# Chapter 14 Solutions

***14.1** For two 70.0-kg persons, modeled as spheres,

$$F_g = \frac{Gm_1m_2}{r^2} = \frac{(6.67 \times 10^{-11}\ \text{N} \cdot \text{m}^2/\text{kg}^2)(70.0\ \text{kg})(70.0\ \text{kg})}{(2.00\ \text{m})^2} = \boxed{\sim 10^{-7}\ \text{N}}$$

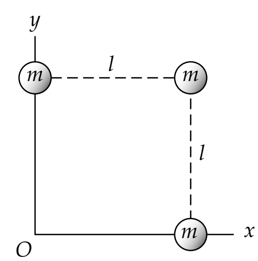
**14.2** (a) At the midpoint between the two masses, the forces exerted by the 200-kg and 500-kg masses are oppositely directed, and from $F_g = \frac{Gm_1m_2}{r^2}$ we have

$$\Sigma F = \frac{G(50.0\ \text{kg})(500\ \text{kg} - 200\ \text{kg})}{(0.200\ \text{m})^2} = \boxed{2.50 \times 10^{-5}\ \text{N}}\ \text{toward the 500-kg mass}$$

(b) At a point between the two masses at a distance $d$ from the 500-kg mass, the net force will be zero when

$$\frac{G(50.0\ \text{kg})(200\ \text{kg})}{(0.400\ \text{m} - d)^2} = \frac{G(50.0\ \text{kg})(500\ \text{kg})}{d^2} \quad \text{or} \quad d = \boxed{0.245\ \text{m}}$$

**14.3** $$\mathbf{g} = \frac{Gm}{l^2}\,\mathbf{i} + \frac{Gm}{l^2}\,\mathbf{j} + \frac{Gm}{2l^2}(\cos 45.0°\,\mathbf{i} + \sin 45.0°\,\mathbf{j})$$




so $$\mathbf{g} = \frac{GM}{l^2}\left(1 + \frac{1}{2\sqrt{2}}\right)(\mathbf{i} + \mathbf{j}) \quad \text{or}$$

$$\mathbf{g} = \boxed{\frac{Gm}{l^2}\left(\sqrt{2} + \frac{1}{2}\right)\text{toward the opposite corner}}$$

**14.4** $m_1 + m_2 = 5.00$ kg $m_2 = 5.00\ \text{kg} - m_1$

$$F = G\frac{m_1m_2}{r^2} \Rightarrow 1.00 \times 10^{-8}\ \text{N} = \left(6.67 \times 10^{-11}\ \frac{\text{N} \cdot \text{m}^2}{\text{kg}^2}\right)\frac{m_1(5.00\ \text{kg} - m_1)}{(0.200\ \text{m})^2}$$

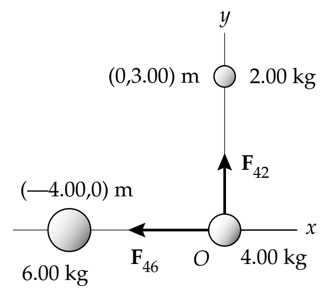
$$(5.00\ \text{kg})m_1 - m_1^2 = \frac{(1.00 \times 10^{-8}\ \text{N})(0.0400\ \text{m}^2)}{6.67 \times 10^{-11}\ \text{N} \cdot \text{m}^2/\text{kg}^2} = 6.00\ \text{kg}^2$$

Thus, $m_1^2 - (5.00\ \text{kg})m_1 + 6.00\ \text{kg} = 0$

or $(m_1 - 3.00\ \text{kg})(m_1 - 2.00\ \text{kg}) = 0$

giving $\boxed{m_1 = 3.00\ \text{kg, so } m_2 = 2.00\ \text{kg}}$. The answer $m_1 = 2.00$ kg and $m_2 = 3.00$ kg is physically equivalent.

**14.5** The force exerted on the 4.00-kg mass by the 2.00-kg mass is directed upward and given by




$$\mathbf{F}_{42} = G\frac{m_4 m_2}{r_{42}^2}\mathbf{j}$$

$$= \left(6.67 \times 10^{-11}\ \frac{\text{N}\cdot\text{m}^2}{\text{kg}^2}\right)\frac{(4.00\ \text{kg})(2.00\ \text{kg})}{(3.00\ \text{m})^2}\ \mathbf{j}$$

$$= 5.93 \times 10^{-11}\ \mathbf{j}\ \text{N}$$

The force exerted on the 4.00-kg mass by the 6.00-kg mass is directed to the left:

$$\mathbf{F}_{46} = G\frac{m_4 m_6}{r_{46}^2}(-\mathbf{i})$$

$$= \left(-6.67 \times 10^{-11}\ \frac{\text{N}\cdot\text{m}^2}{\text{kg}^2}\right)\frac{(4.00\ \text{kg})(6.00\ \text{kg})}{(4.00\ \text{m})^2}\ \mathbf{i} = -10.0 \times 10^{-11}\ \mathbf{i}\ \text{N}$$

Therefore, the resultant force on the 4.00-kg mass is

$$\mathbf{F}_4 = \mathbf{F}_{42} + \mathbf{F}_{46} = \boxed{(-10.0\mathbf{i} + 5.93\mathbf{j}) \times 10^{-11}\ \text{N}}$$

**14.6** $g = \frac{GM}{R^2} = \frac{G\rho(4\pi R^3/3)}{R^2} = \frac{4}{3}\pi G\rho R$

If $\frac{g_M}{g_E} = \frac{1}{6} = \frac{4\pi G\rho_M R_M/3}{4\pi G\rho_E R_E/3}$ then $\frac{\rho_M}{\rho_E} = \left(\frac{g_M}{g_E}\right)\left(\frac{R_E}{R_M}\right) = \left(\frac{1}{6}\right)(4) = \boxed{\frac{2}{3}}$

**14.7** (a) The Sun-Earth distance is $1.496 \times 10^{11}$ m, and the Earth-Moon distance is $3.84 \times 10^8$ m, so the distance from the Sun to the Moon during a solar eclipse is $1.496 \times 10^{11}$ m $- 3.84 \times 10^8$ m $= 1.492 \times 10^{11}$ m.

The mass of the Sun, Earth , and Moon are $M_s = 1.99 \times 10^{30}$ kg, $M_E = 5.98 \times 10^{24}$ kg, and $M_M = 7.36 \times 10^{22}$ kg. We have

$$F_{SM} = \frac{Gm_1m_2}{r^2} = \frac{(6.67 \times 10^{-11})(1.99 \times 10^{30})(7.36 \times 10^{22})}{(1.492 \times 10^{11})^2} = \boxed{4.39 \times 10^{20}\ \text{N}}$$

(b) $$F_{EM} = \frac{(6.67 \times 10^{-11}\ \text{Nm}^2/\text{kg}^2)(5.98 \times 10^{24})(7.36 \times 10^{22})}{(3.84 \times 10^8)^2} = \boxed{1.99 \times 10^{20}\ \text{N}}$$

(c) $$F_{SE} = \frac{(6.67 \times 10^{-11}\ \text{N}\cdot\text{m}^2/\text{kg}^2)(1.99 \times 10^{30})(5.98 \times 10^{24})}{(1.496 \times 10^{11})^2} = \boxed{3.55 \times 10^{22}\ \text{N}}$$

**14.8** (a) $v = \frac{2\pi r}{T} = \frac{2\pi(384\,400)\times 10^3\ \text{m}}{27.3\times(86\,400\ \text{s})} = \boxed{1.02\times 10^3\ \text{m/s}}$

(b) In one second, the Moon falls a distance

$$x = \frac{1}{2}at^2 = \frac{1}{2}\frac{v^2}{r}t^2 = \frac{1}{2}\frac{(1.02\times 10^3)^2}{(3.844\times 10^8)}\times(1.00)^2 = 1.35\times 10^{-3}\ \text{m} = \boxed{1.35\ \text{mm}}$$

The Moon only moves inward 1.35 mm for every 1020 meters it moves along a straight-line path.

**14.9** $a = \frac{MG}{(4R_E)^2} = \frac{9.80\ \text{m/s}^2}{16.0} = \boxed{0.613\ \text{m/s}^2}$ toward the Earth

***14.10** $F = m_1 g = \frac{Gm_1m_2}{r^2}$

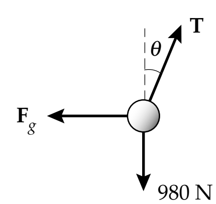
$$g = \frac{Gm_2}{r^2} = \frac{(6.67\times 10^{-11}\ \text{N}\cdot\text{m}^2/\text{kg}^2)(4.00\times 10^4\times 10^3\ \text{kg})}{(100\ \text{m})^2} = \boxed{2.67\times 10^{-7}\ \text{m/s}^2}$$

**14.11** The separation is nearly 1.000 m, so one ball attracts the other with force

$$F_g = \frac{Gm_1m_2}{r^2} = \frac{6.67\times 10^{-11}\ \text{N}\cdot\text{m}^2(100\ \text{kg})^2}{\text{kg}^2(1.000\ \text{m})^2} = 6.67\times 10^{-7}\ \text{N}$$

Call $\theta$ the angle of each cable from the vertical and $T$ its tension.

