac{Q}{C} = \frac{(4.00\times10^{-6}\text{ C})}{1.56\times10^{-11}\text{ F}} = 2.56\times10^5\text{ V} = 256\text{ kV}$$

L: The capacitance agrees with our prediction, but the voltage seems rather high. We can check this voltage by approximating the configuration as the electric field between two charged parallel plates separated by $d = 7.00$ cm, so

$$E \sim \frac{\Delta V}{d} = \frac{2.56\times10^5\text{ V}}{0.0700\text{ m}} = 3.66\times10^6\text{ V/m}$$

This electric field barely exceeds the dielectric breakdown strength of air $\left(3\times10^6\text{ V/m}\right)$, so it may not even be possible to place $4.00\ \mu$C of charge on this capacitor!

26.16 $C = 4\pi\epsilon_0 R = 4\pi\left(8.85\times10^{-12}\ \text{C/N}\cdot\text{m}^2\right)\left(6.37\times10^{6}\ \text{m}\right) = \boxed{7.08\times10^{-4}\ \text{F}}$

***26.17** (a) Capacitors in parallel add. Thus, the equivalent capacitor has a value of

$C_{eq} = C_1 + C_2 = 5.00\ \mu\text{F} + 12.0\ \mu\text{F} = \boxed{17.0\ \mu\text{F}}$

(b) The potential difference across each branch is the same and equal to the voltage of the battery.

$\Delta V = \boxed{9.00\ \text{V}}$

(c) $Q_5 = C(\Delta V) = (5.00\ \mu\text{F})(9.00\ \text{V}) = \boxed{45.0\ \mu\text{C}}$ and $Q_{12} = C(\Delta V) = (12.0\ \mu\text{F})(9.00\ \text{V}) = \boxed{108\ \mu\text{C}}$

***26.18** (a) In series capacitors add as

$\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2} = \frac{1}{5.00\ \mu\text{F}} + \frac{1}{12.0\ \mu\text{F}}$ and $C_{eq} = \boxed{3.53\ \mu\text{F}}$

(c) The charge on the equivalent capacitor is

$Q_{eq} = C_{eq}(\Delta V) = (3.53\ \mu\text{F})(9.00\ \text{V}) = 31.8\ \mu\text{C}$

Each of the series capacitors has this same charge on it. So $Q_1 = Q_2 = \boxed{31.8\ \mu\text{C}}$

(b) The voltage across each is

$\Delta V_1 = \frac{Q_1}{C_1} = \frac{31.8\ \mu\text{C}}{5.00\ \mu\text{F}} = \boxed{6.35\ \text{V}}$ and $\Delta V_2 = \frac{Q_2}{C_2} = \frac{31.8\ \mu\text{C}}{12.0\ \mu\text{F}} = \boxed{2.65\ \text{V}}$

26.19 $C_p = C_1 + C_2$ $\frac{1}{C_s} = \frac{1}{C_1} + \frac{1}{C_2}$

Substitute $C_2 = C_p - C_1$ $\frac{1}{C_s} = \frac{1}{C_1} + \frac{1}{C_p - C_1} = \frac{C_p - C_1 + C_1}{C_1(C_p - C_1)}$

Simplifying, $C_1^2 - C_1C_p + C_pC_s = 0$

$$C_1 = \frac{C_p \pm \sqrt{C_p^2 - 4C_pC_s}}{2} = \frac{1}{2}C_p \pm \sqrt{\frac{1}{4}C_p^2 - C_pC_s}$$

We choose arbitrarily the + sign. (This choice can be arbitrary, since with the case of the minus sign, we would get the same two answers with their names interchanged.)

$$C_1 = \frac{1}{2}C_p + \sqrt{\frac{1}{4}C_p^2 - C_pC_s} = \frac{1}{2}(9.00\ \text{pF}) + \sqrt{\frac{1}{4}(9.00\ \text{pF})^2 - (9.00\ \text{pF})(2.00\ \text{pF})} = \boxed{6.00\ \text{pF}}$$

$$C_2 = C_p - C_1 = \frac{1}{2}C_p - \sqrt{\frac{1}{4}C_p^2 - C_pC_s} = \frac{1}{2}(9.00\ \text{pF}) - 1.50\ \text{pF} = \boxed{3.00\ \text{pF}}$$

26.20 $C_p = C_1 + C_2$ and $\frac{1}{C_s} = \frac{1}{C_1} + \frac{1}{C_2}$

Substitute $C_2 = C_p - C_1$: $\frac{1}{C_s} = \frac{1}{C_1} + \frac{1}{C_p - C_1} = \frac{C_p - C_1 + C_1}{C_1(C_p - C_1)}$

Simplifying, $C_1^2 - C_1 C_p + C_p C_s = 0$

and $C_1 = \frac{C_p \pm \sqrt{C_p^2 - 4C_p C_s}}{2} = \boxed{\frac{1}{2}C_p + \sqrt{\frac{1}{4}C_p^2 - C_p C_s}}$

where the positive sign was arbitrarily chosen (choosing the negative sign gives the same values for the capacitances, with the names reversed). Then, from $C_2 = C_p - C_1$

$$C_2 = \boxed{\frac{1}{2}C_p - \sqrt{\frac{1}{4}C_p^2 - C_p C_s}}$$

26.21 (a) $\frac{1}{C_s} = \frac{1}{15.0} + \frac{1}{3.00}$ $C_s = 2.50\ \mu\text{F}$

$C_p = 2.50 + 6.00 = 8.50\ \mu\text{F}$

$C_{eq} = \left(\frac{1}{8.50\ \mu\text{F}} + \frac{1}{20.0\ \mu\text{F}}\right)^{-1} = \boxed{5.96\ \mu\text{F}}$

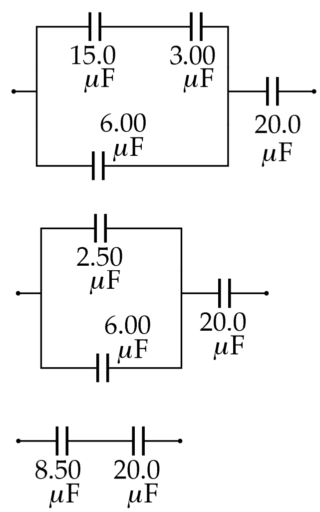
(b) $Q = (\Delta V)C = (15.0\ \text{V})(5.96\ \mu\text{F}) = \boxed{89.5\ \mu\text{C}}$ on $20.0\ \mu\text{F}$

$\Delta V = \frac{Q}{C} = \frac{89.5\ \mu\text{C}}{20.0\ \mu\text{F}} = 4.47\ \text{V}$

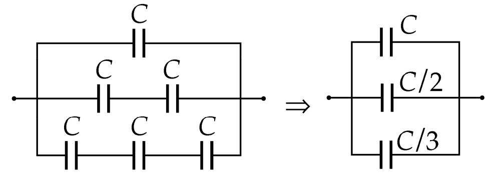
$15.0 - 4.47 = 10.53\ \text{V}$

$Q = (\Delta V)C = (10.53)(6.00\ \mu\text{F}) = \boxed{63.2\ \mu\text{C}}$ on $6.00\ \mu\text{F}$

$89.5 - 63.2 = \boxed{26.3\ \mu\text{C}}$ on $15.0\ \mu\text{F}$ and $3.00\ \mu\text{F}$



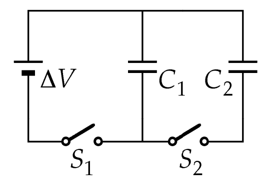
26.22 The circuit reduces first according to the rule for capacitors in series, as shown in the figure, then according to the rule for capacitors in parallel, shown below.



