Chapter 19 Solutions

***19.1** (a) To convert from Fahrenheit to Celsius, we use

$$T_C = \frac{5}{9}(T_F - 32.0) = \frac{5}{9}(98.6 - 32.0) = \boxed{37.0°C}$$

and the Kelvin temperature is found as

$$T = T_C + 273 = \boxed{310\text{ K}}$$

(b) In a fashion identical to that used in (a), we find

$$T_C = \boxed{-20.6°C} \quad \text{and} \quad T = \boxed{253\text{ K}}$$

19.2 $P_1 V = nRT_1$ and $P_2 V = nRT_2$

imply that $\frac{P_2}{P_1} = \frac{T_2}{T_1}$

(a) $P_2 = \frac{P_1 T_2}{T_1} = \frac{(0.980\text{ atm})(273 + 45.0)\text{K}}{(273 + 20.0)\text{K}} = \boxed{1.06\text{ atm}}$

(b) $T_3 = \frac{T_1 P_3}{P_1} = \frac{(293\text{ K})(0.500\text{ atm})}{(0.980\text{ atm})} = 149\text{ K} = \boxed{-124°C}$

19.3 Since we have a linear graph, the pressure is related to the temperature as $P = A + BT$, where A and B are constants. To find A and B, we use the data

$0.900\text{ atm} = A + (-80.0°C)B$ (1)

$1.635\text{ atm} = A + (78.0°C)B$ (2)

Solving (1) and (2) simultaneously, we find

$A = 1.272\text{ atm}$

and $B = 4.652 \times 10^{-3}\text{ atm}/°C$

Therefore, $P = 1.272\text{ atm} + (4.652 \times 10^{-3}\text{ atm}/°C)T$

(a) At absolute zero

$$P = 0 = 1.272\text{ atm} + (4.652 \times 10^{-3}\text{ atm}/°C)T$$

which gives $\boxed{T = -274°C}$

(b) At the freezing point of water

$$P = 1.272 \text{ atm} + 0 = \boxed{1.27 \text{ atm}}$$

(c) and at the boiling point

$$P = 1.272 \text{ atm} + (4.652 \times 10^{-3} \text{ atm}/°\text{C})(100°\text{C}) = \boxed{1.74 \text{ atm}}$$

19.4 Let us use $T_C = \frac{5}{9}(T_F - 32.0)$ with $T_F = -40.0°C$. We find

$$T_C = \frac{5}{9}(-40.0 - 32.0) = -40.0°\text{C}$$

19.5 (a) $T_F = \frac{9}{5} T_C + 32.0°\text{F} = \frac{9}{5}(-195.81) + 32.0 = \boxed{-320°\text{F}}$

(b) $T = T_C + 273.15 = -195.81 + 273.15 = \boxed{77.3 \text{ K}}$

19.6 Require $0.00°\text{C} = a(-15.0°\text{S}) + b$

$100°\text{C} = a(60.0°\text{S}) + b$

Subtracting, $100°\text{C} = a(75.0°\text{S})$

$a = 1.33 \text{ C°}/\text{S°}$

Then $0.00°\text{C} = 1.33(-15.0°\text{S})\text{C°} + b$

$b = 20.0°\text{C}$

So the conversion is $\boxed{T_C = (1.33 \text{ C°}/\text{S°})T_S + 20.0°\text{C}}$

19.7 (a) $\Delta T = 450 \text{ C°} = 450 \text{ C°}\left(\frac{212°\text{F} - 32.0°\text{F}}{100°\text{C} - 0.00°\text{C}}\right) = \boxed{810 \text{ F°}}$

(b) $\Delta T = 450 \text{ C°} = \boxed{450 \text{ K}}$

19.8 (a) $T = 1064 + 273 = \boxed{1337 \text{ K}}$ melting point

$T = 2660 + 273 = \boxed{2933 \text{ K}}$ boiling point

(b) $\Delta T = \boxed{1596 \text{ C°}} = \boxed{1596 \text{ K}}$ The differences are the same.

19.9 The wire is 35.0 m long when $T_C = -20.0°C$

$$\Delta L = L_i \bar{\alpha} (T - T_i)$$

$$\bar{\alpha} \approx \alpha(20.0°C) = 1.70 \times 10^{-5} (C°)^{-1} \text{ for Cu.}$$

$$\Delta L = (35.0 \text{ m})(1.70 \times 10^{-5} (C°)^{-1}\big(35.0°C - (-20.0°C)\big) = \boxed{+3.27 \text{ cm}}$$

Goal Solution

G: Based on everyday observations of telephone wires, we might expect the wire to expand by less than a meter since the change in length of these wires is generally not noticeable.

O: The change in length can be found from the linear expansion of copper wire (we will assume that the insulation around the copper wire can stretch more easily than the wire itself). From Table 19.2, the coefficient of linear expansion for copper is $17 \times 10^{-6} (°C)^{-1}$.

A: The change in length between cold and hot conditions is

$$\Delta L = \alpha L_0 \Delta T = [17 \times 10^{-6} (°C)^{-1}](35.0 \text{ m})(35.0°C - (-20.0°C))$$

$$\Delta L = 3.27 \times 10^{-2} \text{ m} \quad \text{or} \quad \Delta L = 3.27 \text{ cm}$$

L: This expansion is well under our expected limit of a meter. From ΔL, we can find that the wire sags 0.757 m at its midpoint on the hot summer day, which also seems reasonable based on everyday observations.

19.10 $\Delta L = L_i \alpha \; \Delta T = (25.0 \text{ m})(12.0 \times 10^{-6} / C°)(40.0 \text{ C°}) = \boxed{1.20 \text{ cm}}$

19.11 (a) $\Delta L = \alpha L_i \Delta T = 24.0 \times 10^{-6} (C°)^{-1}(3.0000 \text{ m})(80.0°C) = 0.00576 \text{ m}$

$$L_f = \boxed{3.0058 \text{ m}}$$

(b) $\Delta L = 24.0 \times 10^{-6} (C°)^{-1}(3.0000 \text{ m})(-20.0°C) = -0.0014$

$$\boxed{L_f = 2.9986 \text{ m}}$$

19.12 (a) $L_{Al}(1 + \alpha_{Al} \Delta T) = L_{Brass} (1 + \alpha_{Brass} \Delta T)$

$$\Delta T = \frac{L_{Al} - L_{Brass}}{L_{Brass} \, \alpha_{Brass} - L_{Al} \, \alpha_{Al}}$$

$$\Delta T = \frac{(10.01 - 10.00)}{(10.00)(19.0 \times 10^{-6}) - (10.01)(24.0 \times 10^{-6})}$$

$$\Delta T = -199 \text{ C°} \quad \text{so } T = \boxed{-179°C \quad \text{This is attainable.}}$$

(b) $\Delta T = \dfrac{(10.02 - 10.00)}{(10.00)(19.0 \times 10^{-6}) - (10.02)(24.0 \times 10^{-6})}$

$\Delta T = -396\ \text{C°}$ so $T = \boxed{-376\text{°C which is below 0 K so it cannot be reached}}$

19.13 For the dimensions to increase,

$\Delta L = \alpha L_i\, \Delta T$

$1.00 \times 10^{-2}\ \text{cm} = (1.30 \times 10^{-4}/\text{°C})(2.20\ \text{cm})(T - 20.0\text{°C})$

$T = \boxed{55.0\text{°C}}$

19.14 $\alpha = 1.10 \times 10^{-5}\ \text{deg}^{-1}$ for steel

$\Delta L = (518\ \text{m})(1.10 \times 10^{-5}\ \text{deg}^{-1})[35.0\text{°C} - (-20.0\text{°C})] = \boxed{0.313\ \text{m}}$

***19.15** (a) $\Delta A = 2\alpha A_i(\Delta T)$

$\Delta A = 2(17.0 \times 10^{-6}/\text{°C})(0.0800\ \text{m})^2(50.0\text{°C})$

$\Delta A = 1.09 \times 10^{-5}\ \text{m}^2 = \boxed{0.109\ \text{cm}^2}$

(b) The length of each side of the hole has increased. Thus, this represents an $\boxed{\text{increase}}$ in the area of the hole.

