Chapter 25 Solutions

25.1 $\Delta V = -14.0$ V and

$$Q = -N_A e = -(6.02 \times 10^{23})(1.60 \times 10^{-19} \text{ C}) = -9.63 \times 10^4 \text{ C}$$

$$\Delta V = \frac{W}{Q}, \quad \text{so} \quad W = Q(\Delta V) = (-9.63 \times 10^4 \text{ C})(-14.0 \text{ J/C}) = \boxed{1.35 \text{ MJ}}$$

25.2 $\Delta K = q|\Delta V|$ $\qquad$ $7.37 \times 10^{-17} = q(115)$

$$\boxed{q = 6.41 \times 10^{-19} \text{ C}}$$

25.3 $W = \Delta K = q|\Delta V|$

$$\frac{1}{2} mv^2 = e(120 \text{ V}) = 1.92 \times 10^{-17} \text{ J}$$

Thus, $v = \sqrt{\dfrac{3.84 \times 10^{-17} \text{ J}}{m}}$

(a) For a proton, this becomes $\quad v = \sqrt{\dfrac{3.84 \times 10^{-17} \text{ J}}{1.67 \times 10^{-27} \text{ kg}}} = 1.52 \times 10^5 \text{ m/s} = \boxed{152 \text{ km/s}}$

(b) If an electron, $\quad v = \sqrt{\dfrac{3.84 \times 10^{-17} \text{ J}}{9.11 \times 10^{-31} \text{ kg}}} = 6.49 \times 10^6 \text{ m/s} = \boxed{6.49 \text{ Mm/s}}$

Goal Solution

(a) Calculate the speed of a proton that is accelerated from rest through a potential difference of 120 V.
(b) Calculate the speed of an electron that is accelerated through the same potential difference.

G: Since 120 V is only a modest potential difference, we might expect that the final speed of the particles will be substantially less than the speed of light. We should also expect the speed of the electron to be significantly greater than the proton because, with $m_e \ll m_p$, an equal force on both particles will result in a much greater acceleration for the electron.

O: Conservation of energy can be applied to this problem to find the final speed from the kinetic energy of the particles. (Review this work-energy theory of motion from Chapter 8 if necessary.)

A: (a) Energy is conserved as the proton moves from high to low potential, which can be defined for this problem as moving from 120 V down to 0 V:

$$K_i + U_i + \Delta E_{nc} = K_f + U_f$$
$$0 + qV + 0 = \tfrac{1}{2} m v_p^{\,2} + 0$$
$$(1.60 \times 10^{-19}\ \text{C})(120\ \text{V})\left(\frac{1\ \text{J}}{1\ \text{V}\cdot\text{C}}\right) = \tfrac{1}{2}(1.67 \times 10^{-27}\ \text{kg}) v_p^{\,2}$$
$$v_p = 1.52 \times 10^5\ \text{m / s}$$

(b) The electron will gain speed in moving the other way, from $V_i = 0$ to $V_f = 120$ V:

$$K_i + U_i + \Delta E_{nc} = K_f + U_f$$
$$0 + 0 + 0 = \tfrac{1}{2} m v_e^{\,2} + qV$$
$$0 = \tfrac{1}{2}(9.11 \times 10^{-31}\ \text{kg}) v_e^{\,2} + (-1.60 \times 10^{-19}\ \text{C})(120\ \text{J / C})$$
$$v_e = 6.49 \times 10^6\ \text{m / s}$$

L: Both of these speeds are significantly less than the speed of light as expected, which also means that we were justified in not using the relativistic kinetic energy formula. (For precision to three significant digits, the relativistic formula is only needed if *v* is greater than about 0.1 *c*.)

25.4 For speeds larger than one-tenth the speed of light, $\frac{1}{2} mv^2$ gives noticeably wrong answers for kinetic energy, so we use

$$K = mc^2\left(\frac{1}{\sqrt{1 - v^2 / c^2}} - 1\right) = \left(9.11 \times 10^{-31}\ \text{kg}\right)\left(3.00 \times 10^8\ \text{m / s}\right)^2\left(\frac{1}{\sqrt{1 - 0.400^2}} - 1\right) = 7.47 \times 10^{-15}\ \text{J}$$

Energy is conserved during acceleration: $K_i + U_i + \Delta E = K_f + U_f$

$$0 + qV_i + 0 = 7.47 \times 10^{-15}\ \text{J} + qV_f$$

The change in potential is $V_f - V_i$: $$V_f - V_i = \frac{-7.47 \times 10^{-15}\ \text{J}}{q} = \frac{-7.47 \times 10^{-15}\ \text{J}}{-1.60 \times 10^{-19}\ \text{C}} = \boxed{+46.7\ \text{kV}}$$

The positive answer means that the electron speeds up in moving toward higher potential.

25.5 $$W = \Delta K = -q\Delta V$$

$$0 - \tfrac{1}{2}\left(9.11 \times 10^{-31}\ \text{kg}\right)\left(4.20 \times 10^5\ \text{m / s}\right)^2 = -\left(-1.60 \times 10^{-19}\ \text{C}\right)\Delta V$$

From which, $\Delta V = \boxed{-0.502\ \text{V}}$

***25.6** (a) We follow the path from (0, 0) to (20.0 cm, 0) to (20.0 cm, 50.0 cm).

$\Delta U = -$ (work done)

$\Delta U = -\left(\text{work from origin to } (20.0\text{ cm}, 0)\right) - \left(\text{work from } (20.0\text{ cm}, 0) \text{ to } (20.0\text{ cm}, 50.0\text{ cm})\right)$

Note that the last term is equal to 0 because the force is perpendicular to the displacement.

$$\Delta U = -(qE_x)(\Delta x) = -(12.0 \times 10^{-6}\text{ C})(250\text{ V/m})(0.200\text{ m}) = \boxed{-6.00 \times 10^{-4}\text{ J}}$$

(b) $$\Delta V = \frac{\Delta U}{q} = -\frac{6.00 \times 10^{-4}\text{ J}}{12.0 \times 10^{-6}\text{ C}} = -50.0\text{ J/C} = \boxed{-50.0\text{ V}}$$

***25.7** $$E = \frac{|\Delta V|}{d} = \frac{25.0 \times 10^3\text{ J/C}}{1.50 \times 10^{-2}\text{ m}} = 1.67 \times 10^6\text{ N/C} = \boxed{1.67\text{ MN/C}}$$

***25.8** (a) $$|\Delta V| = Ed = (5.90 \times 10^3\text{ V/m})(0.0100\text{ m}) = \boxed{59.0\text{ V}}$$

(b) $\frac{1}{2} m v_f^2 = |q(\Delta V)|$; $\frac{1}{2}(9.11 \times 10^{-31})\ v_f^2 = (1.60 \times 10^{-19})(59.0)$

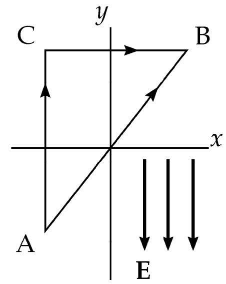
$$\boxed{v_f = 4.55 \times 10^6\text{ m/s}}$$

25.9 $$\Delta U = -\tfrac{1}{2} m\left(v_f^2 - v_i^2\right) = -\tfrac{1}{2}\left(9.11 \times 10^{-31}\text{ kg}\right)\left[\left(1.40 \times 10^5\text{ m/s}\right)^2 - \left(3.70 \times 10^6\text{ m/s}\right)^2\right] = 6.23 \times 10^{-18}\text{ J}$$

$\Delta U = q\Delta V$: $+6.23 \times 10^{-18} = (-1.60 \times 10^{-19})\Delta V$

$\boxed{\Delta V = -38.9\text{ V}}$ The origin is at higher potential.

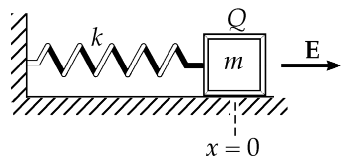
***25.10** $V_B - V_A = -\int_A^B \mathbf{E}\cdot d\mathbf{s} = -\int_A^C \mathbf{E}\cdot d\mathbf{s} - \int_C^B \mathbf{E}\cdot d\mathbf{s}$



$$V_B - V_A = (-E\cos 180°)\int_{-0.300}^{0.500} dy - (E\cos 90.0°)\int_{-0.200}^{0.400} dx$$

$$V_B - V_A = (325)(0.800) = \boxed{+260\text{ V}}$$

25.11 (a) Arbitrarily choose $V = 0$ at $x = 0$. Then at other points, $V = -Ex$ and $U_e = QV = -QEx$. Between the endpoints of the motion,



$$(K + U_s + U_e)_i = (K + U_s + U_e)_f$$

$$0 + 0 + 0 = 0 + \tfrac{1}{2}kx_{\max}^2 - QEx_{\max}$$

so the block comes to rest when the spring is stretched by an amount

$$x_{\max} = \frac{2QE}{k} = \frac{2\left(50.0\times10^{-6}\text{ C}\right)\left(5.00\times10^{5}\text{ V/m}\right)}{100\text{ N/m}} = \boxed{0.500\text{ m}}$$

(b) At equilibrium, $\Sigma F_x = -F_s + F_e = 0$ or $kx = QE$. Thus, the equilibrium position is at

$$x = \frac{QE}{k} = \frac{\left(50.0\times10^{-6}\text{ C}\right)\left(5.00\times10^{5}\text{ N/C}\right)}{100\text{ N/m}} = \boxed{0.250\text{ m}}$$