$C_{\text{eq}} = C\left(1 + \frac{1}{2} + \frac{1}{3}\right) = \frac{11}{6}C = \boxed{1.83C}$

26.23 $C = \dfrac{Q}{\Delta V}$ so $6.00 \times 10^{-6} = \dfrac{Q}{20.0}$ and $Q = \boxed{120\ \mu\text{C}}$

$Q_1 = 120\ \mu\text{C} - Q_2$ and $\Delta V = \dfrac{Q}{C}$



$\dfrac{120 - Q_2}{C_1} = \dfrac{Q_2}{C_2}$ or $\dfrac{120 - Q_2}{6.00} = \dfrac{Q_2}{3.00}$

$(3.00)(120 - Q_2) = (6.00)Q_2$

$Q_2 = \dfrac{360}{9.00} = \boxed{40.0\ \mu\text{C}}$

$Q_1 = 120\ \mu\text{C} - 40.0\ \mu\text{C} = \boxed{80.0\ \mu\text{C}}$

***26.24** (a) In $\boxed{\text{series}}$, to reduce the effective capacitance:

$$\frac{1}{32.0\ \mu\text{F}} = \frac{1}{34.8\ \mu\text{F}} + \frac{1}{C_s}$$

$$C_s = \frac{1}{2.51 \times 10^{-3}/\mu\text{F}} = \boxed{398\ \mu\text{F}}$$

(b) In $\boxed{\text{parallel}}$, to increase the total capacitance:

$$29.8\ \mu\text{F} + C_p = 32.0\ \mu\text{F}$$

$$C_p = \boxed{2.20\ \mu\text{F}}$$

26.25 With switch closed, distance $d' = 0.500d$ and capacitance $C' = \dfrac{\epsilon_0 A}{d'} = \dfrac{2\epsilon_0 A}{d} = 2C$

(a) $Q = C'(\Delta V) = 2C(\Delta V) = 2(2.00 \times 10^{-6}\ \text{F})(100\ \text{V}) = \boxed{400\ \mu\text{C}}$

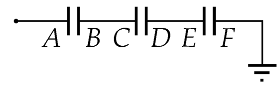
(b) The force stretching out one spring is

$$\mathbf{F} = \frac{Q^2}{2\epsilon_0 A} = \frac{4C^2(\Delta V)^2}{2\epsilon_0 A} = \frac{2C^2(\Delta V)^2}{(\epsilon_0 A / d)d} = \frac{2C(\Delta V)^2}{d}$$

One spring stretches by distance $x = d/4$, so

$$k = \frac{F}{x} = \frac{2C(\Delta V)^2}{d}\left(\frac{4}{d}\right) = \frac{8C(\Delta V)^2}{d^2} = \frac{8(2.00 \times 10^{-6}\ \text{F})(100\ \text{V})^2}{(8.00 \times 10^{-3}\ \text{m})^2} = \boxed{2.50\ \text{kN/m}}$$

26.26 Positive charge on A will induce equal negative charges on B, D, and F, and equal positive charges on C and E. The nesting spheres form three capacitors in series. From Example 26.3,



$$C_{AB} = \frac{ab}{k_e(b-a)} = \frac{R(2R)}{k_e R} = \frac{2R}{k_e}$$

$$C_{CD} = \frac{(3R)(4R)}{k_e R} = \frac{12R}{k_e}$$

$$C_{EF} = \frac{(5R)(6R)}{k_e R} = \frac{30R}{k_e}$$

$$C_{\text{eq}} = \frac{1}{k_e / 2R + k_e / 12R + k_e / 30R} = \boxed{\frac{60R}{37k_e}}$$

26.27 $$nC = \frac{100}{\underbrace{\frac{1}{C}+\frac{1}{C}+\frac{1}{C}+\ldots}_{\text{n capacitors}}} = \frac{100}{n/C}$$

$nC = \dfrac{100C}{n}$ so $n^2 = 100$ and $n = \boxed{10}$

Goal Solution

A group of identical capacitors is connected first in series and then in parallel. The combined capacitance in parallel is 100 times larger than for the series connection. How many capacitors are in the group?

G: Since capacitors in parallel add and ones in series add as inverses, 2 capacitors in parallel would have a capacitance 4 times greater than if they were in series, and 3 capacitors would give a ratio $C_p / C_s = 9$, so maybe $n = \sqrt{C_p / C_s} = \sqrt{100} = 10$.

O: The ratio reasoning above seems like an efficient way to solve this problem, but we should check the answer with a more careful analysis based on the general relationships for series and parallel combinations of capacitors.

A: Call C the capacitance of one capacitor and n the number of capacitors. The equivalent capacitance for n capacitors in parallel is

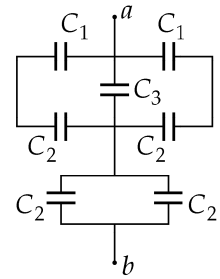
$$C_p = C_1 + C_2 + \ldots + C_n = nC$$

The relationship for n capacitors in series is $\dfrac{1}{C_s} = \dfrac{1}{C_1} + \dfrac{1}{C_2} + \ldots + \dfrac{1}{C_n} = \dfrac{n}{C}$

Therefore $\dfrac{C_p}{C_s} = \dfrac{nC}{C/n} = n^2$ or $n = \sqrt{\dfrac{C_p}{C_s}} = \sqrt{100} = 10$

L: Our prediction appears to be correct. A qualitative reason that $C_p / C_s = n^2$ is because the amount of charge that can be stored on the capacitors increases according to the area of the plates for a parallel combination, but the total charge remains the same for a series combination.

26.28 $C_s = \left(\frac{1}{5.00} + \frac{1}{10.0}\right)^{-1} = 3.33\ \mu\text{F}$



$C_{p1} = 2(3.33) + 2.00 = 8.66\ \mu\text{F}$

$C_{p2} = 2(10.0) = 20.0\ \mu\text{F}$

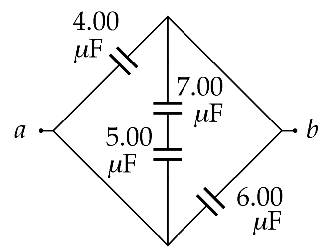
$C_{\text{eq}} = \left(\frac{1}{8.66} + \frac{1}{20.0}\right)^{-1} = \boxed{6.04\ \mu\text{F}}$

26.29 $Q_{\text{eq}} = C_{\text{eq}}(\Delta V) = \left(6.04 \times 10^{-6}\text{ F}\right)(60.0\text{ V}) = 3.62 \times 10^{-4}\text{ C}$

$Q_{p1} = Q_{\text{eq}}$, so $\Delta V_{p1} = \frac{Q_{\text{eq}}}{C_{p1}} = \frac{3.62 \times 10^{-4}\text{ C}}{8.66 \times 10^{-6}\text{ F}} = 41.8\text{ V}$

$Q_3 = C_3\left(\Delta V_{p1}\right) = \left(2.00 \times 10^{-6}\text{ F}\right)(41.8\text{ V}) = \boxed{83.6\ \mu\text{C}}$

26.30 $C_s = \left(\frac{1}{5.00} + \frac{1}{7.00}\right)^{-1} = 2.92\ \mu\text{F}$



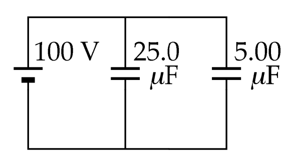
$C_p = 2.92 + 4.00 + 6.00 = \boxed{12.9\ \mu\text{F}}$

***26.31** (a) $U = \frac{1}{2}C(\Delta V)^2 = \frac{1}{2}(3.00\ \mu\text{F})(12.0\text{ V})^2 = \boxed{216\ \mu\text{J}}$

(b) $U = \frac{1}{2}C(\Delta V)^2 = \frac{1}{2}(3.00\ \mu\text{F})(6.00\text{ V})^2 = \boxed{54.0\ \mu\text{J}}$

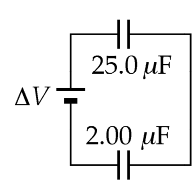
***26.32** $U = \frac{1}{2}C(\Delta V)^2$

The circuit diagram is shown at the right.