Each ball is in equilibrium, with




$$T\cos\theta = mg = 980\ \text{N}$$

$$T\sin\theta = 6.67\times 10^{-7}\ \text{N}$$

$$\tan\theta = \frac{6.67\times 10^{-7}\ \text{N}}{980\ \text{N}} = 6.81\times 10^{-10}$$

Each ball scrunches in by

$$(45.0\ \text{m})\sin\theta = 45.0\ \text{m}(6.81\times 10^{-10}) = 3.06\times 10^{-8}\ \text{m}$$

so their separation is $\boxed{1.000\ \text{m} - 61.3\ \text{nm}}$

**14.12** (a) At the zero-total field point, $\frac{GmM_E}{r_E^2} = \frac{GmM_M}{r_M^2}$ so

$$r_M = r_E\sqrt{\frac{M_M}{M_E}} = r_E\sqrt{\frac{7.36 \times 10^{22}}{5.98 \times 10^{24}}} = \frac{r_E}{9.01}$$

$$r_E + r_M = 3.84 \times 10^8 \text{ m} = r_E + \frac{r_E}{9.01}$$

$$r_E = \frac{3.84 \times 10^8 \text{ m}}{1.11} = \boxed{3.46 \times 10^8 \text{ m}}$$

(b) At this distance the acceleration due to the Earth's gravity is

$$g_E = \frac{GM_E}{r_E^2} = \frac{(6.67 \times 10^{-11} \text{ N} \cdot \text{m}^2/\text{kg}^2)(5.98 \times 10^{24} \text{ kg})}{(3.46 \times 10^8 \text{ m})^2}$$

$$g_E = \boxed{3.34 \times 10^{-3} \text{ m/s}^2 \text{ directed toward the Earth}}$$

**14.13** Since speed is constant, the distance traveled between $t_1$ and $t_2$ is equal to the distance traveled between $t_3$ and $t_4$. The area of a triangle is equal to one-half its (base) width across one side times its (height) dimension perpendicular to that side.

So $\frac{1}{2}\, bv(t_2 - t_1) = \frac{1}{2}\, bv(t_4 - t_3)$

states that the particle's radius vector sweeps out equal areas in equal times.

***14.14** (a) For the geosynchronous satellite, $\Sigma F_r = \frac{GmM_E}{r^2} = ma_r = \frac{mv^2}{r}$ becomes

$$\frac{GM_E}{r} = \left(\frac{2\pi r}{T}\right)^2 \quad \text{or} \quad r^3 = \frac{GM_E T^2}{4\pi^2}$$

Thus, the radius of the satellite orbit is

$$r = \left[\frac{(6.67 \times 10^{-11} \text{ N} \cdot \text{m}^2/\text{kg}^2)(5.98 \times 10^{24} \text{ kg})(86\,400 \text{ s})^2}{4\pi^2}\right]^{1/3} = \boxed{4.23 \times 10^7 \text{ m}}$$

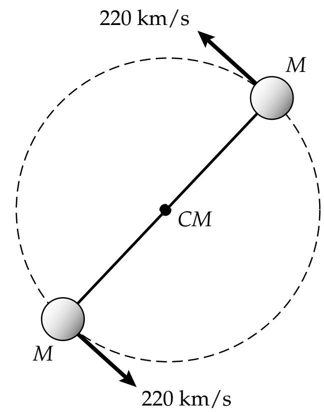
(b) The satellite is so far out that its distance from the north pole,

$$d = \sqrt{(6.37 \times 10^6 \text{ m})^2 + (4.23 \times 10^7 \text{ m})^2} = 4.27 \times 10^7 \text{ m}$$

is nearly the same as its orbital radius. The travel time for the radio signal is

$$t = \frac{2d}{c} = \frac{2(4.27 \times 10^7 \text{ m})}{3.00 \times 10^8 \text{ m/s}} = \boxed{0.285 \text{ s}}$$

**14.15** Centripetal force = Gravitational force between stars' centers




$$\frac{Mv^2}{r} = \frac{GMM}{(2r)^2}$$

For a circular orbit, $v = \frac{2\pi r}{T}$ for each star.

Solving,

$$M = \boxed{12.6 \times 10^{31}\text{ kg} = 63.3\text{ solar masses}}$$

for each star.

The two blue giant stars comprising Plaskett's binary system are among the most massive known.

**Goal Solution**

**G:** From the given data, it is difficult to estimate a reasonable answer to this problem without actually working through the details to actually solve it. A reasonable guess might be that each star has a mass equal to or larger than our Sun since our Sun happens to be less massive than many stars in the universe.

**O:** The only force acting on the two stars is the central gravitational force of attraction which results in a centripetal acceleration. When we solve Newton's 2nd law, we can find the unknown mass in terms of the variables given in the problem.

**A:** Applying Newton's 2nd Law, $\sum F = ma$ yields $F_g = ma_c$ for each star:

$$\frac{GMM}{(2r)^2} = \frac{Mv^2}{r} \quad \text{or} \quad M = \frac{4v^2r}{G}$$

We can write $r$ in terms of the period, $T$, by considering the time and distance of one complete cycle. The distance traveled in one orbit is the circumference of the stars' common orbit , so $2\pi r = vT$. Therefore

$$M = \frac{4v^2r}{G} = \frac{4v^2}{G}\left(\frac{vT}{2\pi}\right)$$

so, $$M = \frac{2v^3T}{\pi G} = \frac{2(220 \times 10^3\text{ m/s})^3(14.4\text{ d})(86\,400\text{ s/d})}{\pi(6.67 \times 10^{-11}\text{ N} \cdot \text{m}^2/\text{kg}^2)} = 1.26 \times 10^{32}\text{ kg}$$

**L:** The mass of each star is about 63 solar masses, much more than our initial guess! A quick check in an astronomy book reveals that stars over 8 solar masses are considered to be *heavyweight* stars, and astronomers estimate that the maximum theoretical limit is about 100 solar masses before a star becomes unstable. So these 2 stars are exceptionally massive!

**14.16** Centripetal force = Gravitational force between stars' centers

$$\frac{Mv^2}{r} = \frac{GMM}{(2r)^2} \quad \text{which reduces to} \quad M = \frac{4rv^2}{G}$$

For a circular orbit, $v = \frac{2\pi r}{T}$ for each star. Hence, the radius of the orbit is given by $r = \frac{vT}{2\pi}$, and the mass of each star is then $M = \boxed{\frac{2v^3T}{\pi G}}$

**14.17** By conservation of angular momentum,

$$r_p v_p = r_a v_a$$

$$\frac{v_p}{v_a} = \frac{r_a}{r_p} = \frac{2289 \text{ km} + 6.37 \times 10^3 \text{ km}}{459 \text{ km} + 6.37 \times 10^3 \text{ km}} = \frac{8659 \text{ km}}{6829 \text{ km}} = \boxed{1.27}$$

We do not need to know the period.

**14.18** By Kepler's Third Law,

$$T^2 = ka^3 \quad (a = \text{semi-major axis})$$

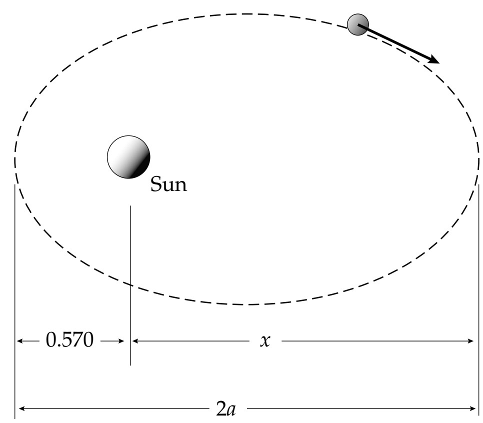
For any object orbiting the Sun, with $T$ in years and $a$ in A.U., $k = 1.00$

Therefore, for Comet Halley

$$(75.6)^2 = (1.00)\left(\frac{0.570 + x}{2}\right)^3$$

The farthest distance the comet gets from the Sun is

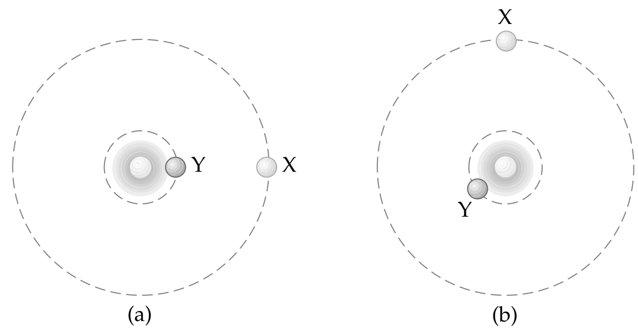
$$x = 2(75.6)^{2/3} - 0.570 = \boxed{35.2 \text{ A.U.}} \quad \text{(out around the orbit of Pluto)}$$