19.16 $\Delta V = (\beta - 3\alpha)V_i\, \Delta T$

$= [(5.81 \times 10^{-4} - 3(11.0 \times 10^{-6})](50.0\ \text{gal})(20.0)$

$= \boxed{0.548\ \text{gal}}$

19.17 (a) $\Delta L = \alpha L_i\, \Delta T = 9.00 \times 10^{-6}(\text{C°})^{-1}(30.0\ \text{cm})(65.0\text{°C}) = \boxed{0.176\ \text{mm}}$

(b) $\Delta L = 9.00 \times 10^{-6}(\text{C°})^{-1}(1.50\ \text{cm})(65.0\text{°C}) = \boxed{8.78 \times 10^{-4}\ \text{cm}}$

(c) $\Delta V = 3\alpha V_i\ \Delta T = 3(9.00 \times 10^{-6}/\text{°C})\dfrac{(30.0)(\pi)(1.50)^2}{4}\ \text{cm}^3(65.0\text{°C}) = \boxed{0.0930\ \text{cm}^3}$

19.18 (a) $V_f = V_i(1 + \beta\,\Delta T) = 100[1 + 1.50 \times 10^{-4}(-15.0)] = \boxed{99.8\text{ mL}}$

(b) $\Delta V_{\text{acetone}} = (\beta V_i\,\Delta T)_{\text{acetone}}$

$\Delta V_{\text{flask}} = (\beta V_i\,\Delta T)_{\text{Pyrex}} = (3\alpha V_i\,\Delta T)_{\text{Pyrex}}$

for same V_i, ΔT,

$$\frac{\Delta V_{\text{acetone}}}{\Delta V_{\text{flask}}} = \frac{\beta_{\text{acetone}}}{\beta_{\text{flask}}} = \frac{1.50 \times 10^{-4}}{3(3.20 \times 10^{-6})} = \frac{1}{6.40 \times 10^{-2}}$$

The volume change of flask is

$\boxed{\text{about 6\% of the change in the acetone's volume}}$.

19.19 (a) and (b) The material would expand by $\Delta L = \alpha L_i\,\Delta T$,

$\frac{\Delta L}{L_i} = \alpha\,\Delta T$, but instead feels stress

$$\frac{F}{A} = \frac{Y\,\Delta L}{L_i} = Y\alpha\,\Delta T = (7.00 \times 10^9\text{ N/m}^2)\,12.0 \times 10^{-6}\,(\text{C°})^{-1}(30.0\text{ C°})$$

$$= \boxed{2.52 \times 10^6\text{ N/m}^2}$$ This will $\boxed{\text{not break}}$ concrete.

19.20 (a) and (b) The gap width is a linear dimension, so it $\boxed{\text{increases}}$ in "thermal enlargement" by

$$\Delta L = \alpha L_i\,\Delta T = (11.0 \times 10^{-6}/\text{C°})(1.60\text{ cm})(160\text{ C°}) = 2.82 \times 10^{-3}\text{ cm}$$

so $L_f = \boxed{1.603\text{ cm}}$

19.21 In $\frac{F}{A} = \frac{Y\,\Delta L}{L_i}$ require $\Delta L = \alpha L_i\,\Delta T$

$$\frac{F}{A} = Y\alpha\,\Delta T$$

$$\Delta T = \frac{F}{AY\alpha} = \frac{500\text{ N}}{(2.00 \times 10^{-4}\text{ m}^2)(20.0 \times 10^{-10}\text{ N/m}^2)(11.0 \times 10^{-6}/\text{C°})}$$

$\Delta T = \boxed{1.14\text{°C}}$

19.22 $\Delta L = \alpha L_i(\Delta T)$ and $\dfrac{F}{A} = Y\dfrac{\Delta L}{L_i}$

$F = AY\dfrac{\Delta L}{L_i} = AY\alpha\,(\Delta T)$

$= \pi(0.0200\text{ m})^2\left(20.6\times10^{10}\ \dfrac{\text{N}}{\text{m}^2}\right)(11.0\times10^{-6}\,/\,\text{C°})(70.0\text{°C})$

$F = \boxed{199\text{ kN}}$

19.23 (a) $\Delta V = V_t\beta_t\,\Delta T - V_{\text{Al}}\beta_{\text{Al}}\Delta T = (\beta_t - 3\alpha_{\text{Al}})V_i\,\Delta T$

$= (9.00\times10^{-4} - 0.720\times10^{-4})(\text{°C})^{-1}(2000\text{ cm}^3)(60.0\text{°C})$

$\Delta V = \boxed{99.4\text{ cm}^3}$ overflows

(b) The whole new volume of turpentine is

$2000\text{ cm}^3 + (9.00\times10^{-4}\,/\,\text{°C})(2000\text{ cm}^3)(60.0\text{°C}) = 2108\text{ cm}^3$

so the fraction lost is $\dfrac{99.4\text{ cm}^3}{2108\text{ cm}^3} = 4.71\times10^{-2}$

and this fraction of the cylinder's depth will be empty upon cooling:

$(4.71\times10^{-2})(20.0\text{ cm}) = \boxed{0.943\text{ cm}}$

19.24 (a) $L = L_i\,(1 + \alpha\Delta T)$

$5.050\text{ cm} = (5.000\text{ cm})\,[1 + 24.0\times10^{-6}\ \text{°C}^{-1}(T - 20.0\text{°C})]$

$T = \boxed{437\text{°C}}$

(b) We must get $L_{\text{Al}} = L_{\text{Brass}}$ for some ΔT, or

$L_{i,\text{Al}}(1 + \alpha_{\text{Al}}\,\Delta T) = L_{i,\text{brass}}(1 + \alpha_{\text{brass}}\Delta T)$

$(5.000\text{ cm})[1 + (24.0\times10^{-6}\ \text{°C}^{-1})\Delta T] = (5.050\text{ cm})[1 + (19.0\times10^{-6}\ \text{°C}^{-1})\,\Delta T]$

Solving for ΔT gives $\Delta T = 2080\text{°C}$, so $\boxed{T = 3000\text{°C}}$

This will not work because $\boxed{\text{aluminum melts at 660°C}}$

***19.25** (a) $n = \dfrac{PV}{RT} = \dfrac{(9.00\text{ atm})(1.013\times10^5\text{ Pa}\,/\,\text{atm})(8.00\times10^{-3}\text{ m}^3)}{(8.315\text{ N}\cdot\text{mol K})(293\text{ K})} = \boxed{3.00\text{ mol}}$

(b) $N = nN_{\text{A}} = (3.00\text{ mol})(6.02\times10^{23}\text{ molecules}\,/\,\text{mol}) = \boxed{1.80\times10^{24}\text{ molecules}}$

19.26 $PV = NP'V' = \frac{4}{3}\pi r^3 NP'$

$$N = \frac{3PV}{4\pi r^3 P'} = \frac{(3)(150)(0.100)}{(4\pi)(0.150)^3(1.20)} = \boxed{884 \text{ balloons}}$$

19.27 $(1.01 \times 10^5)(6000) = n(8.315)(293)$

$n = 2.49 \times 10^5$ mol

$N = \boxed{1.50 \times 10^{29} \text{ molecules}}$

Goal Solution

G: The given room conditions are close to Standard Temperature and Pressure (STP is 0°C and 101.3 kPa), so we can use the estimate that one mole of an ideal gas at STP occupies a volume of about 22 L. The volume of the auditorium is 6000 m^3 and 1 m^3 = 1000 L , so we can estimate the number of molecules to be:

$$N \approx (6 \times 10^3 \text{ m}^3)\left(\frac{1000 \text{ L}}{1 \text{ m}^3}\right)\left(\frac{1 \text{ mol}}{22 \text{ L}}\right)\left(\frac{6.02 \times 10^{23} \text{ molecules}}{1 \text{ mol}}\right) \approx 1.6 \times 10^{29} \text{ molecules of air}$$

O: The number of molecules can be found more precisely by applying $PV = nRT$.