(a) $C_p = C_1 + C_2 = 25.0\ \mu\text{F} + 5.00\ \mu\text{F} = 30.0\ \mu\text{F}$

$U = \frac{1}{2}(30.0 \times 10^{-6})(100)^2 = \boxed{0.150\text{ J}}$



(b) $C_s = \left(\frac{1}{C_1} + \frac{1}{C_2}\right)^{-1} = \left(\frac{1}{25.0\ \mu\text{F}} + \frac{1}{5.00\ \mu\text{F}}\right)^{-1} = 4.17\ \mu\text{F}$

$U = \frac{1}{2}C(\Delta V)^2$

$\Delta V = \sqrt{\frac{2U}{C}} = \sqrt{\frac{(0.150)(2)}{4.17 \times 10^{-6}}} = \boxed{268\text{ V}}$

***26.33** Use $U = \frac{1}{2}\frac{Q^2}{C}$ and $C = \frac{\epsilon_0 A}{d}$

If $d_2 = 2d_1$, $C_2 = \frac{1}{2} C_1$. Therefore, the $\boxed{\text{stored energy doubles}}$.

26.34 $u = \frac{U}{V} = \frac{1}{2}\epsilon_0 E^2$

$$\frac{1.00\times10^{-7}}{V} = \frac{1}{2}(8.85\times10^{-12})(3000)^2$$

$$V = \boxed{2.51\times10^{-3}\text{ m}^3} = \left(2.51\times10^{-3}\text{ m}^3\right)\left(\frac{1000\text{ L}}{\text{m}^3}\right) = \boxed{2.51\text{ L}}$$

26.35 $W = U = \int F\,dx$ so $F = \frac{dU}{dx} = \frac{d}{dx}\left(\frac{Q^2}{2c}\right) = \frac{d}{dx}\left(\frac{Q^2 x}{2\epsilon_0 A}\right) = \boxed{\frac{Q^2}{2\epsilon_0 A}}$

26.36 Plate a experiences force $-kx\mathbf{i}$ from the spring and force $QE\mathbf{i}$ due to the electric field created by plate b according to $E = \sigma / 2\epsilon_0 = Q / 2A\epsilon_0$. Then,

$$kx = \frac{Q^2}{2A\epsilon_0} \qquad x = \boxed{\frac{Q^2}{2A\epsilon_0 k}}$$

where A is the area of one plate.

26.37 The energy transferred is $W = \frac{1}{2}Q(\Delta V) = \frac{1}{2}(50.0\text{ C})(1.00\times10^8\text{ V}) = 2.50\times10^9\text{ J}$ and 1% of this (or $W' = 2.50\times10^7$ J) is absorbed by the tree. If m is the amount of water boiled away, then

$$W' = m(4186\text{ J/kg °C})(100\text{ °C} - 30.0\text{ °C}) + m(2.26\times10^6\text{ J/kg}) = 2.50\times10^7\text{ J}$$

giving $\boxed{m = 9.79\text{ kg}}$

26.38 $U = \frac{1}{2}C(\Delta V)^2$ where $C = 4\pi \epsilon_0 R = \frac{R}{k_e}$ and $\Delta V = \frac{k_e Q}{R} - 0 = \frac{k_e Q}{R}$

$$U = \frac{1}{2}\left(\frac{R}{k_e}\right)\left(\frac{k_e Q}{R}\right)^2 = \boxed{\frac{k_e Q^2}{2R}}$$

26.39 $\frac{k_e Q^2}{2R} = mc^2$

$$R = \frac{k_e e^2}{2mc^2} = \frac{(8.99\times 10^9 \text{ N}\cdot\text{m}^2/\text{C})(1.60\times 10^{-19}\text{ C})^2}{2(9.11\times 10^{-31}\text{ kg})(3.00\times 10^8\text{ m/s})^2} = \boxed{1.40 \text{ fm}}$$

***26.40** $C = \frac{\kappa \in_0 A}{d} = \frac{4.90(8.85\times 10^{-12}\text{ F/m})(5.00\times 10^{-4}\text{ m}^2)}{2.00\times 10^{-3}\text{ m}} = 1.08\times 10^{-11}\text{ F} = \boxed{10.8 \text{ pF}}$

***26.41** (a) $C = \frac{\kappa \in_0 A}{d} = \frac{2.10(8.85\times 10^{-12}\text{ F/m})(1.75\times 10^{-4}\text{ m}^2)}{4.00\times 10^{-5}\text{ m}} = 8.13\times 10^{-11}\text{ F} = \boxed{81.3 \text{ pF}}$

(b) $\Delta V_{max} = E_{max} d = (60.0\times 10^6\text{ V/m})(4.00\times 10^{-5}\text{ m}) = \boxed{2.40 \text{ kV}}$

***26.42** $Q_{max} = C(\Delta V_{max})$, but $\Delta V_{max} = E_{max} d$

Also, $C = \frac{\kappa \in_0 A}{d}$

Thus, $Q_{max} = \frac{\kappa \in_0 A}{d}(E_{max} d) = \kappa \in_0 A E_{max}$

(a) With air between the plates, $\kappa = 1.00$ and $E_{max} = 3.00\times 10^6$ V/m. Therefore,

$Q_{max} = \kappa \in_0 A E_{max} = (8.85\times 10^{-12}\text{ F/m})(5.00\times 10^{-4}\text{ m}^2)(3.00\times 10^6\text{ V/m}) = \boxed{13.3 \text{ nC}}$

(b) With polystyrene between the plates, $\kappa = 2.56$ and $E_{max} = 24.0\times 10^6$ V/m.

$Q_{max} = \kappa \in_0 A E_{max} = 2.56(8.85\times 10^{-12}\text{ F/m})(5.00\times 10^{-4}\text{ m}^2)(24.0\times 10^6\text{ V/m}) = \boxed{272 \text{ nC}}$

26.43 $C = \frac{\kappa \in_0 A}{d}$ or $95.0\times 10^{-9} = \frac{3.70(8.85\times 10^{-12})(0.0700)\ell}{(0.0250\times 10^{-3})}$

= 1.04 m

***26.44** Consider two sheets of aluminum foil, each 40 cm by 100 cm, with one sheet of plastic between them. Suppose the plastic has $\kappa \cong 3$, $E_{\max} \sim 10^7$ V/m and thickness 1 mil = 2.54 cm/1000. Then,

$$C = \frac{\kappa \epsilon_0 A}{d} \sim \frac{3(8.85\times10^{-12}\ \mathrm{C^2/N\cdot m^2})(0.4\ \mathrm{m^2})}{2.54\times10^{-5}\ \mathrm{m}} \sim \boxed{10^{-6}\ \mathrm{F}}$$

$$\Delta V_{\max} = E_{\max} d \sim \left(10^7\ \frac{\mathrm{V}}{\mathrm{m}}\right)\left(2.54\times10^{-5}\ \mathrm{m}\right) \sim \boxed{10^2\ \mathrm{V}}$$

***26.45** (a) With air between the plates, we find $C_0 = \dfrac{Q}{\Delta V} = \dfrac{48.0\ \mu\mathrm{C}}{12.0\ \mathrm{V}} = \boxed{4.00\ \mu\mathrm{F}}$

(b) When Teflon is inserted, the charge remains the same (48.0 μC) because the plates are isolated. However, the capacitance, and hence the voltage, changes. The new capacitance is

$$C' = \kappa C_0 = 2.10(4.00\ \mu\mathrm{F}) = \boxed{8.40\ \mu\mathrm{F}}$$

(c) The voltage on the capacitor now is $\Delta V' = \dfrac{Q}{C'} = \dfrac{48.0\ \mu\mathrm{C}}{8.40\ \mu\mathrm{F}} = \boxed{5.71\ \mathrm{V}}$

and the charge is $\boxed{48.0\ \mu\mathrm{C}}$

26.46 Originally, $C = \epsilon_0 A/d = Q/(\Delta V)_i$

(a) The charge is the same before and after immersion, with value $Q = \epsilon_0 A(\Delta V)_i / d$.