**14.19** $T^2 = \frac{4\pi^2 d^2}{GM}$ (Kepler's Third Law with $m << M$)

$$M = \frac{4\pi^2 d^3}{GT^2} = \boxed{1.90 \times 10^{27} \text{ kg}} \quad \text{(approximately 316 Earth masses)}$$

**14.20** $\sum F = ma$




$$\frac{Gm_{\text{planet}} M_{\text{star}}}{r^2} = \frac{m_{\text{planet}} v^2}{r}$$

$$\frac{GM_{\text{star}}}{r} = v^2 = r^2 \omega^2$$

$$GM_{\text{star}} = r^3 \omega^2 = r_x^3 \omega_x^2 = r_y^3 \omega_y^2$$

$$\omega_y = \omega_x \left(\frac{r_x}{r_y}\right)^{3/2}$$

$$\omega_y = \left(\frac{90.0°}{5.00 \text{ yr}}\right) 3^{3/2} = \frac{468°}{5.00 \text{ yr}}$$

So $\boxed{\text{planet } Y \text{ has turned through 1.30 revolutions}}$

**14.21** $$\frac{GM_J}{(R_J + d)^2} = \frac{4\pi^2 (R_J + d)}{T^2}$$

$$GM_J T^2 = 4\pi^2 (R_J + d)^3$$

$$(6.67 \times 10^{-11}) \frac{\text{N} \cdot \text{m}^2}{\text{kg}^2} (1.90 \times 10^{27} \text{ kg})(9.84 \times 3600)^2 = 4\pi^2 (6.99 \times 10^7 + d)^3$$

$d = \boxed{8.92 \times 10^7 \text{ m}} = \boxed{89\,200 \text{ km}}$ above the planet

***14.22** The gravitational force on a small parcel of material at the star's equator supplies the necessary centripetal force:

$$\frac{GM_s m}{R_s^2} = \frac{mv^2}{R_s} = mR_s \omega^2$$

so $$\omega = \sqrt{\frac{GM_s}{R_s^3}} = \sqrt{\frac{(6.67 \times 10^{-11} \text{ N} \cdot \text{m}^2/\text{kg}^2)[2(1.99 \times 10^{30} \text{ kg})]}{(10.0 \times 10^3 \text{ m})^3}}$$

$$\omega = \boxed{1.63 \times 10^4 \text{ rad/s}}$$

***14.23** Let $m$ represent the mass of the spacecraft, $r_E$ the radius of the Earth's orbit, and $x$ the distance from Earth to the spacecraft.

The Sun exerts a radial inward force of $F_S = \dfrac{GM_S m}{(r_E - x)^2}$ on the spacecraft while the Earth exerts a radial outward force of $F_E = \dfrac{GM_E m}{x^2}$ on it. The net force on the spacecraft must produce the correct centripetal acceleration for it to have an orbital period of 1.000 year. Thus,

$$F_S - F_E = \frac{GM_S m}{(r_E - x)^2} - \frac{GM_E m}{x^2} = \frac{mv^2}{(r_E - x)} = \frac{m}{(r_E - x)}\left[\frac{2\pi(r_E - x)}{T}\right]^2$$

which reduces to

$$\frac{GM_S}{(r_E - x)^2} - \frac{GM_E}{x^2} = \frac{4\pi^2(r_E - x)}{T^2} \qquad (1)$$

This equation is fifth degree in $x$, so we do not solve it algebraically. We may test the assertion that $x$ is between $1.47 \times 10^9$ m and $1.48 \times 10^9$ m by substituting both of these as trial solutions, along with the following data: $M_S = 1.991 \times 10^{30}$ kg, $M_E = 5.983 \times 10^{24}$ kg, $r_E = 1.496 \times 10^{11}$ m, and $T = 1.000$ yr $= 3.156 \times 10^7$ s.

With $x = 1.47 \times 10^9$ m substituted into Equation (1), we obtain

$$6.052 \times 10^{-3}\ \text{m/s}^2 - 1.85 \times 10^{-3}\ \text{m/s}^2 \approx 5.871 \times 10^{-3}\ \text{m/s}^2$$

or $5.868 \times 10^{-3}\ \text{m/s}^2 \approx 5.871 \times 10^{-3}\ \text{m/s}^2$

With $x = 1.48 \times 10^9$ m substituted into the same equation, the result is

$$6.053 \times 10^{-3}\ \text{m/s}^2 - 1.82 \times 10^{-3}\ \text{m/s}^2 \approx 5.8708 \times 10^{-3}\ \text{m/s}^2$$

or $5.8709 \times 10^{-3}\ \text{m/s}^2 \approx 5.808 \times 10^{-3}\ \text{m/s}^2$

Since the first trial solution makes the left-hand side of Equation (1) slightly less than the right-hand side, and the second trial solution does the opposite, the true solution is determined as between the trial values. To three-digit precision, it is $1.48 \times 10^9$ m.

**14.24** (a) $F = \dfrac{GMm}{r^2} = \dfrac{(6.67 \times 10^{-11}\ \text{N}\cdot\text{m}^2/\text{kg}^2)[100(1.99 \times 10^{30}\ \text{kg})(10^3\ \text{kg})]}{(1.00 \times 10^4\ \text{m} + 50.0\ \text{m})^2}$

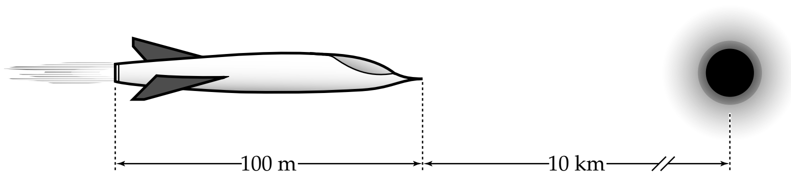
$$F = \boxed{1.31 \times 10^{17}\ \text{N}}$$

(b) $\Delta F = \frac{GMm}{r_{\text{front}}^2} - \frac{GMm}{r_{\text{back}}^2}$

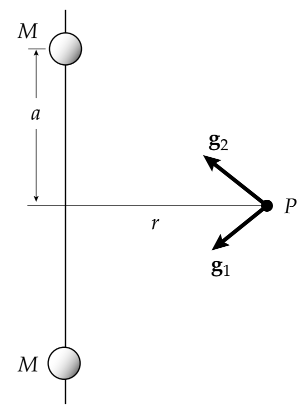
$$\Delta g = \frac{\Delta F}{m} = \frac{GM(r_{\text{back}}^2 - r_{\text{front}}^2)}{r_{\text{front}}^2 r_{\text{back}}^2}$$

$$= \frac{(6.67 \times 10^{-11})[100(1.99 \times 10^{30})][(1.01 \times 10^4\ \text{m})^2 - (1.00 \times 10^4\ \text{m})^2]}{(1.00 \times 10^4\ \text{m})^2(1.01 \times 10^4\ \text{m})^2}$$

$$\Delta g = \boxed{2.62 \times 10^{12}\ \text{N/kg}}$$




**14.25** $g_1 = g_2 = \frac{MG}{r^2 + a^2}$




$g_{1y} = -g_{2y}$

$g_y = g_{1y} + g_{2y} = 0$

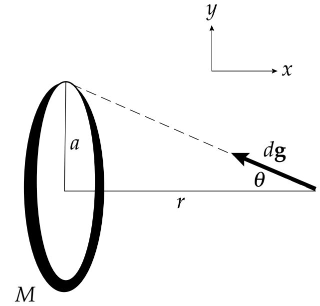
$g_{1x} = g_{2x} = g_2 \cos\theta$

$\cos\theta = \frac{r}{(a^2 + r^2)^{1/2}}$

$\mathbf{g} = 2g_{2x}(-\mathbf{i})$ or

$$\mathbf{g} = \boxed{\frac{2MGr}{(r^2 + a^2)^{3/2}} \text{ toward the center of mass}}$$

**14.26** $\cos\theta = \frac{r}{(a^2 + r^2)^{1/2}}$




$dg_x = dg\cos\theta \qquad g_y = 0$

$$\int dg_x = \int \frac{GdM}{a^2 + r^2}\cos\theta$$

$$g_x = \int \frac{GdM}{(a^2 + r^2)} \frac{r}{(a^2 + r^2)^{1/2}}$$

$$g_x = \boxed{\frac{GMr}{(a^2 + r^2)^{3/2}} \text{ inward along } r}$$

**14.27** (a) $U = -\frac{GM_E m}{r}$

$$U = -\frac{(6.67 \times 10^{-11}\ \text{N} \cdot \text{m}^2/\text{kg}^2)(5.98 \times 10^{24}\ \text{g})(100\ \text{kg})}{(6.37 + 2.00) \times 10^6\ \text{m}} = \boxed{-4.77 \times 10^9\ \text{J}}$$

(b) and (c) Planet and satellite exert forces of equal magnitude on each other, directed downward on the satellite and upward on the planet.

$$F = \frac{GM_E m}{r^2}$$

$$F = \frac{(6.67 \times 10^{-11}\ \text{N} \cdot \text{m}^2/\text{kg}^2)(5.98 \times 10^{24}\ \text{kg})(100\ \text{kg})}{(8.37 \times 10^6\ \text{m})^2} = \boxed{569\ \text{N}}$$