A: The equation of state of an ideal gas is $PV = nRT$ so we need to solve for the number of moles to find N.

$$n = \frac{PV}{RT} = \frac{(1.01 \times 10^5 \text{ N/m}^2)[(10.0 \text{ m})(20.0 \text{ m})(30.0 \text{ m})]}{(8.315 \text{ J/mol} \cdot \text{K})(293 \text{ K})} = 2.49 \times 10^5 \text{ mol}$$

$$N = n(N_A) = (2.49 \times 10^5 \text{ mol})\left(6.022 \times 10^{23} \frac{\text{molecules}}{\text{mol}}\right) = \boxed{1.50 \times 10^{29} \text{ molecules}}$$

L: This result agrees quite well with our initial estimate. The numbers would match even better if the temperature of the auditorium was 0°C.

19.28 $$P = \frac{nRT}{V} = \left(\frac{9.00 \text{ g}}{18.0 \text{ g/mol}}\right)\left(\frac{8.315 \text{ J}}{\text{mol K}}\right)\left(\frac{773 \text{ K}}{2.00 \times 10^{-3} \text{ m}^3}\right) = \boxed{1.61 \text{ MPa}} = 15.9 \text{ atm}$$

19.29 $\rho_{\text{out}}\, gV - \rho_{\text{in}}\, gV - (200\text{ kg})g = 0$

$(\rho_{\text{out}} - \rho_{\text{in}})(400\text{ m}^3) = 200\text{ kg}$

$$\left(1.25\ \frac{\text{kg}}{\text{m}^3}\right)\left(1 - \frac{283\text{ K}}{T_{\text{in}}}\right)(400\text{ m}^3) = 200\text{ kg}$$

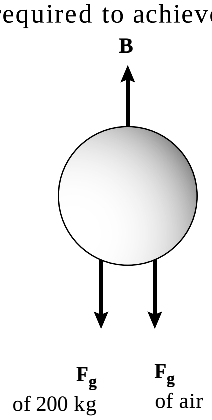
$$1 - \frac{283}{T_{\text{in}}} = 0.400$$

$$0.600 = \frac{283}{T_{\text{in}}} \qquad T_{\text{in}} = \boxed{472\text{ K}}$$

Goal Solution

G: The air inside the balloon must be significantly hotter than the outside air in order for the balloon to have a net upward force, but the temperature must also be less than the melting point of the nylon used for the balloon's envelope (rip-stop nylon melts around 200°C), otherwise the results could be disastrous!

O: The density of the air inside the balloon must be sufficiently low so that the buoyant force is greater than the weight of the balloon, its cargo, and the air inside. The temperature of the air required to achieve this density can be found from the equation of state of an ideal gas.



A: The buoyant force equals the weight of the air at 10.0°C displaced by the balloon:

$$B = m_{\text{air}}g = \rho_a Vg = (1.25\text{ kg/m}^3)(400\text{ m}^3)(9.8\text{ m/s}^2) = 4900\text{ N}$$

The weight of the balloon and its cargo is

$$F_g = m_b g = (200\text{ kg})(9.80\text{ m/s}^2) = 1960\text{ N}$$

Since $B > F_g$, the balloon has a chance of lifting off as long as the weight of the air inside the balloon is less than the difference in these forces:

$$F_{g(\text{air})} < B - F_{g(\text{balloon})} = 4900\text{ N} - 1960\text{ N} = 2940\text{ N}$$

The mass of this air is $m_{\text{air}} = \dfrac{F_{g(\text{air})}}{g} = \dfrac{2940\text{ N}}{9.80\text{ m/s}^2} = 300\text{ kg}$

To find the required temperature of this air from $PV = nRT$, we must find the corresponding number of moles of air. Dry air is approximately 20% O_2, and 80% N_2. Using data from a periodic table, we can calculate the molar mass of the air to be approximately

$$M = 0.80(28 \text{ g/mol}) + 0.20(32 \text{ g/mol}) = 29 \text{ g/mol}$$

so the number of moles is $n = \frac{m}{M} = \frac{300 \text{ kg}}{29 \text{ g/mol}}\left(\frac{10^3 \text{ g}}{1 \text{ kg}}\right) = 1.0 \times 10^4 \text{ mol}$

The pressure of this air is the ambient pressure; from $PV = nRT$, we can now find the minimum temperature required for lift off:

$$T = \frac{PV}{nR} = \frac{(1.013 \times 10^5 \text{ N/m}^2)(400 \text{ m}^3)}{(1.0 \times 10^4 \text{ mol})(8.315 \text{ J/(mol K)})} = 471 \text{ K} = 198°\text{C}$$

L: The average temperature of the air inside the balloon required for lift off appears to be close to the melting point of the nylon fabric, so this seems like a dangerous situation! A larger balloon would be better suited for the given weight of the balloon. (A quick check on the internet reveals that this balloon is only about 1/10 the size of most sport balloons, which have a volume of about 3000 m^3).

If the buoyant force were less than the weight of the balloon and its cargo, the balloon would not lift off no matter how hot the air inside might be! If this were the case, then either the weight would have to be reduced or a bigger balloon would be required.

Even though our result for T is shown with 3 significant figures, the answer should probably be rounded to 2 significant figures to reflect the approximate value of the molar mass of the air.

***19.30** (a) $T_2 = T_1\frac{P_2}{P_1} = (300 \text{ K})(3) = \boxed{900 \text{ K}}$

(b) $T_2 = T_1\frac{P_2 V_2}{P_1 V_1} = 300(2)(2) = \boxed{1200 \text{ K}}$

***19.31** (a) $PV = nRT$

$$n = \frac{PV}{RT} = \frac{(1.013 \times 10^5 \text{ Pa})(1.00 \text{ m}^3)}{(8.315 \text{ J/mol}\cdot\text{K})(293 \text{ K})} = \boxed{41.6 \text{ mol}}$$

(b) $m = nM = (41.6 \text{ mol})(28.9 \text{ g/mol}) = \boxed{1.20 \text{ kg}}$, in agreement with the tabulated density of 1.20 kg/m^3 at 20.0°C.

***19.32** (a) $PV = nRT$ $\quad n = \frac{PV}{RT}$

$$m = nM = \frac{PVM}{RT} = \frac{(1.013 \times 10^5 \text{ Pa})(0.100 \text{ m})^3(28.9 \times 10^{-3} \text{ kg/mol})}{(8.315 \text{ J/mol}\cdot\text{K})(300 \text{ K})}$$

$$m = \boxed{1.17 \times 10^{-3} \text{ kg}}$$

(b) $F_g = mg = (1.17 \times 10^{-3} \text{ kg})(9.80 \text{ m/s}^2) = \boxed{11.5 \text{ mN}}$

(c) $F = PA = (1.013 \times 10^5 \text{ N/m}^2)(0.100 \text{ m})^2 = \boxed{1.01 \text{ kN}}$

(d) The $\boxed{\text{molecules must be moving very fast}}$ to hit the walls hard.