$$Q = \frac{(8.85\times10^{-12}\ \mathrm{C^2})(25.0\times10^{-4}\ \mathrm{m^2})(250\ \mathrm{V})}{\mathrm{N\cdot m^2}\ \ (1.50\times10^{-2}\ \mathrm{m})} = \boxed{369\ \mathrm{pC}}$$

(b) Finally, $C_f = \kappa \epsilon_0 A/d = Q/(\Delta V)_f$

$$C_f = \frac{80.0(8.85\times10^{-12}\ \mathrm{C^2})(25.0\times10^{-4}\ \mathrm{m^2})}{\mathrm{N\cdot m^2}\ (1.50\times10^{-2}\ \mathrm{m})} = \boxed{118\ \mathrm{pF}}$$

$$(\Delta V)_f = \frac{Qd}{\kappa\epsilon_0 A} = \frac{\epsilon_0 A(\Delta V)_i d}{\kappa\epsilon_0 A d} = \frac{(\Delta V)_i}{\kappa} = \frac{250\ \mathrm{V}}{80.0} = \boxed{3.12\ \mathrm{V}}$$

(c) Originally, $U = \frac{1}{2}C(\Delta V)_i^2 = \dfrac{\epsilon_0 A(\Delta V)_i^2}{2d}$

Finally, $U_f = \frac{1}{2}C_f(\Delta V)_f^2 = \dfrac{\kappa\epsilon_0 A(\Delta V)_i^2}{2d\ \ \kappa^2} = \dfrac{\epsilon_0 A(\Delta V)_i^2}{2d\kappa}$

So, $\Delta U = U_f - U = -\dfrac{\epsilon_0 A(\Delta V)_i^2(\kappa - 1)}{2d\ \kappa}$

$$\Delta U = \frac{(-8.85\times10^{-12}\ \mathrm{C^2})(25.0\times10^{-4}\ \mathrm{m^2})(250\ \mathrm{V})^2(79.0)}{\mathrm{N\cdot m^2}\ 2(1.50\times10^{-2}\ \mathrm{m})80} = \boxed{-45.5\ \mathrm{nJ}}$$

26.47

$$\frac{1}{C}=\frac{1}{\left(\dfrac{\kappa_1 ab}{k_e(b-a)}\right)}+\frac{1}{\left(\dfrac{\kappa_2 bc}{k_e(c-b)}\right)}=\frac{k_e(b-a)}{\kappa_1 ab}+\frac{k_e(c-b)}{\kappa_2 bc}$$

$$C=\frac{1}{\dfrac{k_e(b-a)}{\kappa_1 ab}+\dfrac{k_e(c-b)}{\kappa_2 bc}}=\frac{\kappa_1\kappa_2\, abc}{k_e\,\kappa_2(bc-ac)+k_e\,\kappa_1(ac-ab)}=\boxed{\frac{4\pi\,\kappa_1\kappa_2\, abc\epsilon_0}{\kappa_2 bc-\kappa_1 ab+(\kappa_1-\kappa_2)ac}}$$

26.48 (a) $C=\kappa C_0=\dfrac{\kappa\epsilon_0 A}{d}=\dfrac{(173)(8.85\times10^{-12})(1.00\times10^{-4}\text{ m}^2)}{0.100\times10^{-3}\text{ m}}=\boxed{1.53\text{ nF}}$

(b) The battery delivers the free charge

$Q=C(\Delta V)=(1.53\times10^{-9}\text{ F})(12.0\text{ V})=\boxed{18.4\text{ nC}}$

(c) The surface density of free charge is

$$\sigma=\frac{Q}{A}=\frac{18.4\times10^{-9}\text{ C}}{1.00\times10^{-4}\text{ m}^2}=\boxed{1.84\times10^{-4}\text{ C/m}^2}$$

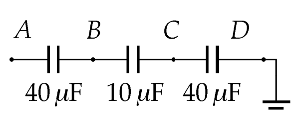
The surface density of polarization charge is

$$\sigma_p=\sigma\left(1-\frac{1}{\kappa}\right)=\sigma\left(1-\frac{1}{173}\right)=\boxed{1.83\times10^{-4}\text{ C/m}^2}$$

(d) We have $E=E_0/\kappa$ and $E_0=\Delta V/d$; hence,

$$E=\frac{\Delta V}{\kappa d}=\frac{12.0\text{ V}}{(173)(1.00\times10^{-4}\text{ m})}=\boxed{694\text{ V/m}}$$

26.49 The given combination of capacitors is equivalent to the circuit diagram shown to the right.



Put charge Q on point A. Then,

$$Q=(40.0\ \mu\text{F})\Delta V_{AB}=(10.0\ \mu\text{F})\Delta V_{BC}=(40.0\ \mu\text{F})\Delta V_{CD}$$

So, $\Delta V_{BC}=4\Delta V_{AB}=4\Delta V_{CD}$, and the center capacitor will break down first, at $\Delta V_{BC}=15.0$ V. When this occurs,

$$\Delta V_{AB}=\Delta V_{CD}=\tfrac{1}{4}(\Delta V_{BC})=3.75\text{ V}$$

and $V_{AD}=V_{AB}+V_{BC}+V_{CD}=3.75\text{ V}+15.0\text{ V}+3.75\text{ V}=\boxed{22.5\text{ V}}$

***26.50** (a) The displacement from negative to positive charge is

$$2\mathbf{a} = (-1.20\,\mathbf{i} + 1.10\,\mathbf{j})\,\text{mm} - (1.40\,\mathbf{i} - 1.30\,\mathbf{j})\,\text{mm} = (-2.60\,\mathbf{i} + 2.40\,\mathbf{j}) \times 10^{-3}\ \text{m}$$

The electric dipole moment is

$$\mathbf{p} = 2\mathbf{a}q = (3.50 \times 10^{-9}\ \text{C})(-2.60\,\mathbf{i} + 2.40\,\mathbf{j}) \times 10^{-3}\ \text{m} = \boxed{(-9.10\,\mathbf{i} + 8.40\,\mathbf{j}) \times 10^{-12}\ \text{C}\cdot\text{m}}$$

(b) $\tau = \mathbf{p} \times \mathbf{E} = \left[(-9.10\,\mathbf{i} + 8.40\,\mathbf{j}) \times 10^{-12}\ \text{C}\cdot\text{m}\right] \times \left[(7.80\,\mathbf{i} - 4.90\,\mathbf{j}) \times 10^{3}\ \text{N/C}\right]$

$$\tau = (+44.6\,\mathbf{k} - 65.5\,\mathbf{k}) \times 10^{-9}\ \text{N}\cdot\text{m} = \boxed{-2.09 \times 10^{-8}\ \text{N}\cdot\text{m}\,\mathbf{k}}$$

(c) $U = -\mathbf{p}\cdot\mathbf{E} = -\left[(-9.10\,\mathbf{i} + 8.40\,\mathbf{j}) \times 10^{-12}\ \text{C}\cdot\text{m}\right] \cdot \left[(7.80\,\mathbf{i} - 4.90\,\mathbf{j}) \times 10^{3}\ \text{N/C}\right]$

$$U = (71.0 + 41.2) \times 10^{-9}\ \text{J} = \boxed{112\ \text{nJ}}$$

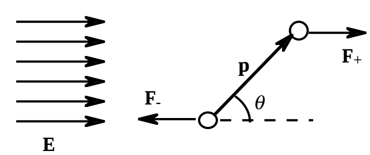
(d) $|\mathbf{p}| = \sqrt{(9.10)^2 + (8.40)^2} \times 10^{-12}\ \text{C}\cdot\text{m} = 12.4 \times 10^{-12}\ \text{C}\cdot\text{m}$

$$|\mathbf{E}| = \sqrt{(7.80)^2 + (4.90)^2} \times 10^{3}\ \text{N C} = 9.21 \times 10^{3}\ \text{N/C}$$

$$U_{\text{max}} = |\mathbf{p}|\,|\mathbf{E}| = 114\ \text{nJ}, \qquad U_{\text{min}} = -114\ \text{nJ}$$

$$U_{\text{max}} - U_{\text{min}} = \boxed{228\ \text{nJ}}$$

***26.51** (a) Let x represent the coordinate of the negative charge. Then $x + 2a\cos\theta$ is the coordinate of the positive charge. The force on the negative charge is $\mathbf{F}_- = -qE(x)\mathbf{i}$. The force on the positive charge is



$$\mathbf{F}_+ = +qE(x + 2a\cos\theta)\mathbf{i} \cong qE(x)\mathbf{i} + q\frac{dE}{dx}(2a\cos\theta)\mathbf{i}$$

The force on the dipole is altogether $\quad \mathbf{F} = \mathbf{F}_- + \mathbf{F}_+ = q\dfrac{dE}{dx}(2a\cos\theta)\mathbf{i} = \boxed{p\dfrac{dE}{dx}\cos\theta\,\mathbf{i}}$

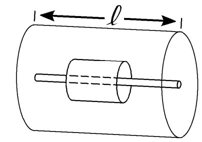
(b) The balloon creates field along the x-axis of $\dfrac{k_e q}{x^2}\mathbf{i}$.