(c) The equation of motion for the block is $\Sigma F_x = -kx + QE = m\dfrac{d^2x}{dt^2}$. Let $x' = x - \dfrac{QE}{k}$, or $x = x' + \dfrac{QE}{k}$ so the equation of motion becomes:

$$-k\left(x' + \frac{QE}{k}\right) + QE = m\frac{d^2\left(x' + QE/k\right)}{dt^2},\ \text{ or }\ \frac{d^2x'}{dt^2} = -\left(\frac{k}{m}\right)x'$$

This is the equation for simple harmonic motion $\left(a_{x'} = -\omega^2x'\right)$, with $\omega = \sqrt{k/m}$. The period of the motion is then

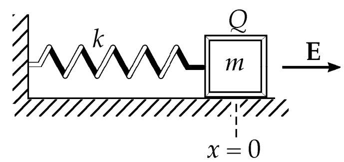
$$T = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{m}{k}} = 2\pi\sqrt{\frac{4.00\text{ kg}}{100\text{ N/m}}} = \boxed{1.26\text{ s}}$$

(d) $(K + U_s + U_e)_i + \Delta E = (K + U_s + U_e)_f$

$$0 + 0 + 0 - \mu_k mgx_{\max} = 0 + \tfrac{1}{2}kx_{\max}^2 - QEx_{\max}$$

$$x_{\max} = \frac{2(QE - \mu_k mg)}{k} = \frac{2\left[\left(50.0\times10^{-6}\ \text{C}\right)\left(5.00\times10^{5}\ \text{N/C}\right) - 0.200\left(4.00\ \text{kg}\right)\left(9.80\ \text{m/s}^2\right)\right]}{100\ \text{N/m}} = \boxed{0.343\ \text{m}}$$

25.12 (a) Arbitrarily choose $V = 0$ at 0. Then at other points $V = -Ex$ and $U_e = QV = -QEx$. Between the endpoints of the motion,



$$(K + U_s + U_e)_i = (K + U_s + U_e)_f$$

$$0 + 0 + 0 = 0 + \tfrac{1}{2}kx_{\max}^2 - QEx_{\max} \qquad \text{so} \qquad x_{\max} = \boxed{\frac{2QE}{k}}$$

(b) At equilibrium, $\Sigma F_x = -F_s + F_e = 0$ or $kx = QE$. So the equilibrium position is at $x = \boxed{\dfrac{QE}{k}}$

(c) The block's equation of motion is $\Sigma F_x = -kx + QE = m\dfrac{d^2x}{dt^2}$. Let $x' = x - \dfrac{QE}{k}$, or $x = x' + \dfrac{QE}{k}$, so the equation of motion becomes:

$$-k\left(x' + \frac{QE}{k}\right) + QE = m\frac{d^2\left(x' + QE/k\right)}{dt^2}, \quad \text{or} \quad \frac{d^2x'}{dt^2} = -\left(\frac{k}{m}\right)x'$$

This is the equation for simple harmonic motion $\left(a_{x'} = -\omega^2 x'\right)$, with $\omega = \sqrt{k/m}$

The period of the motion is then $T = \dfrac{2\pi}{\omega} = \boxed{2\pi\sqrt{\dfrac{m}{k}}}$

(d) $(K + U_s + U_e)_i + \Delta E = (K + U_s + U_e)_f$

$$0 + 0 + 0 - \mu_k mgx_{\max} = 0 + \tfrac{1}{2}kx_{\max}^2 - QEx_{\max}$$

$$x_{\max} = \boxed{\frac{2(QE - \mu_k mg)}{k}}$$

25.13 For the entire motion, $y - y_i = v_{yi}t + \tfrac{1}{2}a_y t^2$

$0 - 0 = v_i t + \tfrac{1}{2}a_y t^2$ so $a_y = -\dfrac{2v_i}{t}$

$\sum F_y = ma_y$: $-mg - qE = -\dfrac{2mv_i}{t}$

$E = \dfrac{m}{q}\left(\dfrac{2v_i}{t} - g\right)$ and $\mathbf{E} = -\dfrac{m}{q}\left(\dfrac{2v_i}{t} - g\right)\mathbf{j}$

For the upward flight: $v_{yf}^2 = v_{yi}^2 + 2a_y(y - y_i)$

$0 = v_i^2 + 2\left(-\dfrac{2v_i}{t}\right)(y_{\max} - 0)$ and $y_{\max} = \tfrac{1}{4}v_i t$

$$\Delta V = \int_0^{y_{\max}} \mathbf{E} \cdot d\mathbf{y} = +\frac{m}{q}\left(\frac{2v_i}{t} - g\right)y\Big|_0^{y_{\max}} = \frac{m}{q}\left(\frac{2v_i}{t} - g\right)\left(\tfrac{1}{4}v_i t\right)$$

$$\Delta V = \frac{2.00\text{ kg}}{5.00 \times 10^{-6}\text{ C}}\left(\frac{2(20.1\text{ m/s})}{4.10\text{ s}} - 9.80\text{ m/s}^2\right)\left[\tfrac{1}{4}(20.1\text{ m/s})(4.10\text{ s})\right] = \boxed{40.2\text{ kV}}$$

25.14 Arbitrarily take $V = 0$ at the initial point. Then at distance d downfield, where L is the rod length, $V = -Ed$ and $U_e = -\lambda LEd$

(a) $(K+U)_i = (K+U)_f$

$$0 + 0 = \tfrac{1}{2}\mu L v^2 - \lambda LEd$$

$$v = \sqrt{\frac{2\lambda Ed}{\mu}} = \sqrt{\frac{2(40.0\times10^{-6}\ \text{C / m})(100\ \text{N / C})(2.00\ \text{m})}{(0.100\ \text{kg / m})}} = \boxed{0.400\ \text{m/s}}$$

(b) The same.

25.15 Arbitrarily take $V = 0$ at point P. Then (from Equation 25.8) the potential at the original position of the charge is $-\mathbf{E}\cdot\mathbf{s} = -EL\cos\theta$. At the final point a, $V = -EL$. Suppose the table is frictionless: $(K+U)_i = (K+U)_f$

$$0 - qEL\cos\theta = \tfrac{1}{2}mv^2 - qEL$$

$$v = = \sqrt{\frac{2qEL(1-\cos\theta)}{\text{m}}} = \sqrt{\frac{2(2.00\times10^{-6}\ \text{C})(300\ \text{N / C})(1.50\ \text{m})(1-\cos 60.0°)}{0.0100\ \text{kg}}} = \boxed{0.300\ \text{m/s}}$$

***25.16** (a) The potential at 1.00 cm is

$$V_1 = k_e\frac{q}{r} = \frac{(8.99\times10^{9}\ \text{N}\cdot\text{m}^2/\text{C}^2)(1.60\times10^{-19}\ \text{C})}{1.00\times10^{-2}\ \text{m}} = \boxed{1.44\times10^{-7}\ \text{V}}$$

(b) The potential at 2.00 cm is

$$V_2 = k_e\frac{q}{r} = \frac{(8.99\times10^{9}\ \text{N}\cdot\text{m}^2/\text{C}^2)(1.60\times10^{-19}\ \text{C})}{2.00\times10^{-2}\ \text{m}} = 0.719\times10^{-7}\ \text{V}$$

Thus, the difference in potential between the two points is

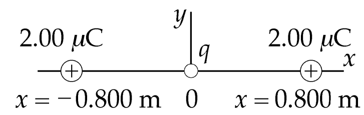
$$\Delta V = V_2 - V_1 = \boxed{-7.19\times10^{-8}\ \text{V}}$$

(c) The approach is the same as above except the charge is -1.60×10^{-19} C. This changes the sign of all the answers, with the magnitudes remaining the same.

That is, the potential at 1.00 cm is $\boxed{-1.44\times10^{-7}\ \text{V}}$

The potential at 2.00 cm is -0.719×10^{-7} V, so $\Delta V = V_2 - V_1 = \boxed{7.19\times10^{-8}\ \text{V}}$.

25.17 (a) Since the charges are equal and placed symmetrically, $\boxed{F = 0}$



(b) Since $F = qE = 0$, $\boxed{E = 0}$

(c) $V = 2k_e \dfrac{q}{r} = 2\left(8.99\times 10^{9}\ \dfrac{\text{N}\cdot\text{m}^2}{\text{C}^2}\right)\left(\dfrac{2.00\times 10^{-6}\ \text{C}}{0.800\ \text{m}}\right)$

$V = 4.50\times 10^{4}\ \text{V} = \boxed{45.0\ \text{kV}}$

25.18 (a) $E_x = \dfrac{k_e q_1}{x^2} + \dfrac{k_e q_2}{(x-2.00)^2} = 0$ becomes $E_x = k_e\left(\dfrac{+q}{x^2} + \dfrac{-2q}{(x-2.00)^2}\right) = 0$

Dividing by k_e, $2qx^2 = q(x-2.00)^2$

$x^2 + 4.00x - 4.00 = 0$

Therefore $E = 0$ when $x = \dfrac{-4.00 \pm \sqrt{16.0 + 16.0}}{2} = \boxed{-4.83\ \text{m}}$

(Note that the positive root does not correspond to a physically valid situation.)