19.33 (a) Initially, $P_i V_i = n_i R T_i$

$(1.00 \text{ atm})V_i = n_i R(10.0 + 273.15)\text{K}$

Finally, $P_f V_f = n_f R T_f \qquad P_f(0.280 V_i) = n_i R(40.0 + 273.15)\text{K}$

Dividing these equations,

$$\frac{0.280 \times P_f}{1.00 \text{ atm}} = \frac{313.15 \text{ K}}{283.15 \text{ K}} \quad \text{giving } P_f = 3.95 \text{ atm or}$$

$$P_f = \boxed{4.00 \times 10^5 \text{ Pa(abs.)}}$$

(b) After being driven

$$P_d(1.02)(0.280 V_i) = n_i R(85.0 + 273.15)\text{K}$$

$$P_d = 1.121 P_f = \boxed{4.49 \times 10^5 \text{ Pa}}$$

***19.34** Let us use $V = \frac{4}{3}\pi r^3$ as the volume of the balloon, and the ideal gas law in

the form $\frac{P_f V_f}{T_f} = \frac{P_i V_i}{T_i}$

to give, $r_i^3 = \frac{300 \text{ K}}{200 \text{ K}} \frac{0.0300 \text{ atm}}{1.00 \text{ atm}} (20.0 \text{ m})^3$

$$\boxed{r_i = 7.11 \text{ m}}$$

19.35 $P_1 V_1 = n_1 R T_1$

$P_2 V_2 = n_2 R T_2$

$$n_1 - n_2 = \frac{PV}{RT_1} - \frac{PV}{RT_2}$$

$$n_1 - n_2 = \frac{(101 \times 10^3 \text{ Pa})80.0 \text{ m}^3}{8.315 \text{ J/mol K}}\left(\frac{1}{291 \text{ K}} - \frac{1}{298 \text{ K}}\right)$$

$$n_1 - n_2 = 78.4 \text{ mol}$$

$$\Delta m = \Delta n M = 78.4 \text{ mol}(28.9 \text{ g/mol}) = \boxed{2.27 \text{ kg}}$$

19.36 $P_0V = n_1RT_1 = (m_1 / M)RT_1$

$P_0V = n_2RT_2 = (m_2 / M)RT_2$

$$\boxed{m_1 - m_2 = \frac{P_0VM}{R}\left(\frac{1}{T_1} - \frac{1}{T_2}\right)}$$

19.37 At depth, $P = P_0 + \rho gh$

and $PV_i = nRT_i$

at the surface, $P_0V_f = nRT_f$

$$\frac{P_0V_f}{(P_0 + \rho gh)V_i} = \frac{T_f}{T_i}$$

$$V_f = V_i\frac{T_f}{T_i}\left(\frac{P_0 + \rho gh}{P_0}\right)$$

$$V_f = 1.00 \text{ cm}^3\frac{293 \text{ K}}{278 \text{ K}}\left(\frac{1.013 \times 10^5 \text{ Pa} + 1025 \text{ kg/m}^3 (9.80 \text{ m/s}^2)25.0 \text{ m}}{1.013 \times 10^5 \text{ Pa}}\right)$$

$$V_f = \boxed{3.67 \text{ cm}^3}$$

***19.38** My bedroom is 4.00 m long, 4.00 m wide, and 2.40 m high, enclosing air at 100 kPa and 20.0°C = 293 K. Think of the air as 80.0% N_2 and 20.0% O_2.

Avogadro's number of molecules has mass

$$0.800 \times 28.0 \text{ g/mol} + 0.200 \times 32.0 \text{ g/mol} = 0.0288 \text{ kg/mol}$$

Then $PV = nRT = (m/M)RT$ gives

$$m = \frac{PVM}{RT} = \frac{(1.00 \times 10^5 \text{ N/m}^2)(38.4 \text{ m}^3)(0.0288 \text{ kg/mol})}{(8.315 \text{ J/mol}\cdot\text{K})(293 \text{ K})}$$

$$m = 45.4 \text{ kg} \quad \boxed{\sim 10^2 \text{ kg}}$$

19.39 $PV = nRT$

$$\frac{m_f}{m_i} = \frac{n_f}{n_i} = \frac{P_fV_f}{RT_f}\frac{RT_i}{P_iV_i} = \frac{P_f}{P_i}, \text{ so } m_f = m_i\left(\frac{P_f}{P_i}\right)$$

$$\Delta m = m_i - m_f = m_i\left(\frac{P_i - P_f}{P_i}\right) = (12.0 \text{ kg})\left(\frac{41.0 \text{ atm} - 26.0 \text{ atm}}{41.0 \text{ atm}}\right) = \boxed{4.39 \text{ kg}}$$

19.40 $$N = \frac{PVN_A}{RT} = \frac{(10^{-9}\text{ Pa})(1.00\text{ m}^3)(6.02\times10^{23})\ \frac{\text{molecule}}{\text{mol}}}{\left(8.315\ \frac{\text{J}}{\text{K}\cdot\text{mol}}\right)(300\text{ K})} = \boxed{2.41\times10^{11}\text{ molecules}}$$

19.41 $PV = nRT$

$$V = \frac{nRT}{P} = \frac{(1.00\text{ mol})(8.315\text{ J/mol}\cdot\text{K})(273\text{ K})}{1.013\times10^5\text{ N/m}^2}\left(\frac{10^3\text{ L}}{1.00\text{ m}^3}\right) = \boxed{22.4\text{ L}}$$

19.42 (a) Initially the air in the bell satisfies $P_0 V_{\text{bell}} = nRT_i$

or $P_0[(2.50\text{ m})A] = nRT_i$ (1)

When the bell is lowered, the air in the bell satisfies

$P_{\text{bell}}(2.50\text{ m} - x)A = nRT_f$ (2)

where x is the height the water rises in the bell. Also, the pressure in the bell, once it is lowered, is equal to the sea water pressure at the depth of the water level in the bell.

$P_{\text{bell}} = P_0 + \rho g\,(82.3\text{ m} - x) \approx P_0 + \rho g(82.3\text{ m})$ (3)

The approximation is good, as $x < 2.50$ m. Substituting (3) into (2) and substituting nR from (1) into (2),

$$[P_0 + \rho g\,(82.3\text{ m})](2.50\text{ m} - x)A = P_0 V_{\text{bell}}\frac{T_f}{T_i}$$

Using $P_0 = 1\text{ atm} = 1.013\times10^5$ Pa and $\rho = 1.025\times10^3\ \frac{\text{kg}}{\text{m}^3}$

$$x = (2.50\text{ m})\left[1 - \frac{T_f}{T_0}\left(1 + \frac{\rho g(82.3\text{ m})}{P_0}\right)^{-1}\right]$$

$$= (2.50\text{ m})\left[1 - \frac{277.15\text{ K}}{293.15\text{ K}}\left(1 + \frac{(1.025\times10^3\text{ kg/m}^3)(9.80\text{ m/s}^2)(82.3\text{ m})}{1.013\times10^5\text{ N/m}^2}\right)^{-1}\right]$$

$x = \boxed{2.24\text{ m}}$

(b) If the water in the bell is to be expelled, the air pressure in the bell must be raised to the water pressure at the bottom of the bell. That is,

$$P_{\text{bell}} = P_0 + \rho g(82.3\text{ m})$$

$$= 1.013 \times 10^5\text{ Pa} + \left(1.025 \times 10^3\ \frac{\text{kg}}{\text{m}^3}\right)\left(9.80\ \frac{\text{m}}{\text{s}^2}\right)(82.3\text{ m})$$

$$P_{\text{bell}} = 9.28 \times 10^5\text{ Pa} = \boxed{9.16\text{ atm}}$$

19.43 The excess expansion of the brass is

$$\Delta L_{\text{rod}} - \Delta L_{\text{tape}} = (\alpha_{\text{brass}} - \alpha_{\text{steel}})L_i\,\Delta T$$

$$\Delta(\Delta L) = (19.0 - 11.0)10^{-6}\,(\text{C}°)^{-1}\ 0.950\text{ m}(35.0\text{ C}°)$$

$$\Delta(\Delta L) = 2.66 \times 10^{-4}\text{ m}$$

(a) The rod contracts more than tape to

a length reading $0.9500\text{ m} - 0.000266\text{ m} = \boxed{0.9497\text{ m}}$

(b) $0.9500\text{ m} + 0.000266\text{ m} = \boxed{0.9503\text{ m}}$

***19.44** At 0°C, 10.0 gallons of gasoline has mass, from $\rho = m/V$

$$m = \rho V = \left(730\ \frac{\text{kg}}{\text{m}^3}\right)(10.0\text{ gal})\left(\frac{0.00380\text{ m}^3}{1.00\text{ gal}}\right) = 27.7\text{ kg}$$

The gasoline will expand in volume by

$$\Delta V = \beta V_i\,\Delta T = (9.60 \times 10^{-4}/\text{C}°)(10.0\text{ gal})(20.0°\text{C} - 0.0°\text{C}) = 0.192\text{ gal}$$