**14.28** $U = -G\frac{Mm}{r}$ and $g = \frac{GM_E}{R_E^2}$

so that $\Delta U = -GMm\left(\frac{1}{3R_E} - \frac{1}{R_E}\right) = \frac{2}{3}\ mgR_E$

$$\Delta U = \frac{2}{3}(1000\ \text{kg})(9.80\ \text{m/s}^2)(6.37 \times 10^6\ \text{m}) = \boxed{4.17 \times 10^{10}\ \text{J}}$$

**14.29** (a) $\rho = \frac{M_S}{\frac{4}{3}\pi r_E^3} = \frac{3(1.99 \times 10^{30}\ \text{kg})}{4\pi(6.37 \times 10^6\ \text{m})^3} = \boxed{1.84 \times 10^9\ \text{kg/m}^3}$

(b) $g = \frac{GM_S}{r_E^2} = \frac{(6.67 \times 10^{-11}\ \text{N} \cdot \text{m}^2/\text{kg}^2)(1.99 \times 10^{30}\ \text{kg})}{(6.37 \times 10^6\ \text{m})^2} = \boxed{3.27 \times 10^6\ \text{m/s}^2}$

(c) $U_g = -\frac{GM_S m}{r_E} = -\frac{(6.67 \times 10^{-11}\ \text{N} \cdot \text{m}^2/\text{kg}^2)(1.99 \times 10^{30}\ \text{kg})(1.00\ \text{kg})}{6.37 \times 10^6\ \text{m}}$

$U_g = \boxed{-2.08 \times 10^{13}\ \text{J}}$

**Goal Solution**

(a) $\rho = \frac{M_s}{V} = \frac{M_s}{\left(\frac{4}{3}\right)\pi R_E^3} = \frac{1.99 \times 10^{30}\ \text{kg}}{\left(\frac{4}{3}\right)\pi\ (6.37 \times 10^6\ \text{m})^3} = 1.84 \times 10^9\ \text{kg/m}^3$

(This white dwarf is on the order of 1 million times more dense than concrete!)

(b) For an object of mass $m$ on its surface, $mg = GM_s m/R_E^2$. Thus,

$$G = \frac{GM_S}{R_E^2} = \frac{(6.67 \times 10^{-11}\ \text{N} \cdot \text{m}^2/\text{kg}^2)(1.99 \times 10^{30}\ \text{kg})}{(6.37 \times 10^6\ \text{m})^2} = 3.27 \times 10^6\ \text{m/s}^2$$

(This acceleration is about 1 million times more than $g$ earth!)

(c) $U_g = \frac{-GM_s m}{R_E} = \frac{(-6.67 \times 10^{-11}\ \text{N} \cdot \text{m}^2/\text{kg}^2)(1.99 \times 10^{30}\ \text{kg})(1\ \text{kg})}{(6.37 \times 10^6\ \text{m})} = -2.08 \times 10^{13}\ \text{J}$

(Such a large loss of potential energy could yield a big gain in kinetic energy. For example, dropping the 1.00-kg object from a height of 1.00 m would result in a final velocity of 2 560 m/s!).

***14.30** The height attained is not small compared to the radius of the Earth, so $U = mgy$ does not apply; $U = -\frac{GM_1M_2}{r}$ does. From launch to apogee at height $h$,

$$K_i + U_i + \Delta E = K_f + U_f$$

$$\frac{1}{2} M_p v_i^2 - \frac{GM_EM_p}{R_E} + 0 = 0 - \frac{GM_EM_p}{R_E + h}$$

$$\frac{1}{2}(10.0 \times 10^3\ \text{m/s})^2 - (6.67 \times 10^{-11}\ \text{N} \cdot \text{m}^2/\text{kg}^2)\left(\frac{5.98 \times 10^{24}\ \text{kg}}{6.37 \times 10^6\ \text{m}}\right)$$

$$= -(6.67 \times 10^{-11}\ \text{N} \cdot \text{m}^2/\text{kg}^2)\left(\frac{5.98 \times 10^{24}\ \text{kg}}{6.37 \times 10^6\ \text{m} + h}\right)$$

$$(5.00 \times 10^7\ \text{m}^2/\text{s}^2) - (6.26 \times 10^7\ \text{m}^2/\text{s}^2) = \frac{-3.99 \times 10^{14}\ \text{m}^3/\text{s}^2}{6.37 \times 10^6\ \text{m} + h}$$

$$6.37 \times 10^6\ \text{m} + h = \frac{3.99 \times 10^{14}\ \text{m}^3/\text{s}^2}{1.26 \times 10^7\ \text{m}^2/\text{s}^2} = 3.16 \times 10^7\ \text{m}$$

$$\boxed{h = 2.52 \times 10^7\ \text{m}}$$

***14.31** (a) $U_{\text{Tot}} = U_{12} + U_{13} + U_{23} = 3U_{12} = 3\left(-\frac{Gm_1m_2}{r_{12}}\right)$

$$U_{\text{Tot}} = -\frac{3(6.67 \times 10^{-11}\ \text{N} \cdot \text{m}^2/\text{kg}^2)(5.00 \times 10^{-3}\ \text{kg})^2}{0.300\ \text{m}} = \boxed{-1.67 \times 10^{-14}\ \text{J}}$$

(b) **At the center** of the equilateral triangle

**14.32** $W = -\Delta U = -\left(\frac{-Gm_1m_2}{r} - 0\right)$

$$W = \frac{+\,6.67 \times 10^{-11}\ \text{N} \cdot \text{m}^2(7.36 \times 10^{22}\ \text{kg})(1.00 \times 10^3\ \text{kg})}{\text{kg}^2\,(1.74 \times 10^6\ \text{m})} = \boxed{2.82 \times 10^9\ \text{J}}$$

**14.33** $$\frac{v_i^2}{R_E + h} = \frac{GM_E}{(R_E + h)^2}$$

$$K_i = \frac{1}{2}\,mv_i^2 = \frac{1}{2}\frac{GM_E m}{R_E + h}$$

$$= \frac{1}{2}\frac{(6.67 \times 10^{-11}\ \text{N} \cdot \text{m}^2/\text{kg}^2)(5.98 \times 10^{24}\ \text{kg})(500\ \text{kg})}{(6.37 \times 10^6\ \text{m}) + (0.500 \times 10^6\ \text{m})} = 1.45 \times 10^{10}\ \text{J}$$

The change in gravitational potential energy is

$$\Delta U = \frac{GM_E m}{R_i} - \frac{GM_E m}{R_f} = GM_E m\left(\frac{1}{R_i} - \frac{1}{R_f}\right)$$

$$= (6.67 \times 10^{-11}\ \text{N} \cdot \text{m}^2/\text{kg}^2)(5.98 \times 10^{24}\ \text{kg})(500\ \text{kg})(-1.14 \times 10^{-8}\ \text{m}^{-1})$$

$$= -2.27 \times 10^9\ \text{J}$$

Also, $K_f = \frac{1}{2}\,mv_f^2 = \frac{1}{2}(500\ \text{kg})(2.00 \times 10^3\ \text{m/s})^2 = 1.00 \times 10^9\ \text{J}$

The energy lost to friction is

$$E_f = K_i - K_f - \Delta U = (14.5 - 1.00 + 2.27) \times 10^9\ \text{J} = \boxed{1.58 \times 10^{10}\ \text{J}}$$

**14.34** (a) $$v_{\text{solar escape}} = \sqrt{\frac{2M_{\text{Sun}}G}{R_{E\,\cdot\,\text{Sun}}}} = \boxed{42.1\ \text{km/s}}$$

(b) $$v = \sqrt{\frac{2M_{\text{Sun}}G}{R_{E\,\cdot\,S}\,x}} = \frac{42.1}{\sqrt{x}}$$

If $v = \dfrac{125\,000\ \text{km}}{3\,600\ \text{s}}$, then $x = 1.47$ A.U. $= \boxed{2.20 \times 10^{11}\ \text{m}}$

(at or beyond the orbit of Mars, 125 000 km/h is sufficient for escape)

**14.35** $F_c = F_G$ gives $\dfrac{mv^2}{r} = \dfrac{GmM_E}{r^2}$

which reduces to $v = \sqrt{\dfrac{GM_E}{r}}$

and period $= \dfrac{2\pi r}{v} = 2\pi r\sqrt{\dfrac{r}{GM_E}}$

(a) $r = R_E + 200 \text{ km} = 6370 \text{ km} + 200 \text{ km} = 6570 \text{ km}$

Thus,

$$\text{period} = 2\pi(6.57 \times 10^6 \text{ m})\sqrt{\frac{(6.57 \times 10^6 \text{ m})}{(6.67 \times 10^{-11} \text{ N} \cdot \text{m}^2/\text{kg}^2)(5.98 \times 10^{24} \text{ kg})}}$$

$$T = 5.30 \times 10^3 \text{ s} = 88.3 \text{ min} = \boxed{1.47 \text{ h}}$$

(b) $$v = \sqrt{\frac{GM_E}{r}} = \sqrt{\frac{(6.67 \times 10^{-11} \text{ N} \cdot \text{m}^2/\text{kg}^2)(5.98 \times 10^{24} \text{ kg})}{(6.57 \times 10^6 \text{ m})}} = \boxed{7.79 \text{ km/s}}$$