Thus, $\dfrac{dE}{dx} = \dfrac{(-2)k_e q}{x^3}$

At $x = 16.0$ cm, $\dfrac{dE}{dx} = \dfrac{(-2)(8.99 \times 10^{9})(2.00 \times 10^{-6})}{(0.160)^3} = \boxed{-8.78\ \dfrac{\text{MN}}{\text{C}\cdot\text{m}}}$

$$\mathbf{F} = (6.30 \times 10^{-9}\ \text{C}\cdot\text{m})\left(-8.78 \times 10^{6}\ \frac{\text{N}}{\text{C}\cdot\text{m}}\right)\cos 0^\circ\ \mathbf{i} = \boxed{-55.3\,\mathbf{i}\ \text{mN}}$$

26.52 $2\pi r \ell E = \frac{q_{\text{in}}}{\epsilon_0}$ so $E = \frac{\lambda}{2\pi r \epsilon_0}$



$$\Delta V = -\int_{r_1}^{r_2} \mathbf{E} \cdot d\mathbf{r} = \int_{r_1}^{r_2} \frac{\lambda}{2\pi r\, \epsilon_0} dr = \frac{\lambda}{2\pi\, \epsilon_0} \ln\left(\frac{r_1}{r_2}\right)$$

$$\frac{\lambda_{\max}}{2\pi\epsilon_0} = E_{\max} r_{\text{inner}}$$

$$\Delta V = \left(1.20 \times 10^6 \ \frac{\text{V}}{\text{m}}\right)\left(0.100 \times 10^{-3} \ \text{m}\right)\left(\ln \frac{25.0}{0.200}\right)$$

$\Delta V_{\max} = \boxed{579 \text{ V}}$

***26.53** (a) Consider a gaussian surface in the form of a cylindrical pillbox with ends of area $A' << A$ parallel to the sheet. The side wall of the cylinder passes no flux of electric field since this surface is everywhere parallel to the field. Gauss's law becomes

$$EA' + EA' = \frac{Q}{\in A} A', \quad \text{so} \quad \boxed{E = \frac{Q}{2 \in A}}$$ directed away from the positive sheet.

(b) In the space between the sheets, each creates field $Q/2 \in A$ away from the positive and toward the negative sheet. Together, they create a field of

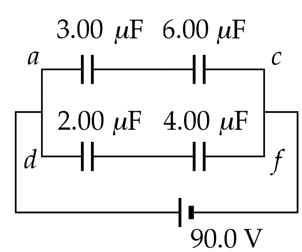
$$\boxed{E = \frac{Q}{\in A}}$$

(c) Assume that the field is in the positive $x-$direction. Then, the potential of the positive plate relative to the negative plate is

$$\Delta V = -\int_{-\text{plate}}^{+\text{plate}} \mathbf{E} \cdot d\mathbf{s} = -\int_{-\text{plate}}^{+\text{plate}} \frac{Q}{\in A} \mathbf{i} \cdot (-\mathbf{i}\, dx) = \boxed{+\frac{Qd}{\in A}}$$

(d) Capacitance is defined by: $C = \frac{Q}{\Delta V} = \frac{Q}{Qd/\in A} = \boxed{\frac{\in A}{d} = \frac{\kappa \epsilon_0 A}{d}}$

26.54 (a) $C = \left[\frac{1}{3.00} + \frac{1}{6.00}\right]^{-1} + \left[\frac{1}{2.00} + \frac{1}{4.00}\right]^{-1} = \boxed{3.33\ \mu\text{F}}$



(c) $Q_{ac} = C_{ac}\,(\Delta V_{ac}) = (2.00\ \mu\text{F})(90.0\ \text{V}) = 180\ \mu\text{C}$

Therefore, $Q_3 = Q_6 = \boxed{180\ \mu\text{C}}$

$Q_{df} = C_{df}\left(\Delta V_{df}\right) = (1.33\ \mu F)(90.0\ \text{V}) = \boxed{120\ \mu\text{C}}$

(b) $\Delta V_3 = \dfrac{Q_3}{C_3} = \dfrac{180\ \mu\text{C}}{3.00\ \mu\text{F}} = \boxed{60.0\text{ V}}$

$\Delta V_6 = \dfrac{Q_6}{C_6} = \dfrac{180\ \mu\text{C}}{6.00\ \mu\text{F}} = \boxed{30.0\text{ V}}$

$\Delta V_2 = \dfrac{Q_2}{C_2} = \dfrac{120\ \mu\text{C}}{2.00\ \mu\text{F}} = \boxed{60.0\text{ V}}$

$\Delta V_4 = \dfrac{Q_4}{C_4} = \dfrac{120\ \mu\text{C}}{4.00\ \mu\text{F}} = \boxed{30.0\text{ V}}$

(d) $U_T = \frac{1}{2}\, C_{\text{eq}}\, (\Delta V)^2 = \frac{1}{2}(3.33\times 10^{-6})(90.0\text{ V})^2 = \boxed{13.4\text{ mJ}}$

***26.55** The electric field due to the charge on the positive wire is perpendicular to the wire, radial, and of magnitude

$$E_+ = \frac{\lambda}{2\pi \in_0 r}$$

The potential difference between wires due to the presence of this charge is

$$\Delta V_1 = -\int_{-\text{wire}}^{+\text{wire}} \mathbf{E}\cdot d\mathbf{r} = -\frac{\lambda}{2\pi \in_0}\int_{D-d}^{d}\frac{dr}{r} = \frac{\lambda}{2\pi\in_0}\ln\left(\frac{D-d}{d}\right)$$

The presence of the linear charge density $-\lambda$ on the negative wire makes an identical contribution to the potential difference between the wires. Therefore, the total potential difference is

$$\Delta V = 2(\Delta V_1) = \frac{\lambda}{\pi\in_0}\ln\left(\frac{D-d}{d}\right)$$

and the capacitance of this system of two wires, each of length ℓ, is

$$C = \frac{Q}{\Delta V} = \frac{\lambda \ell}{\Delta V} = \frac{\lambda\ell}{\left(\frac{\lambda}{\pi\in_0}\right)\ln\left(\frac{D-d}{d}\right)} = \frac{\pi\in_0 \ell}{\ln\left(\frac{D-d}{d}\right)}$$

The capacitance per unit length is: $\boxed{\dfrac{C}{\ell} = \dfrac{\pi\in_0}{\ln\left(\frac{D-d}{d}\right)}}$

26.56 (a) We use Equation 26.11 to find the potential energy. As we will see, the potential difference ΔV changes as the dielectric is withdrawn. The initial and final energies are

$$U_i = \frac{1}{2}\left(\frac{Q^2}{C_i}\right) \quad \text{and} \quad U_f = \frac{1}{2}\left(\frac{Q^2}{C_f}\right)$$

But the initial capacitance (with the dielectric) is $C_i = \kappa C_f$. Therefore, $U_f = \frac{1}{2}\kappa\left(\frac{Q^2}{C_i}\right)$

Since the work done by the external force in removing the dielectric equals the change in potential energy, we have

$$W = U_f - U_i = \frac{1}{2}\kappa\left(\frac{Q^2}{C_i}\right) - \frac{1}{2}\left(\frac{Q^2}{C_i}\right) = \frac{1}{2}\left(\frac{Q^2}{C_i}\right)(\kappa - 1)$$

To express this relation in terms of potential difference ΔV_i, we substitute $Q = C_i(\Delta V_i)$, and evaluate:

$$W = \frac{1}{2}C_i(\Delta V_i)^2(\kappa - 1) = \frac{1}{2}(2.00\times10^{-9}\text{ F})(100\text{ V})^2(5.00 - 1.00) = \boxed{4.00\times10^{-5}\text{ J}}$$

The positive result confirms that the final energy of the capacitor is greater than the initial energy. The extra energy comes from the work done *on* the system by the external force that pulled out the dielectric.