(b) $V = \dfrac{k_e q_1}{x} + \dfrac{k_e q_2}{(2.00 - x)} = 0$ or $V = k_e\left(\dfrac{+q}{x} - \dfrac{2q}{(2.00-x)}\right) = 0$

Again solving for x, $2qx = q(2.00 - x)$

For $0 \le x \le 2.00$ $V = 0$ when $x = \boxed{0.667\ \text{m}}$

and $\dfrac{q}{|x|} = \dfrac{-2q}{|2 - x|}$

For $x < 0$ $x = \boxed{-2.00\ \text{m}}$

25.19 (a) $U = \dfrac{k_e q_1 q_2}{r} = \dfrac{-(8.99\times 10^{9})(1.60\times 10^{-19})^2}{0.0529\times 10^{-9}} = -4.35\times 10^{-18}\ \text{J} = \boxed{-27.2\ \text{eV}}$

(b) $U = \dfrac{k_e q_1 q_2}{r} = \dfrac{-(8.99\times 10^{9})(1.60\times 10^{-19})^2}{2^2(0.0529\times 10^{-9})} = \boxed{-6.80\ \text{eV}}$

(c) $U = \dfrac{k_e q_1 q_2}{r} = \dfrac{-k_e e^2}{\infty} = \boxed{0}$

Goal Solution

The Bohr model of the hydrogen atom states that the single electron can exist only in certain allowed orbits around the proton. The radius of each Bohr orbit is $r = n^2(0.0529\text{ nm})$ where $n = 1, 2, 3, \ldots$. Calculate the electric potential energy of a hydrogen atom when the electron is in the (a) first allowed orbit, $n = 1$; (b) second allowed orbit, $n = 2$; and (c) when the electron has escaped from the atom ($r = \infty$). Express your answers in electron volts.

G: We may remember from chemistry that the lowest energy level for hydrogen is $E_1 = -13.6\text{ eV}$, and higher energy levels can be found from $E_n = E_1 / n^2$, so that $E_2 = -3.40\text{ eV}$ and $E_\infty = 0\text{ eV}$. (see section 42.2) Since these are the total energies (potential plus kinetic), the electric potential energy alone should be lower (more negative) because the kinetic energy of the electron must be positive.

O: The electric potential energy is given by $U = k_e \dfrac{q_1 q_2}{r}$

A: (a) For the first allowed Bohr orbit,

$$U = \left(8.99 \times 10^9 \ \frac{\text{N}\cdot\text{m}^2}{\text{C}^2}\right)\frac{(-1.60 \times 10^{-19}\text{ C})(1.60 \times 10^{-19}\text{ C})}{(0.0529 \times 10^{-9}\text{ m})} = -4.35 \times 10^{-18}\text{ J} = \frac{-4.35 \times 10^{-18}\text{ J}}{1.60 \times 10^{-19}\text{ J / eV}} = -27.2\text{ eV}$$

(b) For the second allowed orbit,

$$U = (8.99 \times 10^9\text{ N}\cdot\text{m}^2 / \text{C}^2)\frac{(-1.60 \times 10^{-19}\text{ C})(1.60 \times 10^{-19}\text{ C})}{2^2(0.0529 \times 10^{-9}\text{ m})} = -1.088 \times 10^{-18}\text{ J} = -6.80\text{ eV}$$

(c) When the electron is at $r = \infty$,

$$U = \left(8.99 \times 10^9\text{ N}\cdot\text{m}^2/\text{C}^2\right)\frac{\left(-1.60 \times 10^{-19}\text{ C}\right)\left(1.60 \times 10^{-19}\text{ C}\right)}{\infty} = 0\text{ J}$$

L: The potential energies appear to be twice the magnitude of the total energy values, so apparently the kinetic energy of the electron has the same absolute magnitude as the total energy.

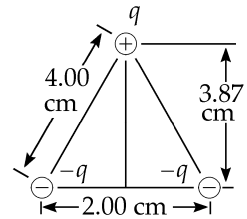
***25.20** (a) $U = \dfrac{qQ}{4\pi\epsilon_0 r} = \dfrac{(5.00 \times 10^{-9}\text{ C})(-3.00 \times 10^{-9}\text{ C})(8.99 \times 10^9\text{ V}\cdot\text{m/C})}{(0.350\text{ m})} = \boxed{-3.86 \times 10^{-7}\text{ J}}$

The minus sign means it takes 3.86×10^{-7} J to pull the two charges apart from 35 cm to a much larger separation.

(b) $V = \dfrac{Q_1}{4\pi\epsilon_0 r_1} + \dfrac{Q_2}{4\pi\epsilon_0 r_2} = \dfrac{(5.00 \times 10^{-9}\text{ C})(8.99 \times 10^9\text{ V}\cdot\text{m / C})}{0.175\text{ m}} + \dfrac{(-3.00 \times 10^{-9}\text{ C})(8.99 \times 10^9\text{ V}\cdot\text{m / C})}{0.175\text{ m}}$

$V = \boxed{103\text{ V}}$

25.21 $V = \sum_i k \frac{q_i}{r_i}$



$V = (8.99 \times 10^9)(7.00 \times 10^{-6})\left[\frac{-1}{0.0100} - \frac{1}{0.0100} + \frac{1}{0.0387}\right]$

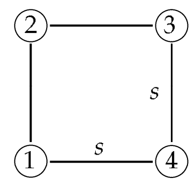
$V = \boxed{-1.10 \times 10^7 \text{ C} = -11.0 \text{ MV}}$

***25.22** $U_e = q_4V_1 + q_4V_2 + q_4V_3 = q_4 \dfrac{1}{4\pi\epsilon_0}\left(\dfrac{q_1}{r_1} + \dfrac{q_2}{r_2} + \dfrac{q_3}{r_3}\right)$

$$U_e = \left(10.0\times10^{-6}\text{ C}\right)^2\left(8.99\times10^9\text{ N}\cdot\text{m}^2/\text{C}^2\right)\left(\frac{1}{0.600\text{ m}} + \frac{1}{0.150\text{ m}} + \frac{1}{\sqrt{(0.600\text{ m})^2 + (0.150\text{ m})^2}}\right)$$

$$U_e = \boxed{8.95\text{ J}}$$

25.23 $U = U_1 + U_2 + U_3 + U_4$



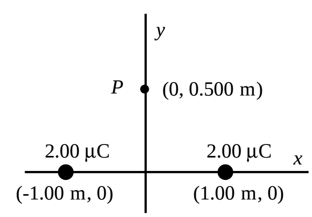
$U = 0 + U_{12} + (U_{13} + U_{23}) + (U_{14} + U_{24} + U_{34})$

$$U = 0 + \frac{k_eQ^2}{s} + \frac{k_eQ^2}{s}\left(\frac{1}{\sqrt{2}} + 1\right) + \frac{k_eQ^2}{s}\left(1 + \frac{1}{\sqrt{2}} + 1\right)$$

$$U = \frac{k_eQ^2}{s}\left(4 + \frac{2}{\sqrt{2}}\right) = \boxed{5.41\,\frac{k_e\,Q^2}{s}}$$

An alternate way to get the term $\left(4 + 2/\sqrt{2}\right)$ is to recognize that there are 4 side pairs and 2 face diagonal pairs.

***25.24** (a) $V = \dfrac{k_eq_1}{r_1} + \dfrac{k_eq_2}{r_2} = 2\left(\dfrac{k_eq}{r}\right) = 2\left(\dfrac{\left(8.99\times10^9\text{ N}\cdot\text{m}^2/\text{C}^2\right)\left(2.00\times10^{-6}\text{ C}\right)}{\sqrt{(1.00\text{ m})^2 + (0.500\text{ m})^2}}\right)$



$V = 3.22\times10^4\text{ V} = \boxed{32.2\text{ kV}}$

(b) $U = qV = \left(-3.00\times10^{-6}\text{ C}\right)\left(3.22\times10^4\ \dfrac{\text{J}}{\text{C}}\right) = \boxed{-9.65\times10^{-2}\text{ J}}$

***25.25** Each charge creates equal potential at the center. The total potential is:

$$V = 5\left[\frac{k_e(-q)}{R}\right] = \boxed{-\frac{5k_eq}{R}}$$

***25.26** (a) Each charge separately creates positive potential everywhere. The total potential produced by the three charges together is then the sum of three positive terms. There is $\boxed{\text{no point}}$ located at a finite distance from the charges, where this total potential is zero.

(b) $V = \frac{k_e q}{a} + \frac{k_e q}{a} = \boxed{\frac{2k_e q}{a}}$

25.27 (a) Conservation of momentum: $0 = m_1 v_1 \mathbf{i} + m_2 v_2 (-\mathbf{i})$ or $v_2 = \frac{m_1 v_1}{m_2}$

By conservation of energy, $0 + \frac{k_e(-q_1)q_2}{d} = \frac{1}{2} m_1 v_1^2 + \frac{1}{2} m_2 v_2^2 + \frac{k_e(-q_1)q_2}{(r_1 + r_2)}$

and $\frac{k_e q_1 q_2}{r_1 + r_2} - \frac{k_e q_1 q_2}{d} = \frac{1}{2} m_1 v_1^2 + \frac{1}{2} \frac{m_1^2 v_1^2}{m_2}$

$$v_1 = \sqrt{\frac{2m_2 k_e q_1 q_2}{m_1(m_1 + m_2)}\left(\frac{1}{r_1 + r_2} - \frac{1}{d}\right)}$$

$$v_1 = \sqrt{\frac{2(0.700\text{ kg})(8.99 \times 10^9\text{ N}\cdot\text{m}^2/\text{C}^2)(2 \times 10^{-6}\text{ C})(3 \times 10^{-6}\text{ C})}{(0.100\text{ kg})(0.800\text{ kg})}\left(\frac{1}{8 \times 10^{-3}\text{ m}} - \frac{1}{1.00\text{ m}}\right)} = \boxed{10.8\text{ m/s}}$$

$$v_2 = \frac{m_1 v_1}{m_2} = \frac{(0.100\text{ kg})(10.8\text{ m/s})}{0.700\text{ kg}} = \boxed{1.55\text{ m/s}}$$

(b) If the spheres are metal, electrons will move around on them with negligible energy loss to place the centers of excess charge on the insides of the spheres. Then just before they touch, the effective distance between charges will be less than $r_1 + r_2$ and the spheres will really be moving $\boxed{\text{faster than calculated in (a)}}$.