At 20.0°C, we have 10.192 gal = 27.7 kg

$$10.0\text{ gal} = \left(\frac{10.0\text{ gal}}{10.192\text{ gal}}\right)(27.7\text{ kg}) = 27.2\text{ kg}$$

The extra mass contained in 10.0 gallons at 0.0°C is

$$27.7\text{ kg} - 27.2\text{ kg} = \boxed{0.523\text{ kg}}$$

19.45 $R_B + \alpha_B R_B(T - 20.0) = R_s + \alpha_s R_s(T - 20.0)$

$$3.994\text{ cm} + (19.0 \times 10^{-6}/\text{C}°)(3.994\text{ cm})(T - 20.0°\text{C})$$

$$= 4.000\text{ cm} + (11.0 \times 10^{-6})(\text{C}°)^{-1}(4.000\text{ cm})(T - 20.0°\text{C})$$

$3.189 \times 10^{-5}\, T = 0.006638$ $\boxed{T = 208°\text{C}}$

***19.46** The frequency played by the cold-walled flute is $f_i = \dfrac{v}{\lambda_i} = \dfrac{v}{2L_i}$.

When the instrument warms up

$$f_f = \frac{v}{\lambda_f} = \frac{v}{2L_f} = \frac{v}{2L_i(1 + \alpha\,\Delta T)} = \frac{f_i}{1 + \alpha\,\Delta T}$$

The final frequency is lower. The change in frequency is

$$\Delta f = f_i - f_f = f_i\left(1 - \frac{1}{1 + \alpha\,\Delta T}\right)$$

$$\Delta f = \frac{v}{2L_i}\left(\frac{\alpha\,\Delta T}{1 + \alpha\,\Delta T}\right) \approx \frac{v}{2L_i}(\alpha\,\Delta T)$$

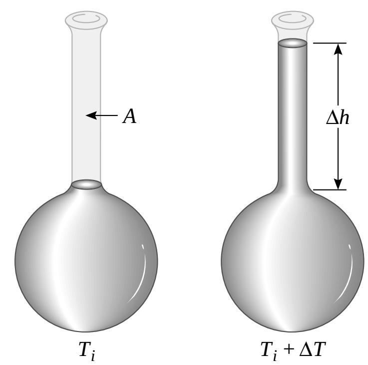
$$\Delta f \approx \frac{(343 \text{ m/s})(24.0 \times 10^{-6}/\text{C°})(15.0 \text{ C°})}{2(0.655 \text{ m})} = \boxed{0.0943 \text{ Hz}}$$

This change in frequency is imperceptibly small.

19.47 Neglecting the expansion of the glass,

$$\Delta h = \frac{V}{A}\beta\Delta T$$

$$\Delta h = \frac{\frac{4}{3}\pi\left(\frac{0.250 \text{ cm}}{2}\right)^3}{\pi(2.00 \times 10^{-3} \text{ cm})^2}(1.82 \times 10^{-4}/\text{C°})(30.0\text{°C}) = \boxed{3.55 \text{ cm}}$$



19.48 (a) The volume of the liquid increases as $\Delta V_1 = V_i\beta\,\Delta T$. The volume of the flask increases as $\Delta V_g = 3\alpha V_i\,\Delta T$. Therefore, the overflow in the capillary is $V_c = V_i\,\Delta T(\beta - 3\alpha)$; and in the capillary $V_c = A\,\Delta h$.

Therefore, $\boxed{\Delta h = \dfrac{V_i}{A}(\beta - 3\alpha)\,\Delta T}$

(b) For a mercury thermometer $\beta(\text{Hg}) = 1.82 \times 10^{-4}/\text{°C}$ and for glass, $3\alpha = 3 \times 3.20 \times 10^{-6}/\text{°C}$. Thus $\beta - 3\alpha \approx \beta$, or $\boxed{\alpha \ll \beta}$

19.49 (a) $\rho = \dfrac{m}{V}$ and $d\rho = -\dfrac{m}{V^2}\,dV$

For very small changes in V and ρ, this can be expressed as

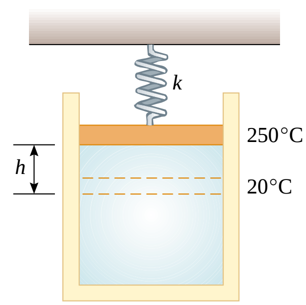
$$\Delta\rho = -\frac{m}{V}\frac{\Delta V}{V} = -\rho\beta\Delta T$$

The negative sign means that any increase in temperature causes the density to decrease and vice versa.

(b) For water we have

$$\beta = \left|\frac{\Delta\rho}{\rho\,\Delta T}\right| = \left|\frac{(1.0000\ \text{g/cm}^3 - 0.9997\ \text{g/cm}^3)}{(1.0000\ \text{g/cm}^3)(10.0 - 4.00)\text{C}^\circ}\right| = \boxed{5\times10^{-5}/^\circ\text{C}}$$

19.50 (a) $\dfrac{P_0V}{T} = \dfrac{P'V'}{T'}$



$V' = V + Ah$

$P' = P_0 + \dfrac{kh}{A}$

$$\left(P_0 + \frac{kh}{A}\right)(V + Ah) = P_0V\left(\frac{T'}{T}\right)$$

$$\left(1.013\times10^5\ \frac{\text{N}}{\text{m}^2} + 2.00\times10^5\ \frac{\text{N}}{\text{m}^3}h\right)(5.00\times10^{-3}\ \text{m}^3 + (0.0100\ \text{m}^2)h)$$

$$= \left(1.013\times10^5\ \frac{\text{N}}{\text{m}^2}\right)(5.00\times10^{-3}\ \text{m}^3)\left(\frac{523\ \text{K}}{293\ \text{K}}\right)$$

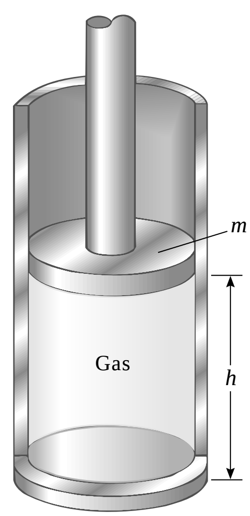
$2000h^2 + 2013h - 397 = 0$

$$h = \frac{-2013 \pm 2689}{4000} = \boxed{0.169\ \text{m}}$$

(b) $$P' = P + \frac{kh}{A} = 1.013\times10^5\ \text{Pa} + \frac{(2.00\times10^3\ \text{N/m})(0.169)}{0.0100\ \text{m}^2}$$

$P' = \boxed{1.35\times10^5\ \text{Pa}}$

19.51 (a) We assume that air at atmospheric pressure is above the piston.



In equilibrium $P_{gas} = \dfrac{mg}{A} + P_0$. Therefore,

$\dfrac{nRT}{hA} = \dfrac{mg}{A} + P_0$ or $\boxed{h = \dfrac{nRT}{(mg + P_0A)}}$ where

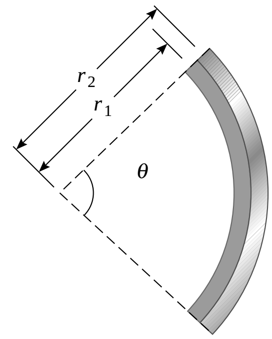
we have used $V = hA$ as the volume of the gas.