(b) The final potential difference across the capacitor is $\Delta V_f = \frac{Q}{C_f}$

Substituting $C_f = \frac{C_i}{\kappa}$ and $Q = C_i(\Delta V_i)$ gives $\Delta V_f = \kappa\Delta V_i = (5.00)(100\text{ V}) = \boxed{500\text{ V}}$

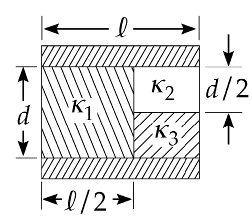
Even though the capacitor is isolated and its charge remains constant, the potential difference across the plates does increase in this case.

26.57 $\kappa = 3.00$, $E_{\max} = 2.00\times10^8\text{ V/m} = \Delta V_{\max}/d$

For $C = \frac{\kappa \epsilon_0 A}{d} = 0.250\times10^{-6}\text{ F}$,

$$A = \frac{Cd}{\kappa\epsilon_0} = \frac{C(\Delta V_{\max})}{\kappa\epsilon_0 E_{\max}} = \frac{(0.250\times10^{-6})(4000)}{(3.00)(8.85\times10^{-12})(2.00\times10^8)} = \boxed{0.188\text{ m}^2}$$

26.58 (a) $C_1 = \frac{\kappa_1\epsilon_0 A/2}{d}$; $C_2 = \frac{\kappa_2\epsilon_0 A/2}{d/2}$; $C_3 = \frac{\kappa_3\epsilon_0 A/2}{d/2}$

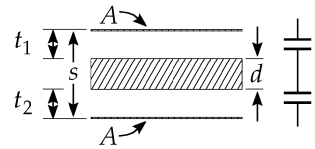


$$\left(\frac{1}{C_2}+\frac{1}{C_3}\right)^{-1} = \frac{C_2C_3}{C_2+C_3} = \frac{\epsilon_0 A}{d}\left(\frac{\kappa_2\kappa_3}{\kappa_2+\kappa_3}\right)$$

$$C = C_1 + \left(\frac{1}{C_2}+\frac{1}{C_3}\right)^{-1} = \boxed{\frac{\epsilon_0 A}{d}\left(\frac{\kappa_1}{2}+\frac{\kappa_2\kappa_3}{\kappa_2+\kappa_3}\right)}$$

(b) Using the given values we find: $C_{\text{total}} = 1.76 \times 10^{-12}\ \text{F} = \boxed{1.76\ \text{pF}}$

26.59 The system may be considered to be two capacitors in series:



$$C_1 = \frac{\epsilon_0 A}{t_1} \quad \text{and} \quad C_2 = \frac{\epsilon_0 A}{t_2}$$

$$\frac{1}{C} = \frac{1}{C_1} + \frac{1}{C_2} = \frac{t_1 + t_2}{\epsilon_0 A}$$

$$C = \frac{\epsilon_0 A}{t_1 + t_2} = \boxed{\frac{\epsilon_0 A}{s - d}}$$

Goal Solution

A conducting slab of a thickness d and area A is inserted into the space between the plates of a parallel-plate capacitor with spacing s and surface area A, as shown in Figure P26.59. The slab is not necessarily halfway between the capacitor plates. What is the capacitance of the system?

G: It is difficult to predict an exact relationship for the capacitance of this system, but we can reason that C should increase if the distance between the slab and plates were decreased (until they touched and formed a short circuit). So maybe $C \propto 1/(s-d)$. Moving the metal slab does not change the amount of charge the system can store, so the capacitance should therefore be independent of the slab position. The slab must have zero net charge, with each face of the plate holding the same magnitude of charge as the outside plates, regardless of where the slab is between the plates.

O: If the capacitor is charged with $+Q$ on the top plate and $-Q$ on the bottom plate, then free charges will move across the conducting slab to neutralize the electric field inside it, with the top face of the slab carrying charge $-Q$ and the bottom face carrying charge $+Q$. Then the capacitor and slab combination is electrically equivalent to two capacitors in series. (We are neglecting the slight fringing effect of the electric field near the edges of the capacitor.) Call x the upper gap, so that $s-d-x$ is the distance between the lower two surfaces.

A: For the upper capacitor, $C_1 = \epsilon_0 A/x$

and the lower has $C_2 = \dfrac{\epsilon_0 A}{s-d-x}$

So the combination has $C = \dfrac{1}{\dfrac{1}{C_1} + \dfrac{1}{C_2}} = \dfrac{1}{\dfrac{x}{\epsilon_0 A} + \dfrac{s-d-x}{\epsilon_0 A}} = \dfrac{\epsilon_0 A}{s-d}$

L: The equivalent capacitance is inversely proportional to $(s-d)$ as expected, and is also proportional to A. This result is the same as for the special case in Example 26.9 when the slab is just halfway between the plates; the only critical factor is the thickness of the slab relative to the plate spacing.

26.60 (a) Put charge Q on the sphere of radius a and $-Q$ on the other sphere. Relative to $V = 0$ at infinity,

the potential at the surface of a is $$V_a = \frac{k_e Q}{a} - \frac{k_e Q}{d}$$

and the potential of b is $$V_b = \frac{-k_e Q}{b} + \frac{k_e Q}{d}$$

The difference in potential is $$V_a - V_b = \frac{k_e Q}{a} + \frac{k_e Q}{b} - \frac{k_e Q}{d} - \frac{k_e Q}{d}$$

and $$C = \frac{Q}{V_a - V_b} = \boxed{\left(\frac{4\pi \in_0}{(1/a) + (1/b) - (2/d)}\right)}$$

(b) As $d \to \infty$, $1/d$ becomes negligible compared to $1/a$. Then,

$$C = \frac{4\pi \in_0}{1/a + 1/b} \quad \text{and} \quad \frac{1}{C} = \boxed{\frac{1}{4\pi \in_0 a} + \frac{1}{4\pi \in_0 b}}$$

as for two spheres in series.

26.61 Note that the potential difference between the plates is held constant at ΔV_i by the battery.

$$C_i = \frac{q_0}{\Delta V_i} \quad \text{and} \quad C_f = \frac{q_f}{\Delta V_i} = \frac{q_0 + q}{\Delta V_i}$$

But $C_f = \kappa C_i$, so $$\frac{q_0 + q}{\Delta V_i} = \kappa\left(\frac{q_0}{\Delta V_i}\right)$$

Thus, $$\kappa = \frac{q_0 + q}{q_0} \quad \text{or} \quad \kappa = \boxed{1 + \frac{q}{q_0}}$$

26.62 (a) $$C = \frac{\in_0}{d}\left[(\ell - x)\ell + \kappa \ell x\right] = \boxed{\frac{\in_0}{d}\left[\ell^2 + \ell x(\kappa - 1)\right]}$$

(b) $$U = \tfrac{1}{2}C(\Delta V)^2 = \boxed{\frac{1}{2}\left(\frac{\in_0(\Delta V)^2}{d}\right)\left[\ell^2 + \ell x(\kappa - 1)\right]}$$

(c) $$|\mathbf{F}| = \left|-\frac{dU}{dx}\right| = \boxed{\quad}$$ to the left

(d) $F = \frac{(2000)^2(8.85\times10^{-12})(0.0500)(4.50-1)}{2(2.00\times10^{-3})} = \boxed{1.55\times10^{-3}\ \text{N}}$

***26.63** The portion of the capacitor nearly filled by metal has capacitance $\kappa \in_0 (\ell x)/d \to \infty$ and stored energy $Q^2/2C \to 0$. The unfilled portion has capacitance $\in_0 \ell\ (\ell - x)/d$. The charge on this portion is $Q = (\ell - x)Q_0 / \ell$.