25.28 (a) Conservation of momentum: $0 = m_1 v_1 \mathbf{i} + m_2 v_2(-\mathbf{i})$ or $v_2 = m_1 v_1 / m_2$

By conservation of energy, $0 + \frac{k_e(-q_1)q_2}{d} = \frac{1}{2} m_1 v_1^2 + \frac{1}{2} m_2 v_2^2 + \frac{k_e(-q_1)q_2}{(r_1 + r_2)}$

and $\frac{k_e q_1 q_2}{r_1 + r_2} - \frac{k_e q_1 q_2}{d} = \frac{1}{2} m_1 v_1^2 + \frac{1}{2} \frac{m_1^2 v_1^2}{m_2}$

$$v_1 = \boxed{\sqrt{\frac{2m_2 k_e q_1 q_2}{m_1(m_1 + m_2)}\left(\frac{1}{r_1 + r_2} - \frac{1}{d}\right)}} \qquad v_2 = \left(\frac{m_1}{m_2}\right) v_1 = \boxed{\sqrt{\frac{2m_1 k_e q_1 q_2}{m_2(m_1 + m_2)}\left(\frac{1}{r_1 + r_2} - \frac{1}{d}\right)}}$$

(b) If the spheres are metal, electrons will move around on them with negligible energy loss to place the centers of excess charge on the insides of the spheres. Then just before they touch, the effective distance between charges will be less than $r_1 + r_2$ and the spheres will really be moving $\boxed{\text{faster than calculated in (a)}}$.

25.29 $V = \frac{k_e Q}{r}$ so $r = \frac{k_e Q}{V} = \frac{(8.99\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)(8.00\times10^{-9}\ \text{C})}{V} = \frac{72.0\ \text{V}\cdot\text{m}}{V}$

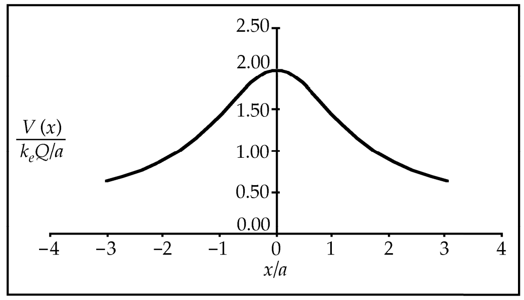
For $V = 100$ V, 50.0 V, and 25.0 V, $\boxed{r = 0.720\ \text{m},\ 1.44\ \text{m, and } 2.88\ \text{m}}$

The radii are $\boxed{\text{inversely proportional}}$ to the potential.

25.30 (a) $V(x) = \frac{k_e Q_1}{r_1} + \frac{k_e Q_2}{r_2} = \frac{k_e(+Q)}{\sqrt{x^2+a^2}} + \frac{k_e(+Q)}{\sqrt{x^2+(-a)^2}}$

$$V(x) = \frac{2k_e Q}{\sqrt{x^2+a^2}} = \frac{k_e Q}{a}\left(\frac{2}{\sqrt{(x/a)^2+1}}\right)$$

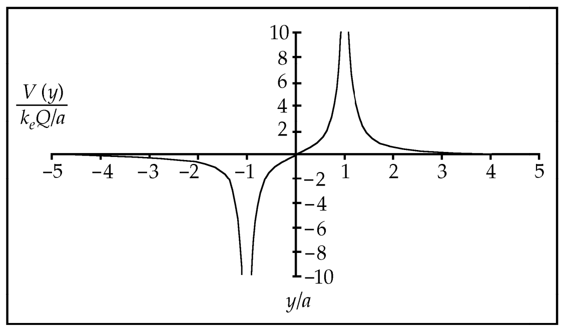
$$\frac{V(x)}{(k_e Q/a)} = \boxed{\frac{2}{\sqrt{(x/a)^2+1}}}$$



(b) $V(y) = \frac{k_e Q_1}{r_1} + \frac{k_e Q_2}{r_2} = \frac{k_e(+Q)}{|y-a|} + \frac{k_e(-Q)}{|y+a|}$

$$V(y) = \frac{k_e Q}{a}\left(\frac{1}{|y/a-1|} - \frac{1}{|y/a+1|}\right)$$

$$\frac{V(y)}{(k_e Q/a)} = \boxed{\left(\frac{1}{|y/a-1|} - \frac{1}{|y/a+1|}\right)}$$



25.31 Using conservation of energy, we have $K_f + U_f = K_i + U_i$.

But $U_i = \frac{k_e q_\alpha q_{\text{gold}}}{r_i}$, and $r_i \approx \infty$. Thus, $U_i = 0$.

Also $K_f = 0$ ($v_f = 0$ at turning point), so $U_f = K_i$, or $\frac{k_e q_\alpha q_{\text{gold}}}{r_{\min}} = \frac{1}{2} m_\alpha v_\alpha^2$

$$r_{\min} = \frac{2k_e q_\alpha q_{\text{gold}}}{m_\alpha v_\alpha^2} = \frac{2(8.99\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)(2)(79)(1.60\times10^{-19}\ \text{C})^2}{(6.64\times10^{-27}\ \text{kg})(2.00\times10^7\ \text{m/s})^2} = 2.74\times10^{-14}\ \text{m} = \boxed{27.4\ \text{fm}}$$

25.32 Using conservation of energy

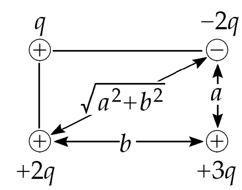
we have: $\dfrac{k_e eQ}{r_1} = \dfrac{k_e eQ}{r_2} + \frac{1}{2}mv^2$

which gives: $v = \sqrt{\dfrac{2k_e eQ}{m}\left(\dfrac{1}{r_1} - \dfrac{1}{r_2}\right)}$

or $v = \sqrt{\dfrac{(2)(8.99\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)(-1.60\times10^{-19}\ \text{C})(10^{-9}\ \text{C})}{9.11\times10^{-31}\ \text{kg}}\left(\dfrac{1}{0.0300\ \text{m}} - \dfrac{1}{0.0200\ \text{m}}\right)}$

Thus, $v = \boxed{7.26\times10^6\ \text{m/s}}$

25.33 $U = \sum \dfrac{k_e q_i q_j}{r_{ij}}$, summed over all pairs of (i, j) where $i \neq j$



$$U = k_e\left[\frac{q(-2q)}{b} + \frac{(-2q)(3q)}{a} + \frac{(2q)(3q)}{b} + \frac{q(2q)}{a} + \frac{q(3q)}{\sqrt{a^2+b^2}} + \frac{2q(-2q)}{\sqrt{a^2+b^2}}\right]$$

$$U = k_e q^2\left[\frac{-2}{0.400} - \frac{6}{0.200} + \frac{6}{0.400} + \frac{2}{0.200} + \frac{3}{0.447} - \frac{4}{0.447}\right]$$

$$U = \left(8.99\times10^9\right)\left(6.00\times10^{-6}\right)^2\left[\frac{4}{0.400} - \frac{4}{0.200} - \frac{1}{0.447}\right] = \boxed{-3.96\ \text{J}}$$

25.34 Each charge moves off on its diagonal line. All charges have equal speeds.

$$\sum(K+U)_i = \sum(K+U)_f$$

$$0 + \frac{4k_e q^2}{L} + \frac{2k_e q^2}{\sqrt{2}L} = 4\left(\tfrac{1}{2}mv^2\right) + \frac{4k_e q^2}{2L} + \frac{2k_e q^2}{2\sqrt{2}L}$$

$$\left(2 + \frac{1}{\sqrt{2}}\right)\frac{k_e q^2}{L} = 2mv^2$$

$$\boxed{v = \sqrt{\left(1 + \frac{1}{\sqrt{8}}\right)\frac{k_e q^2}{mL}}}$$

25.35 A cube has 12 edges and 6 faces. Consequently, there are 12 edge pairs separated by s, 2 × 6 = 12 face diagonal pairs separated by $\sqrt{2}\,s$, and 4 interior diagonal pairs separated $\sqrt{3}\,s$.

$$U = \frac{k_e q^2}{s}\left[12 + \frac{12}{\sqrt{2}} + \frac{4}{\sqrt{3}}\right] = \boxed{22.8\,\frac{k_e q^2}{s}}$$

25.36 $V = a + bx = 10.0\text{ V} + (-7.00\text{ V/m})x$

(a) At $x = 0$, $V = \boxed{10.0\text{ V}}$

At $x = 3.00\text{ m}$, $V = \boxed{-11.0\text{ V}}$

At $x = 6.00\text{ m}$, $V = \boxed{-32.0\text{ V}}$

(b) $E = -\dfrac{dV}{dx} = -b = -(-7.00\text{ V/m}) = \boxed{7.00\text{ N/C in } +x\text{ direction}}$