(b) From the data given,

$$h = \frac{(0.200\ \text{mol})(8.315\ \text{J/K}\cdot\text{mol})(400\ \text{K})}{(20.0\ \text{kg})(9.80\ \text{m/s}^2) + (1.013\times10^5\ \text{N/m}^2)(0.00800\ \text{m}^2)}$$

$$= \boxed{0.661\ \text{m}}$$

19.52 (a) $L_1 = r_1\theta = L_i(1 + \alpha_1 \Delta T)$ $L_2 = r_2\theta = L_i(1 + \alpha_2 \Delta T)$



$$\Delta r = r_2 - r_1 = \frac{L_i(\alpha_2 - \alpha_1)\Delta T}{\theta}$$

$$\therefore r_2 - r_1 = \frac{1}{\theta}[L_i + L_i\alpha_2 \ \Delta T - L_i - L_i\alpha_1 \Delta T]$$

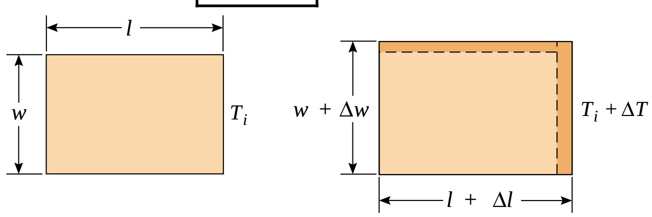
$$\boxed{\theta = L_i(\alpha_2 - \alpha_1)\frac{\Delta T}{(r_2 - r_1)}}$$

(b) $\boxed{\theta \to 0 \text{ as } \Delta T \to 0}$ $\boxed{\theta \to 0 \text{ as } \alpha_1 \to \alpha_2}$

$\theta < 0$ when $\Delta T < 0$ cooling means temperature is decreasing.

(c) It bends the other way.

19.53 From the diagram we see that the change in area is $\Delta A = l\,\Delta w + w\,\Delta l + \Delta w\,\Delta l$. Since Δl and Δw are each small quantities, the product $\Delta w\,\Delta l$ will be very small. Therefore, we assume $\Delta w\,\Delta l \approx 0$. Since $\Delta w = w\alpha\,\Delta T$ *and* $\Delta l = l\alpha\,\Delta T$, we then have $\Delta A = lw\alpha\,\Delta T + wl\alpha\,\Delta T$ and since $A = lw$, we have $\boxed{\Delta A = 2\alpha A\ \Delta T}$. The approximation assumes $\Delta w \Delta l \approx 0$, or $\alpha\,\Delta T \approx 0$. Another way of stating this is $\boxed{\alpha\,\Delta T \ll 1}$.



19.54 (a) $R = R_0(1 + AT_C$

$50.0\ \Omega = R_0(1 + 0) \Rightarrow R_0 = 50.0\ \Omega$

$71.5\ \Omega = (50.0\ \Omega)\ [1 + (231.97°\text{C})A]$

$\boxed{A = 1.85 \times 10^{-3}(\text{C}°)^{-1}}$ $\boxed{R_0 = 50.0\Omega}$

(b) $$T = \frac{1}{A}\left(\frac{R}{R_0} - 1\right) = \frac{1}{1.85 \times 10^{-3}\ (\text{C}°)^{-1}}\left(\frac{89.0}{50.0} - 1\right) = \boxed{422°\text{C}}$$

19.55 (a) $T_i = 2\pi\sqrt{\dfrac{L_i}{g}}$

$$L_i = \frac{T_i^2 g}{4\pi^2} = \frac{(1.000\text{ s})^2(9.80\text{ m/s}^2)}{4\pi^2} = 0.2482\text{ m}$$

$$\Delta L = \alpha L_i\,\Delta T = (19.0\times10^{-6}/\text{C°})(0.2483\text{ m})(10.0\text{°C}) = 4.72\times10^{-5}\text{ m}$$

$$T_f = 2\pi\sqrt{\frac{L_i+\Delta L}{g}} = 2\pi\sqrt{\frac{0.2483\text{ m}}{9.80\text{ m/s}^2}} = 1.0000949\text{ s}$$

$$\Delta T = \boxed{9.49\times10^{-5}\text{ s}}$$

(b) In one week, the time lost is

time lost = (1 week)(9.49×10^{-5} s lost per second)

$$= \left(7.00\,\frac{\text{d}}{\text{week}}\right)\left(\frac{86\,400\text{ s}}{1.00\text{ d}}\right)\left(9.49\times10^{-5}\,\frac{\text{s lost}}{\text{s}}\right) = \boxed{57.4\text{ s lost}}$$

19.56 $I = \int r^2\,dm$ and since $r(T) = r(T_i)(1+\alpha\,\Delta T)$, for $\alpha\,\Delta T << 1$ we find

$$\frac{I(T)}{I(T_i)} = (1+\alpha\,\Delta T)^2,\text{ thus }\frac{I(T)-I(T_i)}{I(T_i)} \approx 2\alpha\,\Delta T$$

(a) With $\alpha = 17.0\times10^{-6}/\text{°C}$ and $\Delta T = 100\text{°C}$, we find for Cu:

$$\frac{\Delta I}{I} = 2(17.0\times10^{-6}/\text{°C})(100\text{°C}) = \boxed{0.340\%}$$

(b) With $\alpha = 24.0\times10^{-6}/\text{°C}$ and $\Delta T = 100\text{°C}$, we find for Al:

$$\frac{\Delta I}{I} = 2(24.0\times10^{-6}/\text{°C})(100\text{°C}) = \boxed{0.480\%}$$

19.57 (a) $B = \rho g V'$ $P' = P_0 + \rho g d$ $P'V' = P_0 V_i$

$$B = \frac{\rho g P_0 V_i}{P'} = \boxed{\frac{\rho g P_0 V_i}{(P_0+\rho g d)}}$$

(b) Since d is in the denominator, B must $\boxed{\text{decrease}}$ as the depth increases. (The volume of the balloon becomes smaller with increasing pressure.)

(c) $$\frac{1}{2} = \frac{B(d)}{B(0)} = \frac{\rho g P_0 V_i / (P_0 + \rho g d)}{\rho g P_0 V_i / P_0} = \frac{P_0}{P_0 + \rho g d}$$

$$P_0 + \rho g d = 2P_0$$

$$d = \frac{P_0}{\rho g} = \frac{1.013 \times 10^5 \text{ N/m}^2}{(1.00 \times 10^3 \text{ kg/m}^3)(9.80 \text{ m/s}^2)} = \boxed{10.3 \text{ m}}$$

19.58 (a) Let m represent the sample mass. Then the number of moles is $n = m/M$ and the density is $\rho = m/V$. So $PV = nRT$ becomes

$$PV = \frac{m}{M} RT \quad \text{or} \quad PM = \frac{m}{V} RT$$

Then, $\rho = \frac{m}{V} = \boxed{\frac{PM}{RT}}$

(b) $$\rho = \frac{PM}{RT} = \frac{(1.013 \times 10^5 \text{ N/m}^2)(0.0320 \text{ kg/mol})}{(8.315 \text{ J/mol} \cdot \text{K})(293 \text{ K})} = \boxed{1.33 \text{ kg/m}^3}$$

19.59 For each gas alone, $P_1 = \frac{N_1 kT}{V}$ and $P_2 = \frac{N_2 kT}{V}$ and $P_3 = \frac{N_3 kR}{V}$, etc.

For all gases

$$P_1V_1 + P_2V_2 + P_3V_3 \ldots = (N_1 + N_2 + N_3 \ldots)kT \quad \text{and}$$

$$(N_1 + N_2 + N_3 \ldots)kT = PV$$

Also, $V_1 = V_2 = V_3 = \ldots = V$, therefore $\boxed{P = P_1 + P_2 + P_3 \ldots}$

19.60 (a) Using the Periodic Table, we find the molecular masses of the air components to be

$$M(N_2) = 28.01 \text{ u}, \; M(O_2) = 32.00 \text{ u}, \; M(\text{Ar}) = 39.95 \text{ u}$$

and $M(CO_2) = 44.01$ u

Thus, the number of moles of each gas in the sample is

$$n(N_2) = \frac{75.52 \text{ g}}{28.01 \text{ g/mol}} = 2.696 \text{ mol}$$

$$n(O_2) = \frac{23.15 \text{ g}}{32.00 \text{ g/mol}} = 0.7234 \text{ mol}$$

$$n(\text{Ar}) = \frac{1.28 \text{ g}}{39.95 \text{ g/mol}} = 0.0320 \text{ mol}$$

$$n(CO_2) = \frac{0.05 \text{ g}}{44.01 \text{ g/mol}} = 0.0011 \text{ mol}$$

The total number of moles is $n_0 = \sum n_i = 3.453$ mol. Then, the partial pressure of N_2 is

$$P(N_2) = \frac{2.696\text{ mol}}{3.453\text{ mol}} \cdot (1.013\times 10^5\text{ Pa}) = \boxed{79.1\text{ kPa}}$$