(a) The stored energy is $U = \dfrac{Q^2}{2C} = \dfrac{[(\ell - x)Q_0/\ell]^2}{2\in_0 \ell(\ell - x)/d} = \boxed{\dfrac{Q_0^2 d(\ell - x)}{2\in_0 \ell^3}}$

(b) $F = -\dfrac{dU}{dx} = -\dfrac{d}{dx}\left(\dfrac{Q_0^2 d(\ell - x)}{2\in_0 \ell^3}\right) = +\dfrac{Q_0^2 d}{2\in_0 \ell^3}$

F = [ ] to the right

(c) $Stress = \dfrac{F}{\ell d} =$ [ ]

(d) $u = \frac{1}{2}\in_0 E^2 = \frac{1}{2}\in_0\left(\dfrac{\sigma}{\in_0}\right)^2 = \frac{1}{2}\in_0\left(\dfrac{Q_0}{\in_0 \ell^2}\right)^2 =$ [ ]

26.64 Gasoline: $\left(126\,000\ \dfrac{\text{Btu}}{\text{gal}}\right)\left(1054\ \dfrac{\text{J}}{\text{Btu}}\right)\left(\dfrac{1.00\ \text{gal}}{3.786 \times 10^{-3}\ \text{m}^3}\right)\left(\dfrac{1.00\ \text{m}^3}{670\ \text{kg}}\right) = 5.25 \times 10^7\ \dfrac{\text{J}}{\text{kg}}$

Battery: $\dfrac{(12.0\ \text{J/C})(100\ \text{C/s})(3600\ \text{s})}{16.0\ \text{kg}} = 2.70 \times 10^5\ \text{J/kg}$

Capacitor: $\dfrac{\frac{1}{2}(0.100\ \text{F})(12.0\ \text{V})^2}{0.100\ \text{kg}} = 72.0\ \text{J/kg}$

Gasoline has 194 times the specific energy content of the battery and 727 000 times that of the capacitor

26.65 Call the unknown capacitance C_u

$$Q = C_u(\Delta V_i) = (C_u + C)(\Delta V_f)$$

$$\boxed{C_u = \frac{C(\Delta V_f)}{(\Delta V_i) - (\Delta V_f)}} = \frac{(10.0\ \mu\text{F})(30.0\ \text{V})}{(100\ \text{V} - 30.0\ \text{V})} = \boxed{4.29\ \mu\text{F}}$$

Goal Solution

An isolated capacitor of unknown capacitance has been charged to a potential difference of 100 V. When the charged capacitor is then connected in parallel to an uncharged 10.0-μF capacitor, the voltage across the combination is 30.0 V. Calculate the unknown capacitance.

G: The voltage of the combination will be reduced according to the size of the added capacitance. (Example: If the unknown capacitance were $C = 10.0\ \mu\text{F}$, then $\Delta V_1 = 50.0$ V because the charge is now distributed evenly between the two capacitors.) Since the final voltage is less than half the original, we might guess that the unknown capacitor is about $5.00\ \mu\text{F}$.

O: We can use the relationships for capacitors in parallel to find the unknown capacitance, along with the requirement that the charge on the unknown capacitor must be the same as the total charge on the two capacitors in parallel.

A: We name our ignorance and call the unknown capacitance C_u. The charge originally deposited on **each** plate, + on one, – on the other, is

$$Q = C_u \Delta V = C_u(100\ \text{V})$$

Now in the new connection this same conserved charge redistributes itself between the two capacitors according to $Q = Q_1 + Q_2$.

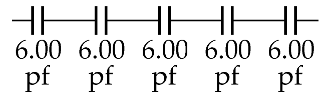
$$Q_1 = C_u(30.0\ \text{V}) \text{ and } \quad Q_2 = (10.0\ \mu\text{F})(30.0\ \text{V}) = 300\ \mu\text{C}$$

We can eliminate Q and Q_1 by substitution:

$$C_u(100\ \text{V}) = C_u(30.0\ \text{V}) + 300\ \mu\text{C} \qquad \text{so} \qquad C_u = \frac{300\ \mu\text{C}}{70.0\ \text{V}} = 4.29\ \mu\text{F}$$

L: The calculated capacitance is close to what we expected, so our result seems reasonable. In this and other capacitance combination problems, it is important not to confuse the charge and voltage of the system with those of the individual components, especially if they have different values. Careful attention must be given to the subscripts to avoid this confusion. It is also important to not confuse the variable "C" for capacitance with the unit of charge, "C" for coulombs.

26.66 Put five 6.00 pF capacitors in series.



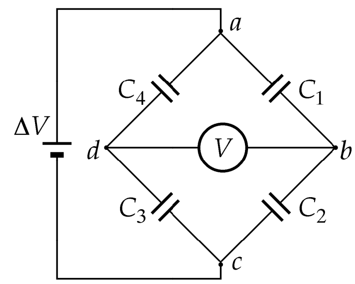
The potential difference across any one of the capacitors will be:

$$\Delta V = \frac{\Delta V_{\text{max}}}{5} = \frac{1000\text{ V}}{5} = 200\text{ V}$$

and the equivalent capacitance is:

$$\frac{1}{C_{\text{eq}}} = 5\left(\frac{1}{6.00\text{ pF}}\right) \quad \text{or} \quad C_{\text{eq}} = \frac{6.00\text{ pF}}{5} = 1.20\text{ pF}$$

26.67 When $\Delta V_{db} = 0$, $\Delta V_{bc} = \Delta V_{dc}$, and $\dfrac{Q_2}{C_2} = \dfrac{Q_3}{C_3}$



Also, $\Delta V_{ba} = \Delta V_{da}$ or $\dfrac{Q_1}{C_1} = \dfrac{Q_4}{C_4}$

From these equations we have $C_2 = \left(\dfrac{C_3}{C_4}\right)\left(\dfrac{Q_2}{Q_1}\right)\left(\dfrac{Q_4}{Q_3}\right)C_1$

However, from the properties of capacitors in series, we have $Q_1 = Q_2$ and $Q_3 = Q_4$

Therefore, $C_2 = \left(\dfrac{C_3}{C_4}\right)C_1 = \dfrac{9.00}{12.0}(4.00\ \mu F) = \boxed{3.00\ \mu\text{F}}$

26.68 Let C = the capacitance of an individual capacitor, and C_s represent the equivalent capacitance of the group in series. While being charged in parallel, each capacitor receives charge

$$Q = C\,\Delta V_{\text{chg}} = (5.00 \times 10^{-4}\text{ F})(800\text{ V}) = 0.400\text{ C}$$

While being discharged in series,

$$\Delta V_{\text{disch}} = \frac{Q}{C_s} = \frac{Q}{C/10} = \frac{0.400\text{ C}}{5.00 \times 10^{-5}\text{ F}} = \boxed{8.00\text{ kV}}$$

or 10 times the original voltage.

26.69 (a) $C_0 = \dfrac{\epsilon_0 A}{d} = \dfrac{Q_0}{\Delta V_0}$

When the dielectric is inserted at constant voltage,

$$C = \kappa C_0 = \frac{Q}{\Delta V_0}; \qquad U_0 = \frac{C_0(\Delta V_0)^2}{2}$$

$$U = \frac{C(\Delta V_0)^2}{2} = \frac{\kappa C_0(\Delta V_0)^2}{2} \quad \text{and} \quad \frac{U}{U_0} = \kappa$$

The extra energy comes from (part of the) electrical work done by the battery in separating the extra charge.