25.37 $V = 5x - 3x^2y + 2yz^2$ Evaluate E at (1, 0 – 2)

$$E_x = -\frac{\partial V}{\partial x} = \boxed{-5 + 6xy} = -5 + 6(1)(0) = -5$$

$$E_y = -\frac{\partial V}{\partial y} = \boxed{+3x^2 - 2z^2} = 3(1)^2 - 2(-2)^2 = -5$$

$$E_z = -\frac{\partial V}{\partial z} = \boxed{-4yz} = -4(0)(-2) = 0$$

$$E = \sqrt{{E_x}^2 + {E_y}^2 + {E_z}^2} = \sqrt{(-5)^2 + (-5)^2 + 0^2} = \boxed{7.07\text{ N/C}}$$

25.38 (a) For $r < R$ $\quad V = \dfrac{k_e Q}{R}$

$$E_r = -\frac{dV}{dr} = \boxed{0}$$

(b) For $r \geq R$ $\quad V = \dfrac{k_e Q}{r}$

$$E_r = -\frac{dV}{dr} = -\left(-\frac{k_e Q}{r^2}\right) = \boxed{\frac{k_e Q}{r^2}}$$

25.39 $$E_y = -\frac{\partial V}{\partial y} = -\frac{\partial}{\partial y}\left[\frac{k_e Q}{\ell}\ln\left(\frac{\ell + \sqrt{\ell^2 + y^2}}{y}\right)\right]$$

$$E_y = \frac{k_e Q}{\ell y}\left[1 - \frac{y^2}{\ell^2 + y^2 + \ell\sqrt{\ell^2 + y^2}}\right] = \boxed{\frac{k_e Q}{y\sqrt{\ell^2 + y^2}}}$$

25.40 Inside the sphere, $E_x = E_y = E_z = 0$.

Outside, $$E_x = -\frac{\partial V}{\partial x} = -\frac{\partial}{\partial x}\left(V_0 - E_0 z + E_0 a^3 z(x^2 + y^2 + z^2)^{-3/2}\right)$$

So $$E_x = -\left[0 + 0 + E_0 a^3 z(-3/2)(x^2 + y^2 + z^2)^{-5/2}(2x)\right] = \boxed{3E_0 a^3 xz(x^2 + y^2 + z^2)^{-5/2}}$$

$$E_y = -\frac{\partial V}{\partial y} = -\frac{\partial}{\partial y}\left(V_0 - E_0 z + E_0 a^3 z(x^2 + y^2 + z^2)^{-3/2}\right)$$

$$E_y = -E_0 a^3 z(-3/2)(x^2 + y^2 + z^2)^{-5/2}\, 2y = \boxed{3E_0 a^3 yz(x^2 + y^2 + z^2)^{-5/2}}$$

$$E_z = -\frac{\partial V}{\partial z} = E_0 - E_0 a^3 z(-3/2)(x^2 + y^2 + z^2)^{-5/2}(2z) - E_0 a^3 (x^2 + y^2 + z^2)^{-3/2}$$

$$E_z = \boxed{E_0 + E_0 a^3 (2z^2 - x^2 - y^2)(x^2 + y^2 + z^2)^{-5/2}}$$

***25.41** $$\Delta V = V_{2R} - V_0 = \frac{k_e Q}{\sqrt{R^2 + (2R)^2}} - \frac{k_e Q}{R} = \frac{k_e Q}{R}\left(\frac{1}{\sqrt{5}} - 1\right) = \boxed{-0.553\frac{k_e Q}{R}}$$

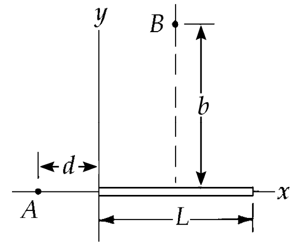
***25.42** $$V = \int dV = \frac{1}{4\pi\epsilon_0}\int\frac{dq}{r}$$

All bits of charge are at the same distance from O, so

$$V = \frac{1}{4\pi\epsilon_0}\frac{Q}{R} = \left(8.99\times 10^9\ \frac{\text{N}\cdot\text{m}^2}{\text{C}^2}\right)\frac{\left(-7.50\times 10^{-6}\ \text{C}\right)}{(0.140\ \text{m}/\pi)} = \boxed{-1.51\ \text{MV}}$$

25.43 (a) $[\alpha] = \left[\frac{\lambda}{x}\right] = \frac{\text{C}}{\text{m}} \cdot \left(\frac{1}{\text{m}}\right) = \boxed{\frac{\text{C}}{\text{m}^2}}$

(b) $V = k_e \int \frac{dq}{r} = k_e \int \frac{\lambda\, dx}{r} = k_e \alpha \int_0^L \frac{x\, dx}{(d+x)} = \boxed{k_e \alpha \left[L - d \ln\left(1 + \frac{L}{d}\right)\right]}$



25.44 $V = \int \frac{k_e\, dq}{r} = k_e \int \frac{\alpha x\, dx}{\sqrt{b^2 + (L/2 - x)^2}}$

Let $z = \frac{L}{2} - x$. Then $x = \frac{L}{2} - z$, and $dx = -dz$

$$V = k_e\alpha \int \frac{(L/2 - z)(-dz)}{\sqrt{b^2 + z^2}} = -\frac{k_e \alpha L}{2} \int \frac{dz}{\sqrt{b^2+z^2}} + k_e\alpha \int \frac{z\, dz}{\sqrt{b^2+z^2}} = -\frac{k_e\alpha L}{2} \ln\left(z + \sqrt{z^2+b^2}\right) + k_e\alpha\sqrt{z^2+b^2}$$

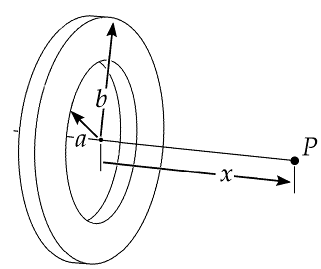
$$V = -\frac{k_e\alpha L}{2} \ln\left[(L/2 - x) + \sqrt{(L/2 - x)^2 + b^2}\,\right]\Bigg|_0^L + k_e\alpha\sqrt{(L/2-x)^2 + b^2}\,\Bigg|_0^L$$

$$V = -\frac{k_e\alpha L}{2} \ln\left[\frac{L/2 - L + \sqrt{(L/2)^2 + b^2}}{L/2 + \sqrt{(L/2)^2+b^2}}\right] + k_e\alpha\left[\sqrt{(L/2 - L)^2 + b^2} - \sqrt{(L/2)^2 + b^2}\right]$$

$$V = \boxed{-\frac{k_e\alpha L}{2} \ln\left[\frac{\sqrt{b^2 + (L^2/4)} - L/2}{\sqrt{b^2 + (L^2/4)} + L/2}\right]}$$

25.45 $dV = \frac{k_e\, dq}{\sqrt{r^2 + x^2}}$ where $dq = \sigma\, dA = \sigma 2\pi r\, dr$

$$V = 2\pi\sigma k_e \int_a^b \frac{r\, dr}{\sqrt{r^2 + x^2}} = \boxed{2\pi k_e \sigma\left[\sqrt{x^2 + b^2} - \sqrt{x^2 + a^2}\right]}$$



25.46 $V = k_e \int_{\text{all charge}} \frac{dq}{r} = k_e \int_{-3R}^{-R} \frac{\lambda\, dx}{-x} + k_e \int_{\text{semicircle}} \frac{\lambda\, ds}{R} + k_e \int_R^{3R} \frac{\lambda\, dx}{x}$

$$V = -k_e\lambda \ln(-x)\Big|_{-3R}^{-R} + \frac{k_e\lambda}{R}\pi R + k_e\lambda \ln x\Big|_R^{3R}$$

$$V = k_e\lambda \ln\frac{3R}{R} + k_e\lambda\pi + k_e\lambda \ln 3 = \boxed{k_e\lambda(\pi + 2\ln 3)}$$

25.47 Substituting given values into $V = \frac{k_e q}{r}$, $7.50 \times 10^3 \text{ V} = \frac{(8.99 \times 10^9 \text{ N}\cdot\text{m}^2/\text{C}^2)\, q}{(0.300 \text{ m})}$

Substituting $q = 2.50 \times 10^{-7}$ C, $N = \frac{2.50 \times 10^{-7} \text{ C}}{1.60 \times 10^{-19} \text{ C}/e^-} = \boxed{1.56 \times 10^{12} \text{ electrons}}$

25.48 $q_1 + q_2 = 20.0\ \mu\text{C}$ so $q_1 = 20.0\ \mu\text{C} - q_2$

$\frac{q_1}{q_2} = \frac{r_1}{r_2}$ so $\frac{20.0\ \mu\text{C} - q_2}{q_2} = \frac{4.00 \text{ cm}}{6.00 \text{ cm}}$

Therefore $6.00(20.0\ \mu\text{C} - q_2) = 4.00 q_2$;

Solving, $q_2 = 12.0\ \mu\text{C}$ and $q_1 = 20.0\ \mu\text{C} - 12.0\ \mu\text{C} = 8.00\ \mu\text{C}$

(a) $E_1 = \frac{k_e q_1}{r_1^2} = \frac{(8.99 \times 10^9)(8.00 \times 10^{-6})}{(0.0400)^2} = 4.50 \times 10^7 \text{ V/m} = \boxed{45.0 \text{ MV/m}}$