(c) $K_f + U_f = K_i + U_i +$ energy input, gives

$$\text{input} = \frac{1}{2} mv_f^2 - \frac{1}{2} mv_i^2 + \left(\frac{-GM_E m}{r_f}\right) - \left(\frac{-GM_E m}{r_i}\right) \quad (1)$$

$$r_i = R_E = 6.37 \times 10^6 \text{ m}$$

$$v_i = \frac{2\pi R_E}{86\,400 \text{ s}} = 4.63 \times 10^2 \text{ m/s}$$

Substituting the appropriate values into (1) yields the

$$\text{minimum energy input} = \boxed{6.43 \times 10^9 \text{ J}}$$

**14.36** The gravitational force supplies the needed centripetal acceleration. Thus,

$$\frac{GM_E m}{(R_E + h)^2} = \frac{mv^2}{(R_E + h)} \quad \text{or} \quad v^2 = \frac{GM_E}{R_E + h}$$

(a) $$T = \frac{2\pi r}{v} = \frac{2\pi(R_E + h)}{\sqrt{GM_E/(R_E + h)}} = \boxed{2\pi\sqrt{\frac{(R_E + h)^3}{GM_E}}}$$

(b) $$v = \boxed{\sqrt{\frac{GM_E}{R_E + h}}}$$

(c) minimum energy input $= \Delta E_{\min} = (K_f + U_{gf}) - (K_i + U_{gi})$

where $K_i = \frac{1}{2} mv_i^2$ with $v_i = \frac{2\pi R_E}{1.00 \text{ day}} = \frac{2\pi R_E}{86\,400 \text{ s}}$

and $U_{gi} = -\frac{GM_E m}{R_E}$

Thus,

$$\Delta E_{\min} = \frac{1}{2}\, m\left(\frac{GM_E}{R_E + h}\right) - \frac{GM_E m}{R_E + h} - \frac{1}{2}\, m\left[\frac{4\pi^2 R_E^2}{(86\ 400\ \text{s})^2}\right] + \frac{GM_E m}{R_E}$$

or $$\boxed{\Delta E_{\min} = GM_E m\left[\frac{R_E + 2h}{2R_E(R_E + h)}\right] - \frac{2\pi^2 R_E^2 m}{(86\ 400\ \text{s})^2}}$$

**14.37** $$\frac{mv_1^2}{2} - \frac{GM_E m}{R_E} = -\frac{GM_E m}{r_{\max}} + \frac{mv_f^2}{2}$$

or $$v_f^2 = v_1^2 - \frac{2GM_E}{R_E}$$

and $$v_f = \left(v_1^2 - \frac{2GM_E}{R_E}\right)^{1/2}$$

$$v_f = [(2.00 \times 10^4)^2 - 1.25 \times 10^8]^{1/2} = \boxed{1.66 \times 10^4\ \text{m/s}}$$

**Goal Solution**

Energy is conserved between surface and the distant point:

$$(K + U_g)_i + \Delta E = (K + U_g)_f$$

$$\frac{1}{2}\, mv_i^2 - \frac{GM_E m}{R_E} + 0 = \frac{1}{2}\, mv_f^2 - \frac{GM_E m}{\infty}$$

$$v_f^2 = v_i^2 - \frac{2GM_E}{R_E} \qquad \left(\text{Note: } \frac{2GM_E}{R_E} \text{ is simply } v_{\text{esc}}^2\right)$$

$$v_f^2 = (2.00 \times 10^4\ \text{m/s})^2 - \frac{2(6.67 \times 10^{-11}\ \text{N} \cdot \text{m}^2/\text{kg}^2)(5.98 \times 10^{24}\ \text{kg})}{(6.37 \times 10^6\ \text{m})}$$

$$v_f^2 = 4.00 \times 10^8\ \text{m}^2/\text{s}^2 - 1.25 \times 10^8\ \text{m}^2/\text{s}^2 = 2.75 \times 10^8\ \text{m}^2/\text{s}^2$$

$$v_f = 1.66 \times 10^4\ \text{m/s}$$

**14.38** $E_{\text{tot}} = -\dfrac{GMm}{2r}$

$$\Delta E = \frac{GMm}{2}\left(\frac{1}{r_i} - \frac{1}{r_f}\right)$$

$$= \frac{(6.67\times10^{-11})(5.98\times10^{24})}{2}\frac{10^3\text{ kg}}{10^3\text{ m}}\left(\frac{1}{6370+100} - \frac{1}{6370+200}\right)$$

$\Delta E = 4.69\times10^{8}\text{ J} = \boxed{469\text{ MJ}}$

**14.39** To obtain the orbital velocity, we use

$$\Sigma F = \frac{mMG}{R^2} = \frac{mv^2}{R}$$

or $v = \sqrt{\dfrac{MG}{R}}$

We can obtain the escape velocity from

$$\frac{1}{2}mv_{\text{esc}}^2 = \frac{mMG}{R}$$

or $v_{\text{esc}} = \sqrt{\dfrac{2MG}{R}} = \boxed{\sqrt{2}\,v}$

***14.40** $g_E = \dfrac{Gm_E}{r_E^2}$ $\qquad g_U = \dfrac{Gm_U}{r_U^2}$

(a) $\dfrac{g_U}{g_E} = \dfrac{m_U r_E^2}{m_E r_U^2} = 14.0\left(\dfrac{1}{3.70}\right)^2 = 1.02$

$g_U = (1.02)(9.80\text{ m/s}^2) = \boxed{10.0\text{ m/s}^2}$

(b) $v_{\text{esc},E} = \sqrt{\dfrac{2Gm_E}{r_E}}$ $\qquad v_{\text{esc},U} = \sqrt{\dfrac{2Gm_U}{r_U}}$

$$\frac{v_{\text{esc},U}}{v_{\text{esc},E}} = \sqrt{\frac{m_U r_E}{m_E r_U}} = \sqrt{\frac{14.0}{3.70}} = 1.95$$

For the Earth,

$v_{\text{esc},E} = 11.2\text{ km/s}$ (from Table 14.3)

$\therefore v_{\text{esc},U} = (1.95)(11.2\text{ km/s}) = \boxed{21.8\text{ km/s}}$

**14.41** The rocket is in a potential well at Ganymede's surface with energy

$$U_1 = -\frac{Gm_1m_2}{r} = -\frac{6.67 \times 10^{-11}\ \mathrm{N \cdot m^2}\ m_2\ (1.495 \times 10^{23}\ \mathrm{kg})}{\mathrm{kg^2}\ (2.64 \times 10^6\ \mathrm{m})}$$

$$U_1 = -3.78 \times 10^6\ m_2\ \mathrm{m^2/s^2}$$

The potential well from Jupiter at the distance of Ganymede is

$$U_2 = -\frac{Gm_1m_2}{r} = -\frac{6.67 \times 10^{-11}\ \mathrm{N \cdot m^2}\ m_2\ (1.90 \times 10^{27}\ \mathrm{kg})}{\mathrm{kg^2}\ (1.071 \times 10^9\ \mathrm{m})}$$

$$U_2 = -1.18 \times 10^8\ m_2\ \mathrm{m^2/s^2}$$

To escape from both requires

$$\frac{1}{2} m_2 v_{\mathrm{esc}}^2 = +(3.78 \times 10^6 + 1.18 \times 10^8) m_2\ \mathrm{m^2/s^2}$$

$$v_{\mathrm{esc}} = \sqrt{2 \times 1.22 \times 10^8\ \mathrm{m^2/s^2}} = \boxed{15.6\ \mathrm{km/s}}$$

**14.42** We interpret "lunar escape speed" to be the escape speed from the surface of a stationary moon alone in the Universe:

$$\frac{1}{2} m v_{\mathrm{esc}}^2 = \frac{GM_m m}{R_m}$$

$$v_{\mathrm{esc}} = \sqrt{\frac{2GM_m}{R_m}}$$

$$v_{\mathrm{launch}} = 2\sqrt{\frac{2GM_m}{R_m}}$$

Now for the flight from moon to Earth

$$(K + U)_i = (K + U)_f$$

$$\frac{1}{2} m v_{\mathrm{launch}}^2 - \frac{GmM_m}{R_m} - \frac{GmM_E}{r_{\mathrm{el}}} = \frac{1}{2} m v_{\mathrm{impact}}^2 - \frac{GmM_m}{r_{m2}} - \frac{GmM_E}{R_E}$$

$$\frac{4GM_m}{R_m} - \frac{GM_m}{R_m} - \frac{GM_E}{r_{\mathrm{el}}} = \frac{1}{2} v_{\mathrm{impact}}^2 - \frac{GM_m}{r_{m2}} - \frac{GM_E}{R_E}$$

$$v_{\text{impact}} = \left[2G\left(\frac{3M_m}{R_m} + \frac{M_m}{r_{m2}} + \frac{M_E}{R_E} - \frac{M_E}{r_{\text{el}}}\right)\right]^{1/2}$$

$$= \left[2G\left(\frac{3 \times 7.36 \times 10^{22}\text{ kg}}{1.74 \times 10^6\text{ m}} + \frac{7.36 \times 10^{22}\text{ kg}}{3.84 \times 10^8\text{ m}}\right.\right.$$

$$\left.\left. + \frac{5.98 \times 10^{24}\text{ kg}}{6.37 \times 10^6\text{ m}} - \frac{5.98 \times 10^{24}\text{ kg}}{3.84 \times 10^8\text{ m}}\right)\right]^{1/2}$$