(b) $Q_0 = C_0 \Delta V_0$ and $Q = C \Delta V_0 = \kappa C_0 \Delta V_0$ so $\boxed{Q / Q_0 = \kappa}$

26.70 (a) A slice of width (dx) at coordinate x in $0 \le x \le L$ has thickness xd/L filled with dielectric κ_2, and $d - xd/L$ is filled with the material having constant κ_1. This slice has a capacitance given by

$$\frac{1}{dC} = \frac{1}{\left(\frac{\kappa_2 \epsilon_0 (dx) W}{xd/L}\right)} + \frac{1}{\left(\frac{\kappa_1 \epsilon_0 (dx) W}{d - xd/L}\right)} = \frac{xd}{\kappa_2 \epsilon_0 W L(dx)} + \frac{dL - xd}{\kappa_1 \epsilon_0 W L(dx)} = \frac{\kappa_1 xd + \kappa_2 dL - \kappa_2 xd}{\kappa_1 \kappa_2 \epsilon_0 W L(dx)}$$

$$dC = \frac{\kappa_1 \kappa_2 \epsilon_0 W L(dx)}{\kappa_2 dL + (\kappa_1 - \kappa_2) xd}$$

The whole capacitor is all the slices in parallel:

$$C = \int dC = \int_{x=0}^{L} \frac{\kappa_1 \kappa_2 \epsilon_0 W L(dx)}{\kappa_2 Ld + (\kappa_1 - \kappa_2) xd} = \frac{\kappa_1 \kappa_2 \epsilon_0 W L}{(\kappa_1 - \kappa_2) d} \int_{x=0}^{L} \left(\kappa_2 Ld + (\kappa_1 - \kappa_2) xd\right)^{-1} (\kappa_1 - \kappa_2) d(dx)$$

$$C = \frac{\kappa_1 \kappa_2 \epsilon_0 W L}{(\kappa_1 - \kappa_2) d} \ln\left[\kappa_2 Ld + (\kappa_1 - \kappa_2) xd\right]\Big|_0^L = \frac{\kappa_1 \kappa_2 \epsilon_0 W L}{(\kappa_1 - \kappa_2) d} \left[\ln \kappa_1 Ld - \ln \kappa_2 Ld\right] = \boxed{\frac{\kappa_1 \kappa_2 \epsilon_0 W L}{(\kappa_1 - \kappa_2) d} \ln \frac{\kappa_1}{\kappa_2}}$$

(b) To take the limit $\kappa_1 \to \kappa_2$, write $\kappa_1 = \kappa_2 (1 + x)$ and let $x \to 0$. Then

$$C = \frac{\kappa_2^2 (1 + x) \epsilon_0 W L}{(\kappa_2 + \kappa_2 x - \kappa_2) d} \ln (1 + x)$$

Use the expansion of $\ln(1 + x)$ from Appendix B.5.

$$C = \frac{\kappa_2^2 (1 + x) \epsilon_0 W L}{\kappa_2 xd} \left(x - \tfrac{1}{2} x^2 + \tfrac{1}{3} x^3 \ldots\right) = \frac{\kappa_2 (1 + x) \epsilon_0 W L}{d} \left(1 - \tfrac{1}{2} x + \ldots\right)$$

$$\lim_{x \to 0} C = \frac{\kappa_2 \epsilon_0 W L}{d} = \boxed{\frac{\kappa \epsilon_0 A}{d}}$$

26.71 The vertical orientation sets up two capacitors in parallel, with equivalent capacitance

$$C_p = \frac{\epsilon_0 (A/2)}{d} + \frac{\kappa \epsilon_0 (A/2)}{d} = \left(\frac{\kappa + 1}{2}\right) \frac{\epsilon_0 A}{d}$$

where A is the area of either plate and d is the separation of the plates. The horizontal orientation produces two capacitors in series. If f is the fraction of the horizontal capacitor filled with dielectric, the equivalent capacitance is

$$\frac{1}{C_s} = \frac{fd}{\kappa \epsilon_0 A} + \frac{(1 - f) d}{\epsilon_0 A} = \left[\frac{f + \kappa(1 - f)}{\kappa}\right] \frac{d}{\epsilon_0 A}, \quad \text{or} \quad C_s = \left[\frac{\kappa}{f + \kappa(1 - f)}\right] \frac{\epsilon_0 A}{d}$$

Requiring that $C_p = C_s$ gives $\frac{\kappa + 1}{2} = \frac{\kappa}{f + \kappa(1 - f)}$, or $(\kappa + 1)\left[f + \kappa(1 - f)\right] = 2\kappa$

For $\kappa = 2.00$, this yields $3.00\left[2.00 - (1.00) f\right] = 4.00$, with the solution $f = \boxed{2/3}$.

26.72 Initially (capacitors charged in parallel),

$q_1 = C_1(\Delta V) = (6.00\ \mu\text{F})(250\ \text{V}) = 1500\ \mu\text{C}$

$q_2 = C_2(\Delta V) = (2.00\ \mu\text{F})(250\ \text{V}) = 500\ \mu\text{C}$

After reconnection (positive plate to negative plate),

$q'_{\text{total}} = q_1 - q_2 = 1000\ \mu\text{C}$ and $\Delta V' = \dfrac{q'_{\text{total}}}{C_{\text{total}}} = \dfrac{1000\ \mu\text{C}}{8.00\ \mu\text{F}} = 125\ \text{V}$

Therefore,

$q'_1 = C_1(\Delta V') = (6.00\ \mu\text{F})(125\ \text{V}) = \boxed{750\ \mu\text{C}}$

$q'_2 = C_2(\Delta V') = (2.00\ \mu\text{F})(125\ \text{V}) = \boxed{250\ \mu\text{C}}$

26.73 E_{max} occurs at the inner conductor's surface.

$E_{\text{max}} = \dfrac{2k_e\lambda}{a}$ from Equation 24.7.

$\Delta V = 2k_e\lambda \ln\left(\dfrac{b}{a}\right)$ from Example 26.2

$$E_{\text{max}} = \frac{\Delta V}{a\ln(b/a)}$$

$$\Delta V_{\text{max}} = E_{\text{max}} a \ln\left(\frac{b}{a}\right) = (18.0\times 10^6\ \text{V/m})(0.800\times 10^{-3}\ \text{m})\ln\left(\frac{3.00}{0.800}\right) = \boxed{19.0\ \text{kV}}$$

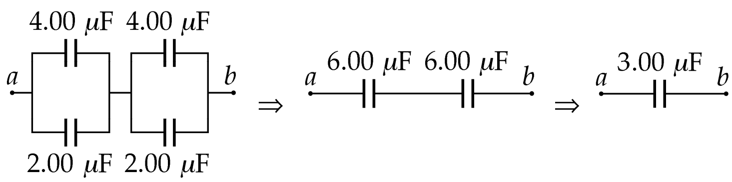
26.74 $E = \dfrac{2\kappa\lambda}{a}$; $\Delta V = 2\kappa\lambda \ln\left(\dfrac{b}{a}\right)$

$$\Delta V_{\text{max}} = E_{\text{max}} a \ln\left(\frac{b}{a}\right)$$

$$\frac{dV_{\text{max}}}{da} = E_{\text{max}}\left[\ln\left(\frac{b}{a}\right) + a\left(\frac{1}{b/a}\right)\left(-\frac{b}{a^2}\right)\right] = 0$$

$\ln\left(\dfrac{b}{a}\right) = 1$ or $\dfrac{b}{a} = e^1$ so $\boxed{a = \dfrac{b}{e}}$

26.75 Assume a potential difference across a and b, and notice that the potential difference across the $8.00\ \mu$F capacitor must be zero by symmetry. Then the equivalent capacitance can be determined from the following circuit:



$C_{ab} = \boxed{3.00\ \mu\text{F}}$

26.76 By symmetry, the potential difference across $3C$ is zero, so the circuit reduces to

$$C_{\text{eq}} = \frac{(2C)(4C)}{2C + 4C} = \frac{8}{6}C = \boxed{\frac{4}{3}C}$$