$E_2 = \frac{k_e q_2}{r_2^2} = \frac{(8.99 \times 10^9)(12.0 \times 10^{-6})}{(0.0600)^2} = 3.00 \times 10^7 \text{ V/m} = \boxed{30.0 \text{ MV/m}}$

(b) $V_1 = V_2 = \frac{k_e q_2}{r_2} = \boxed{1.80 \text{ MV}}$

25.49 (a) $E = \boxed{0}$; $V = \frac{k_e q}{R} = \frac{(8.99 \times 10^9)(26.0 \times 10^{-6})}{0.140} = \boxed{1.67 \text{ MV}}$

(b) $E = \frac{k_e q}{r^2} = \frac{(8.99 \times 10^9)(26.0 \times 10^{-6})}{(0.200)^2} = \boxed{5.84 \text{ MN/C}}$ away

$V = \frac{k_e q}{r} = \frac{(8.99 \times 10^9)(26.0 \times 10^{-6})}{(0.200)} = \boxed{1.17 \text{ MV}}$

(c) $E = \frac{k_e q}{R^2} = \frac{(8.99 \times 10^9)(26.0 \times 10^{-6})}{(0.140)^2} = \boxed{11.9 \text{ MN/C}}$ away

$V = \frac{k_e q}{R} = \boxed{1.67 \text{ MV}}$

25.50 No charge stays on the inner sphere in equilibrium. If there were any, it would create an electric field in the wire to push more charge to the outer sphere. Charge Q is on the outer sphere. Therefore, $\boxed{\text{zero charge is on the inner sphere}}$ and $\boxed{10.0\ \mu\text{C is on the outer sphere}}$.

25.51 (a) $E_{\max} = 3.00 \times 10^6 \text{ V / m} = \dfrac{k_e Q}{r^2} = \dfrac{k_e Q}{r}\dfrac{1}{r} = V_{\max}\dfrac{1}{r}$

$$V_{\max} = E_{\max} r = 3.00 \times 10^6 (0.150) = \boxed{450 \text{ kV}}$$

(b) $\dfrac{k_e Q_{\max}}{r^2} = E_{\max}$ $\left\{\text{or } \dfrac{k_e Q_{\max}}{r} = V_{\max}\right\}$

$$Q_{\max} = \frac{E_{\max} r^2}{k_e} = \frac{3.00 \times 10^6 (0.150)^2}{8.99 \times 10^9} = \boxed{7.51\ \mu\text{C}}$$

Goal Solution

Consider a Van de Graaff generator with a 30.0-cm-diameter dome operating in dry air. (a) What is the maximum potential of the dome? (b) What is the maximum charge on the dome?

G: Van de Graaff generators produce voltages that can make your hair stand on end, somewhere on the order of about 100 kV (see the Puzzler at beginning of Chapter 25). With these high voltages, the maximum charge on the dome is probably more than typical point charge values of about $1\ \mu\text{C}$.

The maximum potential and charge will be limited by the electric field strength at which the air surrounding the dome will ionize. This critical value is determined by the **dielectric strength** of air which, from page 789 or from Table 26.1, is $E_{\text{critical}} = 3 \times 10^6 \text{ V / m}$. An electric field stronger than this will cause the air to act like a conductor instead of an insulator. This process is called dielectric breakdown and may be seen as a spark.

O: From the maximum allowed electric field, we can find the charge and potential that would create this situation. Since we are only given the diameter of the dome, we will assume that the conductor is spherical, which allows us to use the electric field and potential equations for a spherical conductor. With these equations, it will be easier to do part (b) first and use the result for part (a).

A: (b) For a spherical conductor with total charge Q, $|\mathbf{E}| = \dfrac{k_e Q}{r^2}$

$$Q = \frac{Er^2}{k_e} = \frac{\left(3.00 \times 10^6 \text{ V / m}\right)(0.150 \text{ m})^2}{8.99 \times 10^9 \text{ N} \cdot \text{m}^2 / \text{C}^2}(1 \text{ N} \cdot \text{m / V} \cdot \text{C}) = 7.51\ \mu\text{C}$$

(a) $$V = \frac{k_e Q}{r} = \frac{(8.99 \times 10^9 \text{ N} \cdot \text{m}^2 / \text{C}^2)(7.51 \times 10^{-6} \text{ C})}{0.150 \text{ m}} = 450 \text{ kV}$$

L: These calculated results seem reasonable based on our predictions. The voltage is about 4000 times larger than the 120 V found from common electrical outlets, but the charge is similar in magnitude to many of the static charge problems we have solved earlier. This implies that most of these charge configurations would have to be in a vacuum because the electric field near these point charges would be strong enough to cause sparking in air. (Example: A charged ball with $Q = 1\ \mu\text{C}$ and $r = 1 \text{ mm}$ would have an electric field near its surface of

$$E = \frac{k_e Q}{r^2} = \frac{\left(9 \times 10^9 \text{ N} \cdot \text{m}^2/\text{C}^2\right)\left(1 \times 10^{-6} \text{ C}\right)}{(0.001 \text{ m})^2} = 9 \times 10^9 \text{ V / m}$$

which is well beyond the dielectric breakdown of air!)

25.52 $V = \frac{k_e q}{r}$ and $E = \frac{k_e q}{r^2}$ Since $E = \frac{V}{r}$,

(b) $r = \frac{V}{E} = \frac{6.00 \times 10^5 \text{ V}}{3.00 \times 10^6 \text{ V/m}} = \boxed{0.200 \text{ m}}$ and

(a) $q = \frac{Vr}{k_e} = \boxed{13.3\ \mu\text{C}}$

25.53 $U = qV = k_e \frac{q_1 q_2}{r_{12}} = (8.99 \times 10^9) \frac{(38)(54)(1.60 \times 10^{-19})^2}{(5.50 + 6.20) \times 10^{-15}} = 4.04 \times 10^{-11} \text{ J} = \boxed{253 \text{ MeV}}$

***25.54** (a) To make a spark 5 mm long in dry air between flat metal plates requires potential difference

$$V = Ed = \left(3.0 \times 10^6 \text{ V/m}\right)\left(5.0 \times 10^{-3} \text{ m}\right) = 1.5 \times 10^4 \text{ V} \quad \boxed{\sim 10^4 \text{ V}}$$

(b) Suppose your surface area is like that of a 70-kg cylinder with the density of water and radius 12 cm. Its length would be given by

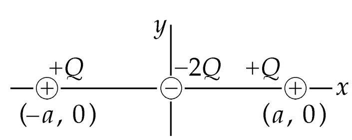
$$70 \times 10^3 \text{ cm}^3 = \pi(12 \text{ cm})^2 l \qquad l = 1.6 \text{ m}$$

The lateral surface area is $A = 2\pi r l = 2\pi(0.12 \text{ m})(1.6 \text{ m}) = 1.2 \text{ m}^2$

The electric field close to your skin is described by $E = \frac{\sigma}{\epsilon_0} = \frac{Q}{A\epsilon_0}$, so

$$Q = EA\epsilon_0 = \left(3.0 \times 10^6 \frac{\text{N}}{\text{C}}\right)\left(1.2 \text{ m}^2\right)\left(8.85 \times 10^{-12} \frac{\text{C}^2}{\text{N} \cdot \text{m}^2}\right) \quad \boxed{\sim 10^{-5} \text{ C}}$$

25.55 (a) $V = k_e Q \left(\frac{1}{x+a} - \frac{2}{x} + \frac{1}{x-a}\right)$



$$V = k_e Q \left[\frac{x(x-a) - 2(x+a)(x-a) + x(x+a)}{x(x+a)(x-a)}\right] = \boxed{\frac{2k_e Q a^2}{x^3 - xa^2}}$$

(b) $V = \boxed{\frac{2k_e Q a^2}{x^3}}$ for $\frac{a}{x} \ll 1$

25.56 (a) $E_x = -\frac{dV}{dx} = -\frac{d}{dx}\left(\frac{2k_eQa^2}{x^3 - xa^2}\right) = \boxed{\frac{(2k_eQa^2)(3x^2 - a^2)}{(x^3 - xa^2)^2}}$ and $\boxed{E_y = E_z = 0}$

(b) $E_x = \frac{2(8.99\times 10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)(3\times 10^{-6}\ \text{C})(2\times 10^{-3}\ \text{m})^2\left[3(6\times 10^{-3}\ \text{m})^2 - (2\times 10^{-3}\ \text{m})^2\right]}{\left[(6\times 10^{-3}\ \text{m})^3 - (6\times 10^{-3}\ \text{m})(2\times 10^{-3}\ \text{m})^2\right]^2}$

$E_x = 609\times 10^6\ \text{N/C} = \boxed{609\ \text{MN/C}}$

25.57 (a) $E = \frac{Q}{4\pi\epsilon_0 r^2}$

$V = \frac{Q}{4\pi\epsilon_0 r}$

$r = \frac{|V|}{|E|} = \frac{3000\ \text{V}}{500\ \text{V / m}} = \boxed{6.00\ \text{m}}$

(b) $V = -3000\ \text{V} = \frac{Q}{4\pi\epsilon_0 (6.00\ \text{m})}$

$Q = \frac{-3000\ \text{V}}{(8.99\times 10^9\ \text{V}\cdot\text{m / C})}(6.00\ \text{m}) = \boxed{-2.00\ \mu\text{C}}$