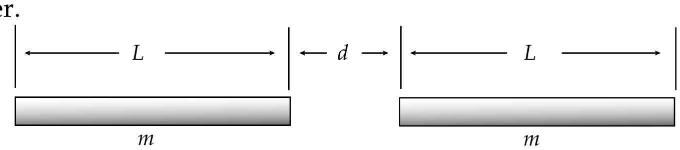
$$= [2G(1.27 \times 10^{17} + 1.92 \times 10^{14} + 9.39 \times 10^{17} - 1.56 \times 10^{16})\text{ kg/m}]^{1/2}$$

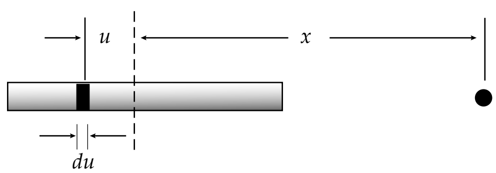
$$= \left[2 \times 6.67 \times 10^{-11}\,\frac{\text{N}\cdot\text{m}^2}{\text{kg}^2}\,10.5 \times 10^{17}\text{ kg/m}\right]^{1/2} = \boxed{11.8\text{ km/s}}$$

**14.43** In a circular orbit of radius $r$, the total energy of an Earth satellite is $E = -\frac{GM_E}{2r}$. Thus, in changing from a circular orbit of radius $r = 2R_E$ to one of radius $r = 3R_E$, the required work is

$$W = \Delta E = -\frac{GM_Em}{2r_f} + \frac{GM_Em}{2r_i} = GM_Em\left[\frac{1}{4R_E} - \frac{1}{6R_E}\right] = \boxed{\frac{GM_Em}{12R_E}}$$

**14.44** First find the acceleration of gravity created by the left-hand rod at a point distant $x$ from its center.




A bit of the left-hand rod of width $du$ has mass $(m/L)du$ and creates field

$$dg = \frac{Gdm}{r^2} = \frac{Gmdu}{L(x+u)^2}$$

Then, $g = \int_{-L/2}^{L/2} \frac{Gmdu}{L(x+u)^2} = \frac{Gm}{L}\,\frac{(x+u)^{-1}}{-1}\Bigg|_{-L/2}^{L/2}$

$$g = \frac{Gm}{L}\left(\frac{-1}{x+L/2} - \frac{-1}{x-L/2}\right) = \frac{Gm}{L}\,\frac{L}{x^2 - L^2/4}$$

$$g = \frac{Gm}{x^2 - L^2/4}$$

Now the force on the right-hand rod is the summation of bits $dF = gdm = gmdx/L$ .

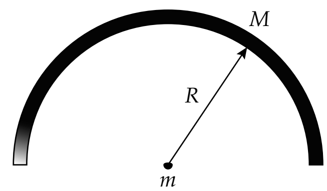
Thus: $F = \int_{x=d+L/2}^{d+3L/2} \frac{Gm\,mdx}{(x^2 - L^2/4)L} = \frac{Gm^2}{L}\int_{x=d+L/2}^{d+3L/2} \frac{dx}{x^2 - L^2/4}$ . Use the table of integrals in the Appendix of the textbook.

$$F = \frac{Gm^2}{L}\frac{1}{L}\ln\left(\frac{x - L/2}{x + L/2}\right)\Bigg|_{d+L/2}^{d+3L/2}$$

$$= \frac{Gm^2}{L^2}\left[\ln\left(\frac{d+L}{d+2L}\right) - \ln\left(\frac{d}{d+L}\right)\right]$$

$$= \frac{Gm^2}{L^2}\left[\ln\left(\frac{d+L}{d+2L}\frac{d+L}{d}\right)\right] = \boxed{\frac{Gm^2}{L^2}\ln\left[\frac{(d+L)^2}{d(d+2L)}\right]}$$

**14.45** By symmetry, **F** is in the $y$ direction.




$$dM = \left(\frac{M}{\pi R}\right) Rd\theta = \left(\frac{M}{\pi}\right) d\theta \quad \text{and} \quad dF = \frac{GmdM}{R^2}$$

$$dF_y = \frac{GmdM\cos\theta}{R^2} = \frac{\left[Gm\left(\frac{M}{\pi}\right) d\theta\cos\theta\right]}{R^2}$$

$$F_y = \int_{-\pi/2}^{\pi/2} \frac{GmM}{\pi R^2}\cos\theta\, d\theta = \frac{GMm}{\pi R^2}\sin\theta\,\Big|_{-\pi/2}^{\pi/2} = \frac{GMm}{\pi R^2}[1-(-1)] = \boxed{\frac{2GmM}{\pi R^2}}$$

**Goal Solution**

**G:** If the rod completely encircled the point mass, the net force on $m$ would be zero (by symmetry), and if all the mass $M$ would be concentrated at the middle of the rod, the force would be $F = \frac{GmM}{R^2}$ directed upwards. Since the given configuration is somewhere between these two extreme cases, we can expect the net force on $m$ to be upwards and $F < \frac{GmM}{R^2}$

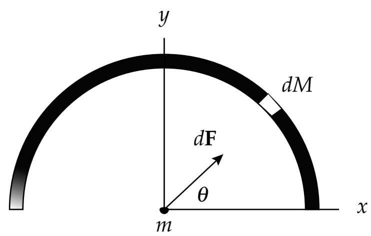
**O:** The net force on the point mass can be found by integrating the contributions from each small piece of the semicircular rod.

**A:** If we consider a segment of the curved rod, $dM$, to act like a point mass, we can apply Eq. 14.1 to find the force exerted on $m$ due to $dM$ as:

$$d\mathbf{F} = \frac{Gm(dM)}{R^2} \text{(directed toward } dM\text{)} .$$

If we could integrate this differential force, we could find the total force on the point mass, $m$, but the differential mass element must first be written in terms of a variable that we can integrate easily, like the angle, $\theta$, which ranges from 0 to 180° for this semicircular rod. One segment of the arc, $dM$, subtends an angle $d\theta$ and has length $R d\theta$. Since the whole rod has length $\pi R$ and mass $M$, this incremental element has mass

$$dM = \left(\frac{M}{\pi R}\right) R d\theta = \frac{M d\theta}{\pi}$$




This mass element exerts a force $d\mathbf{F}$ on the point mass at the center.

$$d\mathbf{F} = \frac{GmdM}{R^2}\hat{\mathbf{r}} = \frac{Gm}{R^2}\left(\frac{M}{\pi}d\theta\right)\hat{\mathbf{r}}$$

where $\hat{\mathbf{r}}$ is directed at an angle $\theta$ above the $x$-axis

or $$d\mathbf{F} = \frac{GmM}{\pi R^2}\, d\theta(\cos\theta\mathbf{i} + \sin\theta\mathbf{j})$$

To find the net force on the point mass, we integrate the contributions for all mass elements from $\theta = 0°$ to 180°:

$$\mathbf{F} = \int_{\text{all } m} d\mathbf{F} = \int_{\theta=0}^{180°}\left(\frac{GmMd\theta}{\pi R^2}\right)(\cos\theta\mathbf{i} + \sin\theta\mathbf{j})$$

$$\mathbf{F} = \left(\frac{GmM\,\mathbf{i}}{\pi R^2}\right)\int_0^{180°}\cos\theta d\theta + \left(\frac{GmM\mathbf{j}}{\pi R^2}\right)\int_0^{180°}\sin\theta d\theta$$

$$\mathbf{F} = \left(\frac{GmM\,\mathbf{i}}{\pi R^2}\right)[\sin\theta]_0^{180°} + \left(\frac{GmM\mathbf{j}}{\pi R^2}\right)[-\cos\theta]_0^{180°}$$

$$\mathbf{F} = \left(\frac{GmM\,\mathbf{i}}{\pi R^2}\right)(0-0) + \left(\frac{GmM\mathbf{j}}{\pi R^2}\right)[-(-1)+1]$$

$$\mathbf{F} = 0\mathbf{i} + \left(\frac{2GmM}{\pi R^2}\right)\mathbf{j}$$

**L:** As predicted, the direction of the force on the point mass at the center is vertically upward. Also, the net force has the same algebraic form as for two point masses, reduced by a factor of $2/\pi$, so this answer agrees with our prediction.