Similarly,

$$P(O_2) = \boxed{21.2\text{ kPa}} \qquad P(\text{Ar}) = \boxed{940\text{ Pa}} \qquad P(CO_2) = \boxed{33.3\text{ Pa}}$$

(b) Solving the ideal gas law equation for V and using $T = 273.15 + 15.00 = 288.15$ K, we find

$$V = \frac{n_0RT}{P} = \frac{(3.453\text{ mol})(8.315\text{ J/mol}\cdot\text{K})(288.15\text{ K})}{1.013\times 10^5\text{ Pa}} = 8.167\times 10^{-2}\text{ m}^3$$

Then, $\rho = \frac{m}{V} = \frac{100\times 10^{-3}\text{kg}}{8.167\times 10^{-2}\text{ m}^3} = \boxed{1.22\text{ kg/m}^3}$

(c) The 100 g sample must have an appropriate molar mass to yield n_0 moles of gas: that is

$$M(\text{air}) = \frac{100\text{ g}}{3.453\text{ mol}} = \boxed{29.0\text{ g/mol}}$$

19.61 In any one section of concrete, length L_i expands by

$$\Delta L = \alpha L_i\,\Delta T$$

$$= (12.0\times 10^{-6}/\text{C}^\circ)L_i\,(25.0\text{ C}^\circ) = 3.00\times 10^{-4}\,L_i$$

The unstressed length of that rail increases by

$$(11.0\times 10^{-6}/\text{C}^\circ)L_i\,(25.0\text{ C}^\circ) = 2.75\times 10^{-4}\,L_i$$

(a) So the rail is stretched elastically by the extra

$$3.00\times 10^{-4}\,L_i - 2.75\times 10^{-4}\,L_i = 2.50\times 10^{-5}\,L_i$$

in $\frac{F}{A} = Y\frac{\Delta L}{L_i} = (20.0\times 10^{10}\text{ N/m})(2.50\times 10^{-5}) = \boxed{5.00\times 10^6\text{ N/m}^2}$

(b) Fraction of yield strength $= \frac{5.00\times 10^6\text{ N/m}^2}{52.2\times 10^7\text{ N/m}^2} = \boxed{9.58\times 10^{-3}}$

19.62 (a) From $PV = nRT$, the volume is: $V = \left(\frac{nR}{P}\right) T$

Therefore, when pressure is held constant,

$$\frac{dV}{dT} = \frac{nR}{P} = \frac{V}{T}$$

Thus, $\beta \equiv \left(\frac{1}{V}\right)\frac{dV}{dT} = \left(\frac{1}{V}\right)\frac{V}{T}$, or $\boxed{\beta = \frac{1}{T}}$

(b) At $T = 0°\text{C} = 273\text{ K}$, this predicts $\beta = \frac{1}{273\text{ K}} = \boxed{3.66 \times 10^{-3}/\text{K}}$

Experimental values are: $\beta_{\text{He}} = 3.665 \times 10^{-3}/\text{K}$ and $\beta_{\text{air}} = 3.67 \times 10^{-3}/\text{K}$

19.63 After expansion, the length of one of the spans is

$$L_f = L_i(1 + \alpha\,\Delta T) = (125\text{ m})[1 + (12 \times 10^{-6}/\text{C°})(20.0\text{ C°})] = 125.03\text{ m}$$

L_f, y, and the original 125 m length of this span form a right triangle with y as the altitude. Using the Pythagorean theorem gives:

$$(125.03\text{ m})^2 = y^2 + (125\text{ m})^2 \quad \text{yielding} \quad y = \boxed{2.74\text{ m}}$$

19.64 After expansion, the length of one of the spans is $L_f = L(1 + \alpha\,\Delta T)$. L_f, y, and the original length L of this span form a right triangle with y as the altitude. Using the Pythagorean theorem gives $L_f^2 = L^2 + y^2$, or

$$y = \sqrt{L_f^2 - L^2} = L\sqrt{(1 + \alpha\,\Delta T)^2 - 1} = L\sqrt{2\alpha\,\Delta T + (\alpha\,\Delta T)^2}$$

Since $\alpha\,\Delta T << 1$, $y \approx \boxed{L\sqrt{2\alpha\,\Delta T}}$

19.65 For $\Delta L = L_s - L_c$ to be constant, the rods must expand by equal amounts:

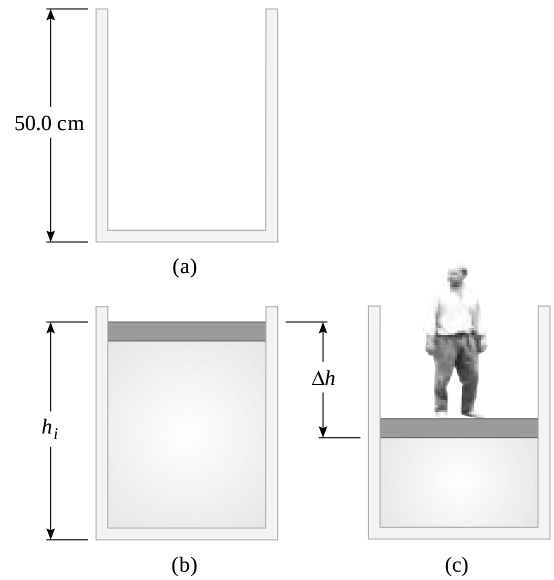
$$\alpha_c L_c \Delta T = \alpha_s L_s \Delta T$$

$$L_s = \frac{\alpha_c L_c}{\alpha_s}$$

$$\Delta L = \frac{\alpha_c L_c}{\alpha_s} - L_c$$

$$\therefore \quad L_c = \frac{\Delta L\,\alpha_s}{(\alpha_c - \alpha_s)} = \frac{5.00\text{ cm}(11.0 \times 10^{-6}/\text{C°})}{(17.0 \times 10^{-6}/\text{C°} - 11.0 \times 10^{-6}/\text{C°})} = \boxed{9.17\text{ cm}}$$

and $$L_s = \frac{\Delta L\,\alpha_c}{(\alpha_c - \alpha_s)} = 5.00\text{ cm}\left(\frac{17.0}{6.00}\right) = \boxed{14.2\text{ cm}}$$



19.66 (a) With piston alone:

T = constant, so $PV = P_0 V_i$

or $P(Ah_0) = P_0(Ah_i)$

With A = constant, $P = P_0\left(\frac{h_i}{h_0}\right)$

But, $P = P_0 + \frac{m_p g}{A}$, where m_p is the mass of the piston.

Thus, $P_0 + \frac{m_p g}{A} = P_0\left(\frac{h_i}{h_0}\right)$, which reduces to

$$h_0 = \frac{h_i}{1 + \frac{m_p g}{P_0 A}} = \frac{50.0 \text{ cm}}{1 + \frac{(20.0 \text{ kg})(9.80 \text{ m/s}^2)}{(1.013 \times 10^5 \text{ Pa})\pi(0.400 \text{ m})^2}} = 49.81 \text{ cm}$$

With the man of mass M on the piston, a very similar calculation (replacing m_p by $m_p + M$) gives:

$$h' = \frac{h_i}{1 + \frac{(m_p + M)g}{P_0 A}} = \frac{50.0 \text{ cm}}{1 + \frac{(95.0 \text{ kg})(9.80 \text{ m/s}^2)}{(1.013 \times 10^5 \text{ Pa})\pi(0.400 \text{ m})^2}} = 49.10 \text{ cm}$$

Thus, when the man steps on the piston, it moves downward by

$$\Delta h = h_0 - h' = 49.81 \text{ cm} - 49.10 \text{ cm} = 0.710 \text{ cm} = \boxed{7.10 \text{ mm}}$$