25.58 From Example 25.5, the potential created by the ring at the electron's starting point is

$V_i = \frac{k_eQ}{\sqrt{x_i^2 + a^2}} = \frac{k_e(2\pi\lambda a)}{\sqrt{x_i^2 + a^2}}$

while at the center, it is $V_f = 2\pi k_e\lambda$. From conservation of energy,

$0 + (-eV_i) = \frac{1}{2}m_e v_f^2 + (-eV_f)$

$v_f^2 = \frac{2e}{m_e}(V_f - V_i) = \frac{4\pi e k_e\lambda}{m_e}\left(1 - \frac{a}{\sqrt{x_i^2 + a^2}}\right)$

$v_f^2 = \frac{4\pi(1.60\times 10^{-19})(8.99\times 10^9)(1.00\times 10^{-7})}{9.11\times 10^{-31}}\left(1 - \frac{0.200}{\sqrt{(0.100)^2 + (0.200)^2}}\right)$

$v_f = \boxed{1.45\times 10^7\ \text{m/s}}$

25.59 (a) Take the origin at the point where we will find the potential. One ring, of width dx, has charge $Q\,dx/h$ and, according to Example 25.5, creates potential

$$dV = \frac{k_e Q\,dx}{h\sqrt{x^2 + R^2}}$$

The whole stack of rings creates potential

$$V = \int_{\text{all charge}} dV = \int_d^{d+h} \frac{k_e Q\,dx}{h\sqrt{x^2+R^2}} = \frac{k_e Q}{h}\ln\left(x + \sqrt{x^2+R^2}\right)\Bigg|_d^{d+h} = \boxed{\frac{k_e Q}{h}\ln\left(\frac{d+h+\sqrt{(d+h)^2+R^2}}{d+\sqrt{d^2+R^2}}\right)}$$

(b) A disk of thickness dx has charge $Q\,dx/h$ and charge-per-area $Q\,dx/\pi R^2 h$. According to Example 25.6, it creates potential

$$dV = 2\pi k_e \frac{Q\,dx}{\pi R^2 h}\left(\sqrt{x^2+R^2} - x\right)$$

Integrating,

$$V = \int_d^{d+h} \frac{2k_e Q}{R^2 h}\left(\sqrt{x^2+R^2}\,dx - x\,dx\right) = \frac{2k_e Q}{R^2 h}\left[\frac{1}{2}x\sqrt{x^2+R^2} + \frac{R^2}{2}\ln\left(x+\sqrt{x^2+R^2}\right) - \frac{x^2}{2}\right]_d^{d+h}$$

$$V = \boxed{\frac{k_e Q}{R^2 h}\left[(d+h)\sqrt{(d+h)^2+R^2} - d\sqrt{d^2+R^2} - 2dh - h^2 + R^2\ln\left(\frac{d+h+\sqrt{(d+h)^2+R^2}}{d+\sqrt{d^2+R^2}}\right)\right]}$$

25.60 The positive plate by itself creates a field $E = \dfrac{\sigma}{2\epsilon_0} = \dfrac{36.0\times10^{-9}\ \text{C/m}^2}{2(8.85\times10^{-12}\ \text{C}^2/\text{N}\cdot\text{m}^2)} = 2.03\ \dfrac{\text{kN}}{\text{C}}$

away from the + plate. The negative plate by itself creates the same size field and between the plates it is in the same direction. Together the plates create a uniform field 4.07 kN/C in the space between.

(a) Take $V = 0$ at the negative plate. The potential at the positive plate is then

$$V - 0 = -\int_0^{12.0\ \text{cm}} (-4.07\ \text{kN/C})\,dx$$

The potential difference between the plates is $V = (4.07\times10^3\ \text{N/C})(0.120\ \text{m}) = \boxed{488\ \text{V}}$

(b) $\left(\frac{1}{2}mv^2 + qV\right)_i = \left(\frac{1}{2}mv^2 + qV\right)_f$

$$qV = (1.60\times10^{-19}\ \text{C})(488\ \text{V}) = \frac{1}{2}mv_f^2 = \boxed{7.81\times10^{-17}\ \text{J}}$$

(c) $v_f = \boxed{306\ \text{km/s}}$

(d) $v_f^2 = v_i^2 + 2a(x - x_i)$

$(3.06 \times 10^5 \text{ m/s})^2 = 0 + 2a(0.120 \text{ m})$

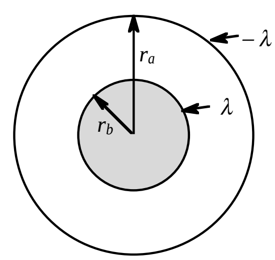
$a = \boxed{3.90 \times 10^{11} \text{ m/s}^2}$

(e) $\sum F = ma = (1.67 \times 10^{-27} \text{ kg})(3.90 \times 10^{11} \text{ m/s}^2) = \boxed{6.51 \times 10^{-16} \text{ N}}$

(f) $E = \dfrac{F}{q} = \dfrac{6.51 \times 10^{-16} \text{ N}}{1.60 \times 10^{-19} \text{ C}} = \boxed{4.07 \text{ kN/C}}$

25.61 $W = \int_0^Q V\,dq$ where $V = \dfrac{k_e q}{R}$; Therefore, $\boxed{W = \dfrac{k_e Q^2}{2R}}$

25.62 (a) $V_B - V_A = -\int_A^B \mathbf{E} \cdot d\mathbf{s}$ and the field at distance r from a uniformly charged rod (where $r >$ radius of charged rod) is



$$E = \frac{\lambda}{2\pi \epsilon_0 r} = \frac{2k_e\lambda}{r}$$

In this case, the field between the central wire and the coaxial cylinder is directed perpendicular to the line of charge so that

$$V_B - V_A = -\int_{r_a}^{r_b} \frac{2k_e\lambda}{r}\,dr = 2k_e\lambda \ln\left(\frac{r_a}{r_b}\right), \quad \text{or} \quad \boxed{\Delta V = 2k_e\lambda \ln\left(\frac{r_a}{r_b}\right)}$$

(b) From part (a), when the outer cylinder is considered to be at zero potential, the potential at a distance r from the axis is

$$V = 2k_e\lambda \ln\left(\frac{r_a}{r}\right)$$

The field at r is given by $E = -\dfrac{\partial V}{\partial r} = -2k_e\lambda\left(\dfrac{r}{r_a}\right)\left(-\dfrac{r_a}{r^2}\right) = \dfrac{2k_e\lambda}{r}$

But, from part (a), $2k_e\lambda = \dfrac{\Delta V}{\ln(r_a/r_b)}$. Therefore, $\boxed{E = \dfrac{\Delta V}{\ln(r_a/r_b)}\left(\dfrac{1}{r}\right)}$

25.63 $V_2 - V_1 = -\int_{r_1}^{r_2} \mathbf{E} \cdot d\mathbf{r} = -\int_{r_1}^{r_2} \frac{\lambda}{2\pi\varepsilon_0 r} dr$

$$V_2 - V_1 = \boxed{\frac{-\lambda}{2\pi\epsilon_0} \ln\left(\frac{r_2}{r_1}\right)}$$

25.64 For the given charge distribution, $V(x,y,z) = \frac{k_e(q)}{r_1} + \frac{k_e(-2q)}{r_2}$

where $r_1 = \sqrt{(x+R)^2 + y^2 + z^2}$ and $r_2 = \sqrt{x^2 + y^2 + z^2}$

The surface on which $V(x,y,z) = 0$

is given by $k_e q\left(\frac{1}{r_1} - \frac{2}{r_2}\right) = 0$, or $2r_1 = r_2$

This gives: $4(x+R)^2 + 4y^2 + 4z^2 = x^2 + y^2 + z^2$

which may be written in the form: $x^2 + y^2 + z^2 + \left(\frac{8}{3}R\right)x + (0)y + (0)z + \left(\frac{4}{3}R^2\right) = 0$ [1]

The general equation for a sphere of radius a centered at (x_0, y_0, z_0) is:

$$(x - x_0)^2 + (y - y_0)^2 + (z - z_0)^2 - a^2 = 0$$

or $x^2 + y^2 + z^2 + (-2x_0)x + (-2y_0)y + (-2z_0)z + \left(x_0^2 + y_0^2 + z_0^2 - a^2\right) = 0$ [2]

Comparing equations [1] and [2], it is seen that the equipotential surface for which $V = 0$ is indeed a sphere and that:

$$-2x_0 = \frac{8}{3}R; \quad -2y_0 = 0; \quad -2z_0 = 0; \quad x_0^2 + y_0^2 + z_0^2 - a^2 = \frac{4}{3}R^2$$

Thus, $x_0 = -\frac{4}{3}R$, $y_0 = z_0 = 0$, and $a^2 = \left(\frac{16}{9} - \frac{4}{3}\right)R^2 = \frac{4}{9}R^2$.