**14.46** (a) From Example 14.10, $T = 2\pi\sqrt{\frac{R_E^3}{GM_E}}$

At the surface $g = GM_E/R_E^2$ so indeed $\boxed{T = 2\pi(\sqrt{R_E/g})}$

(b) $T = 2\pi\sqrt{\frac{R_M^3}{GM_M}}$

$$T = 2\pi\sqrt{\frac{(1.74 \times 10^6 \text{ m})^3}{(6.67 \times 10^{-11} \text{ N} \cdot \text{m}^2/\text{kg}^2)(7.36 \times 10^{22} \text{ kg})}}$$

$T = 6.51 \times 10^3 \text{ s} = \boxed{1.81 \text{ h}}$

(c) The Moon may be hot down deep inside, but $\boxed{\text{it is not molten}}$ .

**14.47** (a) $F = \frac{GmM}{r^2} = \frac{(6.67 \times 10^{-11} \text{ N} \cdot \text{m}^2/\text{kg}^2)(0.0500 \text{ kg})(500 \text{ kg})}{(1.500 \text{ m})^2} = \boxed{7.41 \times 10^{-10} \text{ N}}$

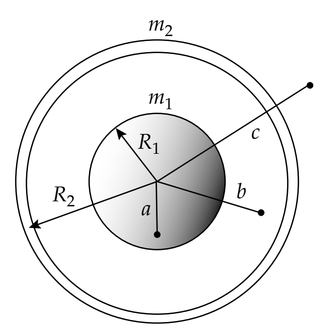
(b) $F = \frac{(6.67 \times 10^{-11} \text{ N} \cdot \text{m}^2/\text{kg}^2)(0.0500 \text{ kg})(500 \text{ kg})}{(0.400 \text{ m})^2} = \boxed{1.04 \times 10^{-8} \text{ N}}$

(c) In this case the mass $m$ is a distance $r$ from a sphere of mass,

$$M = (500 \text{ kg})\left(\frac{0.200 \text{ m}}{0.400 \text{ m}}\right)^3 = 62.5 \text{ kg} \quad \text{and}$$

$$F = \frac{(6.67 \times 10^{-11} \text{ N} \cdot \text{m}^2/\text{kg}^2)(0.0500 \text{ kg})(62.5 \text{ kg})}{(0.200 \text{ m})^2} = \boxed{5.21 \times 10^{-9} \text{ N}}$$

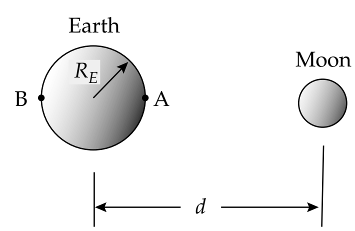
**14.48** (a) $F = \boxed{\frac{Gmm_1 a}{R_1^3}}$ toward the center of the sphere




(b) $F = \boxed{\frac{Gmm_1}{b^2}}$ toward the center of the sphere

(c) $F = \boxed{\frac{Gm(m_1 + m_2)}{c^2}}$ toward the center of the sphere

**14.49** The acceleration of an object at the center of the Earth due to the gravitational force of the Moon is given by $a = G\frac{M_{\text{Moon}}}{d^2}$




At the point nearest the Moon, $a_+ = G\frac{M_M}{(d - r)^2}$

At the point farthest from the Moon, $a_- = G\dfrac{M_M}{(d+r)^2}$

$$\Delta a = a_+ - a = GM_M\left[\frac{1}{(d-r)^2} - \frac{1}{d^2}\right]$$

For $d >> r$, $\Delta a = \dfrac{2GM_M r}{d^3} = 1.11 \times 10^{-6}\ \text{m/s}^2$

Across the planet, $\dfrac{\Delta g}{g} = \dfrac{2\Delta a}{g} = \dfrac{2.22 \times 10^{-6}\ \text{m/s}^2}{9.80\ \text{m/s}^2} = \boxed{2.26 \times 10^{-7}}$

**14.50** Momentum is conserved:

$$m_1\mathbf{v}_{1i} + m_2\mathbf{v}_{2i} = m_1\mathbf{v}_{1f} + m_2\mathbf{v}_{2f}$$

$$0 = M\mathbf{v}_{1f} + 2M\mathbf{v}_{2f}$$

$$\mathbf{v}_{2f} = -\frac{1}{2}\mathbf{v}_{1f}$$

Energy is conserved:

$$(K+U)_i + \Delta E = (K+U)_f$$

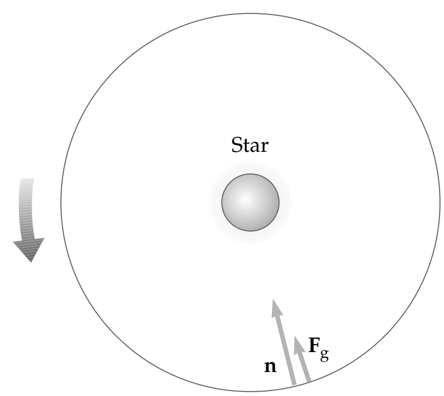
$$0 - \frac{Gm_1m_2}{r_i} + 0 = \frac{1}{2}m_1v_{2f}^2 - \frac{Gm_1m_2}{r_f}$$

$$-\frac{GM(2M)}{12R} = \frac{1}{2}Mv_{1f}^2 + \frac{1}{2}(2M)\left(\frac{1}{2}v_{1f}\right)^2 - \frac{GM(2M)}{4R}$$

$$v_{1f} = \boxed{\frac{2}{3}\sqrt{\frac{GM}{R}}} \qquad v_{2f} = \frac{1}{2}v_{1f} = \boxed{\frac{1}{3}\sqrt{\frac{GM}{R}}}$$

**14.51** (a) $a_r = \dfrac{v^2}{r}$

$$a_r = \frac{(1.25 \times 10^6\ \text{m/s})^2}{1.53 \times 10^{11}\ \text{m}} = \boxed{10.2\ \text{m/s}^2}$$




(b) diff $= 10.2 - 9.90 = 0.312\ \text{m/s}^2 = \dfrac{GM}{r^2}$

$$M = \frac{(0.312\ \text{m/s}^2)(1.53 \times 10^{11}\ \text{m})^2}{6.67 \times 10^{-11}\ \text{N}\cdot\text{m/kg}^2} = \boxed{1.10 \times 10^{32}\ \text{kg}}$$

***14.52** (a) The free-fall acceleration produced by the Earth is

$$g = \frac{GM_E}{r^2} = GM_E r^{-2} \quad \text{(directed downward)}$$

Its rate of change is $\frac{dg}{dr} = GM_E(-2)r^{-3} = -2GM_E r^{-3}$. The minus sign indicates that $g$ decreases with increasing height.

At the Earth's surface, $\boxed{\frac{dg}{dr} = -\frac{2GM_E}{R_E^3}}$

(b) For small differences,

$$\frac{|\Delta g|}{\Delta r} = \frac{|\Delta g|}{h} = \frac{2GM_E}{R_E^3} \qquad \text{Thus, } \boxed{|\Delta g| = \frac{2GM_E h}{R_E^3}}$$

(c) $$|\Delta g| = \frac{2(6.67 \times 10^{-11}\ \text{N} \cdot \text{m}^2/\text{kg}^2)(5.98 \times 10^{24}\ \text{kg})(6.00\ \text{m})}{(6.37 \times 10^6\ \text{m})^3} = \boxed{1.85 \times 10^{-5}\ \text{m/s}^2}$$

**14.53** (a) Initially take the particle from infinity and move it to the sphere's surface. Then,

$$U = \int_\infty^R \frac{GmM}{r^2}\, dr = -\frac{GmM}{R}$$

Now move it to a position $r$ from the center of the sphere. The force in this case is a function of the mass enclosed by $r$ at any point.

Since $\rho = \frac{M}{\frac{4}{3}\pi R^3}$ we have

$$U = \int_R^r \frac{Gm(4\pi r^3)\rho}{3r^2}\, dr = \frac{GmM}{R^3}\left(\frac{r^2 - R^2}{2}\right)$$

and the total gravitational potential energy is

$$U = \boxed{\left(\frac{GmM}{2R^3}\right)r^2 - \frac{3GmM}{2R}}$$

(b) $U(R) = -\frac{GMm}{R}$ and $U(0) = -\frac{3GmM}{2R}$

so $W_g = -[U(0) - U(R)] = \boxed{\frac{GMm}{2R}}$

**14.54** To approximate the height of the sulfur, set

$$\frac{mv^2}{2} = mgh \qquad h = 70\,000 \text{ m} \qquad g_{\text{Io}} = \frac{GM}{r^2} = 1.79 \text{ m/s}^2$$

$$v = \sqrt{2gh}$$

$$v = \sqrt{2(1.79)(70\,000)} \approx 500 \text{ m/s (over 1000 mph)}$$

A more precise answer is given by

$$\frac{1}{2}mv^2 - \frac{GMm}{r_1} = -\frac{GMm}{r_2}$$

$$\frac{1}{2}v^2 = (6.67 \times 10^{-11})(8.90 \times 10^{22})\left(\frac{1}{1.82 \times 10^6} - \frac{1}{1.89 \times 10^6}\right)$$

$$v = \boxed{492 \text{ m/s}}$$

***14.55** From the walk, $2\pi r = 25\,000$ m. Thus, the radius of the planet is

$$r = \frac{25\,000 \text{ m}}{2\pi} = 3.98 \times 10^3 \text{ m}$$

From the drop: $\Delta y = \frac{1}{2}gt^2 = \frac{1}{2}g(29.2 \text{ s})^2 = 1.40 \text{ m}$

so, $g = \frac{2(1.40 \text{ m})}{(29.2 \text{ s})^2} = 3.28 \times 10^{-3} \text{ m/s}^2 = \frac{MG}{r^2}$

$$\therefore M = \boxed{7.79 \times 10^{14} \text{ kg}}$$

**14.56** For a 6.00 km diameter cylinder, $r = 3000$ m and to simulate 1 $g = 9.80 \text{ m/s}^2$

$$g = \frac{v^2}{r} = \omega^2 r$$

$$\omega = \sqrt{\frac{g}{r}} = \boxed{0.0572 \text{ rad/s}}$$

The required rotation rate of the cylinder is $\boxed{1 \text{ rev}/110 \text{ s}}$

(For a description of proposed cities in space, see Gerard K. O'Neill in *Physics Today*, Sept. 1974.)