(b) P = const, so $\frac{V}{T} = \frac{V'}{T_i}$ or $\frac{Ah_0}{T} = \frac{Ah'}{T_i}$, giving

$$T = T_i\left(\frac{h_0}{h'}\right) = (293 \text{ K})\left(\frac{49.81}{49.10}\right) = \boxed{297 \text{ K}} \text{ (or 24°C)}$$

19.67 (a) $\dfrac{dL}{L} = \alpha dT$

$$\int_{T_i}^{T_i} \alpha dT = \int_{L_i}^{L_i} \frac{dL}{L} \Rightarrow \ln\left(\frac{L_f}{L_i}\right) = \alpha \Delta T \Rightarrow \boxed{L_f = L_i e^{\alpha \Delta T}}$$

(b) $L_f = (1.00\text{ m})e^{[2.00\times10^{-5}(\text{C}^\circ)^{-1}](100^\circ\text{C})} = 1.002002\text{ m}$

$L_f' = (1.00\text{ m})\,[1 + (2.00\times10^{-5}/{}^\circ\text{C})(100^\circ\text{C})] = 1.002000\text{ m}$

$\dfrac{L_f - L_f'}{L_f} = 2.00\times10^{-6} = \boxed{2.00\times10^{-4}\%}$

$L_f = (1.00\text{ m})e^{[2.00\times10^{-2}({}^\circ\text{C})^{-1}](100^\circ\text{C})} = 7.389\text{ m}$

$L_f' = (1.00\text{ m})[1 + (0.0200/{}^\circ\text{C})(100^\circ\text{C})] = 3.000\text{ m}$

$\dfrac{L_f - L_f'}{L_f} = \boxed{59.4\%}$

19.68 At 20.0˚C, the unstretched lengths of the steel and copper wires are

$L_s(20.0^\circ\text{C}) = (2.000\text{ m})[1 + 11.0\times10^{-6})(\text{C}^\circ)^{-1}(-20.0^\circ\text{C})] = 1.99956\text{ m}$

$L_c(20.0^\circ\text{C}) = (2.000\text{ m})[1 + 17.0\times10^{-6})(\text{C}^\circ)^{-1}(-20.0^\circ\text{C})] = 1.99932\text{ m}$

Under a tension F, the length of the steel and copper wires are

$$L_s' = L_s\left[1 + \frac{F}{YA}\right]_s \qquad L_c' = L_c\left[1 + \frac{F}{YA}\right]_c \qquad \text{where } L_s' + L_c' = 4.000\text{ m}$$

Since the tension, F, must be the same in each wire, solve for F:

$$F = \frac{(L_s' + L_c') - (L_s + L_c)}{\dfrac{L_s}{Y_s A_s} + \dfrac{L_c}{Y_c A_c}}$$

When the wires are stretched, their areas become

$A_s = \pi(1.000\times10^{-3}\text{ m})^2[1 + (11.0\times10^{-6})(-20.0)]^2 = 3.140\times10^{-6}\text{ m}^2$

$A_c = \pi(1.000\times10^{-3}\text{ m})^2[1 + (17.0\times10^{-6})(-20.0)]^2 = 3.139\times10^{-6}\text{ m}^2$

Recall $Y_s = 20.0\times10^{10}$ Pa and $Y_c = 11.0\times10^{10}$ Pa. Substituting into the equation for F, we obtain

$$F = \frac{4.000\text{ m} - (1.99956\text{ m} + 1.99932\text{ m})}{\dfrac{1.99956\text{ m}}{(20.0\times10^{10}\text{ Pa})(3.140\times10^{-6})\text{ m}^2} + \dfrac{1.99932\text{ m}}{(11.0\times10^{10}\text{ Pa})(3.139\times10^{-6})\text{ m}^2}}$$

$F = \boxed{125\text{ N}}$

To find the *x*-coordinate of the junction,

$$L_s' = (1.99956\text{ m})\left[1 + \frac{125\text{ N}}{(20.0 \times 10^{10}\text{ N/m}^2)(3.140 \times 10^{-6}\text{ m}^2)}\right] = 1.999958\text{ m}$$

Thus the x-coordinate is $-2.000 + 1.999958 =$ $\boxed{-4.20 \times 10^{-5}\text{ m}}$

19.69 (a) $\mu = \pi r^2 \rho = \pi(5.00 \times 10^{-4}\text{ m})^2(7.86 \times 10^3\text{ kg/m}^3) =$ $\boxed{6.17 \times 10^{-3}\text{ kg/m}}$

(b) $f_1 = \dfrac{v}{2L}$ and $v = \sqrt{\dfrac{T}{\mu}}$ so $f_1 = \dfrac{1}{2L}\sqrt{\dfrac{T}{\mu}}$

Therefore, $T = \mu(2Lf_1)^2 = (6.17 \times 10^{-3})(2 \times 0.800 \times 200)^2 =$ $\boxed{632\text{ N}}$

(c) First find the unstressed length of the string at 0°C:

$$L = L_{\text{natural}}\left(1 + \frac{T}{AY}\right) \quad \text{so} \quad L_{\text{natural}} = \frac{L}{1 + T/AY}$$

$$A = \pi(5.00 \times 10^{-4}\text{ m})^2 = 7.854 \times 10^{-7}\text{ m}^2 \quad \text{and} \quad Y = 20.0 \times 10^{10}\text{ Pa}$$

Therefore, $\dfrac{T}{AY} = \dfrac{632}{(7.854 \times 10^{-7})(20.0 \times 10^{10})} = 4.02 \times 10^{-3}$, and

$$L_{\text{natural}} = \frac{(0.800\text{ m})}{(1 + 4.02 \times 10^{-3})} = 0.7968\text{ m}$$

The unstressed length at 30.0°C is $L_{30°\text{C}} = L_{\text{natural}}[1 + \alpha(30.0°\text{C} - 0.0°\text{C})]$,

or $L_{30°\text{C}} = (0.7968\text{ m})[1 + (11.0 \times 10^{-6})(30.0)] = 0.79706\text{ m}$

Since $L = L_{30°\text{C}}\left[1 + \dfrac{T'}{AY}\right]$, where T' is the tension in the string at 30.0°C,

$$T' = AY\left[\frac{L}{L_{30°\text{C}}} - 1\right] = (7.854 \times 10^{-7})(20.0 \times 10^{10})\left[\frac{0.800}{0.79706} - 1\right] = 580\text{ N}$$

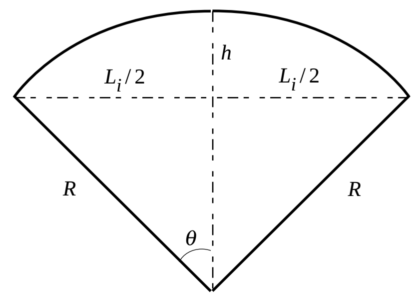
To find the frequency at 30.0°C, realize that

$$\frac{f_1'}{f_1} = \sqrt{\frac{T'}{T}} \quad \text{so} \quad f_1' = (200\text{ Hz})\sqrt{\frac{580\text{ N}}{632\text{ N}}} = \boxed{192\text{ Hz}}$$

19.70 Let 2θ represent the angle the curved rail subtends. We have

$$L_i + \Delta L = 2\theta R = L_i(1 + \alpha\,\Delta T) \quad \text{and}$$

$$\sin\theta = \frac{L_i/2}{R} = \frac{L_i}{2R}$$



Thus, $\theta = \frac{L_i}{2R}(1 + \alpha\,\Delta T) = (1 + \alpha\,\Delta T)\sin\theta$

and we must solve the transcendental equation

$$\theta = (1 + \alpha\,\Delta T)\sin\theta = (1.000\,0055)\sin\theta$$

Homing in on the non-zero solution gives, to four digits,

$$\theta = 0.01816 \text{ rad} = 1.0405°$$

Now, $h = R - R\cos\theta = \frac{L_i(1-\cos\theta)}{2\sin\theta}$

This yields $\boxed{h = 4.54 \text{ m}}$, a remarkably large value compared to $\Delta L = 5.50$ cm.