The equipotential surface is therefore a sphere centered at $\boxed{\left(-\frac{4}{3}R, 0, 0\right)}$, having a radius $\boxed{\frac{2}{3}R}$

25.65 (a) From Gauss's law, $\boxed{E_A = 0}$ (no charge within)

$$E_B = k_e \frac{q_A}{r^2} = (8.99 \times 10^9)\frac{(1.00 \times 10^{-8})}{r^2} = \boxed{\left(\frac{89.9}{r^2}\right) \text{V / m}}$$

$$E_C = k_e \frac{(q_A + q_B)}{r^2} = (8.99 \times 10^9)\frac{(-5.00 \times 10^{-9})}{r^2} = \boxed{\left(-\frac{45.0}{r^2}\right) \text{V / m}}$$

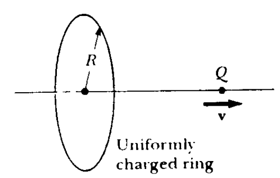
(b) $$V_C = k_e \frac{(q_A + q_B)}{r} = (8.99 \times 10^9)\frac{(-5.00 \times 10^{-9})}{r} = \boxed{\left(-\frac{45.0}{r}\right) \text{V}}$$

$$\therefore \text{ At } r_2, \; V = -\frac{45.0}{0.300} = -150 \text{ V}$$

$$\text{Inside } r_2, \; V_B = -150 \text{ V} + \int_{r_2}^{r} \frac{89.9}{r^2}\, dr = -150 + 89.9\left(\frac{1}{r} - \frac{1}{0.300}\right) = \boxed{\left(-450 + \frac{89.9}{r}\right) \text{V}}$$

$$\therefore \text{ At } r_1, \; V = -450 + \frac{89.9}{0.150} = +150 \text{ V} \quad \text{so} \quad \boxed{V_A = +150 \text{ V}}$$

25.66 From Example 25.5, the potential at the center of the ring is $V_i = k_e Q/R$ and the potential at an infinite distance from the ring is $V_f = 0$. Thus, the initial and final potential energies of the point charge are:



$$U_i = QV_i = \frac{k_e Q^2}{R} \quad \text{and} \quad U_f = QV_f = 0$$

From conservation of energy, $K_f + U_f = K_i + U_i$

$$\text{or} \quad \tfrac{1}{2} M v_f^2 + 0 = 0 + \frac{k_e Q^2}{R} \quad \text{giving} \quad v_f = \boxed{\sqrt{\frac{2k_e Q^2}{MR}}}$$

25.67 The sheet creates a field $\mathbf{E}_1 = \dfrac{\sigma}{2\in_0}\mathbf{i}$ for $x > 0$. Along the $x-$axis, the line of charge creates a field

$$\mathbf{E}_2 = \frac{\lambda}{2\pi r \in_0} \text{ away} = \frac{\lambda}{2\pi \in_0 (3.00 \text{ m} - x)}(-\mathbf{i}) \text{ for } x < 3.00 \text{ m}$$

The total field along the $x-$axis in the region $0 < x < 3.00$ m is then

$$\mathbf{E} = \mathbf{E}_1 + \mathbf{E}_2 = \left[\frac{\sigma}{2\in_0} - \frac{\lambda}{2\pi\in_0 (3.00 - x)}\right]\mathbf{i}$$

(a) The potential at point x follows from

$$V - V_0 = -\int_0^x \mathbf{E}\cdot\mathbf{i}dx = -\int_0^x \left[\frac{\sigma}{2\,\epsilon_0} - \frac{\lambda}{2\pi\epsilon_0\,(3.00 - x)}\right]dx$$

$$V = V_0 - \frac{\sigma x}{2\epsilon_0} - \frac{\lambda}{2\pi\epsilon_0}\ln\left(1 - \frac{x}{3.00}\right)$$

$$V = 1.00\text{ kV} - \frac{(25.0\times 10^{-9}\text{ C / m}^2)x}{2(8.85\times 10^{-12}\text{ C}^2\text{ / N}\cdot\text{m}^2)} - \frac{80.0\times 10^{-9}\text{ C / m}}{2\pi(8.85\times 10^{-12}\text{ C}^2\text{ / N}\cdot\text{m}^2)}\ln\left(1 - \frac{x}{3.00}\right)$$

$$V = \boxed{1.00\text{ kV} - \left(1.41\ \frac{\text{kV}}{\text{m}}\right)x - (1.44\text{ kV})\ln\left(1.00 - \frac{x}{3.00\text{ m}}\right)}$$

(b) At $x = 0.800$ m, $V = 316$ V

and $U = QV = \left(2.00\times 10^{-9}\text{ C}\right)\left(316\text{ J/C}\right) = 6.33\times 10^{-7}\text{ J} = \boxed{633\text{ nJ}}$

25.68 $$V = k_e \int_a^{a+L} \frac{\lambda\,dx}{\sqrt{x^2 + b^2}} = k_e\lambda\,\ln\left[x + \sqrt{(x^2 + b^2)}\right]\Big|_a^{a+L} = \boxed{k_e\lambda\,\ln\left[\frac{a + L + \sqrt{(a+L)^2 + b^2}}{a + \sqrt{a^2 + b^2}}\right]}$$

25.69 (a) $E_r = -\dfrac{\partial V}{\partial r} = \boxed{\dfrac{2k_e p\cos\theta}{r^3}}$

In spherical coordinates, the θ component of the gradient is $\dfrac{1}{r}\left(\dfrac{\partial}{\partial\theta}\right)$.

Therefore, $E_\theta = -\dfrac{1}{r}\left(\dfrac{\partial V}{\partial\theta}\right) = \boxed{\dfrac{k_e p\sin\theta}{r^3}}$

For $r >> a$, $E_r(0°) = \dfrac{2k_e p}{r^3}$ and $E_r(90°) = 0$, $E_\theta(0°) = 0$ and $E_\theta(90°) = \dfrac{k_e p}{r^3}$

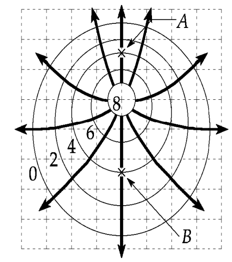
These results are $\boxed{\text{reasonable for } r >> a}$.

However, for $\boxed{r \to 0,\ E(0) \to \infty}$.

(b) $V = \boxed{\dfrac{k_e py}{(x^2 + y^2)^{3/2}}}$ and $\boxed{E_x = -\dfrac{\partial V}{\partial x} = \dfrac{3k_e pxy}{(x^2 + y^2)^{5/2}}}$

$E_y = -\dfrac{\partial V}{\partial y} = \boxed{\dfrac{k_e p(2y^2 - x^2)}{(x^2 + y^2)^{5/2}}}$

25.70 (a) $\boxed{E_A > E_B \text{ since } E = \dfrac{\Delta V}{\Delta s}}$



(b) $E_B = -\dfrac{\Delta V}{\Delta s} = -\dfrac{(6-2)\text{ V}}{2\text{ cm}} = \boxed{200\text{ N/C}}$ down

(c) The figure is shown to the right, with sample field lines sketched in.

25.71 For an element of area which is a ring of radius r and width dr, $dV = \dfrac{k_e\,dq}{\sqrt{r^2+x^2}}$

$dq = \sigma\,dA = Cr(2\pi r\,dr)$ and

$$V = C(2\pi k_e)\int_0^R \frac{r^2\,dr}{\sqrt{r^2+x^2}} = \boxed{C(\pi k_e)\left[R\sqrt{R^2+x^2} + x^2\ln\left(\frac{x}{R+\sqrt{R^2+x^2}}\right)\right]}$$

25.72 $dU = V\,dq$ where the potential $V = \dfrac{k_e q}{r}$.

The element of charge in a shell is $dq = \rho$ (volume element) or $dq = \rho(4\pi r^2\,dr)$ and the charge q in a sphere of radius r is

$$q = 4\pi\rho\int_0^r r^2\,dr = \rho\left(\frac{4\pi r^3}{3}\right)$$

Substituting this into the expression for dU, we have

$$dU = \left(\frac{k_e q}{r}\right)dq = k_e\rho\left(\frac{4\pi r^3}{3}\right)\left(\frac{1}{r}\right)\rho(4\pi r^2\,dr) = k_e\left(\frac{16\pi^2}{3}\right)\rho^2 r^4\,dr$$

$$U = \int dU = k_e\left(\frac{16\pi^2}{3}\right)\rho^2\int_0^R r^4\,dr = k_e\left(\frac{16\pi^2}{15}\right)\rho^2 R^5$$

But the *total* charge, $Q = \rho\frac{4}{3}\pi R^3$. Therefore, $\boxed{U = \dfrac{3}{5}\dfrac{k_e Q^2}{R}}$

***25.73** (a) From Problem 62, $E = \frac{\Delta V}{\ln(r_a/r_b)} \frac{1}{r}$

We require just outside the central wire $5.50 \times 10^6 \ \frac{\text{V}}{\text{m}} = \frac{50.0 \times 10^3 \ \text{V}}{\ln\left(\frac{0.850 \ \text{m}}{r_b}\right)} \left(\frac{1}{r_b}\right)$

or $\left(110 \ \text{m}^{-1}\right) r_b \ln\left(\frac{0.850 \ \text{m}}{r_b}\right) = 1$

We solve by homing in on the required value

r_b (m)	0.0100	0.00100	0.00150	0.00145	0.00143	0.00142
$\left(110 \ \text{m}^{-1}\right) r_b \ln\left(\frac{0.850 \ \text{m}}{r_b}\right)$	4.89	0.740	1.05	1.017	1.005	0.999

Thus, to three significant figures, $\boxed{r_b = 1.42 \ \text{mm}}$

(b) At r_a, $E = \frac{50.0 \ \text{kV}}{\ln(0.850 \ \text{m}/0.00142 \ \text{m})} \left(\frac{1}{0.850 \ \text{m}}\right) = \boxed{9.20 \ \text{kV/m}}$