**14.57** $F = \frac{GMm}{r^2} = \left(6.67 \times 10^{-11} \frac{\text{N} \cdot \text{m}^2}{\text{kg}^2}\right)\frac{(1.50 \text{ kg})(15.0 \times 10^3 \text{ kg})}{(4.50 \times 10^{-2} \text{ m})^2} = \boxed{7.41 \times 10^{-10} \text{ N}}$

***14.58** (a) $G$ has units $\frac{\text{N}\cdot\text{m}^2}{\text{kg}^2} = \frac{\text{kg}\cdot\text{m}\cdot\text{m}^2}{\text{s}^2\cdot\text{kg}^2} = \frac{\text{m}^3}{\text{s}^2\cdot\text{kg}}$

and dimensions $[G] = \frac{\text{L}^3}{\text{T}^2\cdot\text{M}}$

The speed of light has dimensions of $[c] = \frac{\text{L}}{\text{T}}$, and Planck's constant has the same dimensions as angular momentum or $[h] = \frac{\text{M}\cdot\text{L}^2}{\text{T}}$.

We require $[G^p c^q h^r] = \text{L}$, or $\text{L}^{3p}\text{T}^{-2p}\text{M}^{-p}\text{L}^q\text{T}^{-q}\text{M}^r\text{L}^{2r}\text{T}^{-r} = \text{L}^1\text{M}^0\text{T}^0$.

Thus, $3p + q + 2r = 1$

$-2p - q - r = 0$

$-p + r = 0$

which reduces (using $r = p$) to $3p + q + 2p = 1$

$-2p - q - p = 0$

These equations simplify to $5p + q = 1$ and $q = -3p$

Then, $5p - 3p = 1$, yielding $p = \frac{1}{2}$, $q = -\frac{3}{2}$, and $r = \frac{1}{2}$

Therefore, Planck length = $\boxed{G^{1/2}c^{-3/2}h^{1/2}}$

(b) $(6.67\times10^{-11})^{1/2}(3\times10^{8})^{-3/2}(6.63\times10^{-34})^{1/2} = (1.64\times10^{-69})^{1/2} = 4.05\times10^{-35}\text{ m}$ $\boxed{\sim10^{-34}\text{ m}}$

***14.59** $\frac{1}{2}m_0 v_{\text{esc}}^2 = \frac{Gm_p m_0}{R}$

$$v_{\text{esc}} = \sqrt{\frac{2Gm_p}{R}}$$

With $m_p = \rho\frac{4}{3}\pi R^3$, we have

$$v_{\text{esc}} = \sqrt{\frac{2G\rho\frac{4}{3}\pi R^3}{R}}$$

$$= \boxed{\sqrt{\frac{8\pi G\rho}{3}}\,R}$$

So, $v_{\text{esc}} \propto R$

***14.60** (a) To see this, let $M(r)$ be the mass up to radius $r$, $\rho(r)$ be the density at radius $r$, and $\rho_{av}(r)$ be the average density up to radius $r$.

Then, $\frac{dM(r)}{dr} = 4\pi r^2\rho(r)$ and $M(r) = \frac{4}{3}\pi r^3\rho_{av}(r)$

The gravitational acceleration at radius $r$ is $g(r) = \frac{GM(r)}{r^2}$ ; its derivative with respect to $r$ is

$$\frac{dg}{dr} = \frac{-2GM(r)}{r^3} + \frac{G}{r^2}\frac{dM(r)}{dr} = \frac{-2GM(r)}{r^3} + 4\pi G\rho(r)$$

or $$\boxed{\frac{dg}{dr} = 4\pi G\left[\rho(r) - \frac{2}{3}\rho_{av}(r)\right]}$$

This is positive only if $\rho(r) > \frac{2}{3}\rho_{av}(r)$; if $\rho(r) < \frac{2}{3}\rho_{av}(r)$, $g$ will actually decrease with increasing $r$.

(b) From the numbers given, it is clear that at the surface, the average density is less than 2/3 of the average density of the whole Earth. Clearly, then, $\boxed{\text{the value of } g \text{ increases as one descends into the Earth}}$ . Geophysical evidence shows that the maximum value of $g$ inside the Earth is greater than 10.0 N/kg, and occurs about halfway between the center and the surface.

**14.61** (a) At infinite separation $U = 0$ and at rest $K = 0$. Since energy is conserved we have,

$$0 = \frac{1}{2}m_1v_1^2 + \frac{1}{2}m_2v_2^2 - \frac{Gm_1m_2}{d} \qquad (1)$$

The initial momentum is zero and momentum is conserved.

Therefore, $0 = m_1v_1 - m_2v_2$ (2)

Combine Equations (1) and (2) to find

$$\boxed{v_1 = m_2\sqrt{\frac{2G}{d(m_1+m_2)}}} \quad \text{and} \quad \boxed{v_2 = m_1\sqrt{\frac{2G}{d(m_1+m_2)}}}$$

Relative velocity $v_r = v_1 - (-v_2) = \boxed{\sqrt{\frac{2G(m_1+m_2)}{d}}}$

(b) Substitute given numerical values into the equation found for $v_1$ and $v_2$ in part (a) to find

$$v_1 = 1.03 \times 10^4 \text{ m/s} \quad \text{and} \quad v_2 = 2.58 \times 10^3 \text{ m/s}$$

Therefore, $K_1 = \frac{1}{2} m_1 v_1^2 = \boxed{1.07 \times 10^{32} \text{ J}}$

and $K_2 = \frac{1}{2} m_2 v_2^2 = \boxed{2.67 \times 10^{31} \text{ J}}$

**14.62** (a) The net torque exerted on the Earth is zero. Therefore, the angular momentum is conserved;

$$m r_a v_a = m r_p v_p$$

and $v_a = v_p \left(\frac{r_p}{r_a}\right) = (3.027 \times 10^4 \text{ m/s})\left(\frac{1.471}{1.521}\right) = \boxed{2.93 \times 10^4 \text{ m/s}}$

(b) $K_p = \frac{1}{2} m v_p^2 = \frac{1}{2}(5.98 \times 10^{24})(3.027 \times 10^4)^2 = \boxed{2.74 \times 10^{33} \text{ J}}$

$$U_p = -\frac{GmM}{r_p} = -\frac{(6.673 \times 10^{-11})(5.98 \times 10^{24})(1.99 \times 10^{30})}{1.471 \times 10^{11}} = \boxed{-5.40 \times 10^{33} \text{ J}}$$

(c) Using the same form as in part (b), we find

$$K_a = \boxed{2.57 \times 10^{33} \text{ J}} \quad \text{and} \quad U_a = \boxed{-5.22 \times 10^{33} \text{ J}}$$

Compare $K_p + U_p = \boxed{-2.66 \times 10^{33} \text{ J}}$

and $K_a + U_a = \boxed{-2.65 \times 10^{33} \text{ J}}$ They agree.

**14.63** (a) If we consider a hollow shell in the sphere with radius $r$ and thickness $dr$, then $dM = \rho dV = \rho(4\pi r^2 dr)$. The total mass is then

$$M = \int_0^R \rho dV \int_0^R (Ar)(4\pi r^2 dr) = \pi A R^4$$

and $A = \boxed{\frac{M}{\pi R^4}}$

(b) The total mass of the sphere acts as if it were at the center of the sphere and

$F = \boxed{\frac{GmM}{r^2}}$ directed toward the center of the sphere.

(c) Inside the sphere at the distance $r$ from the center, $dF = \left(\frac{Gm}{r^2}\right) dM$ where $dM$ is just the mass of a shell enclosed within the radius $r$.

Therefore, $F = \frac{Gm}{r^2}\int_0^r dM = \frac{Gm}{r^2}\int_0^r Ar\ 4\pi r^2 dr$

$$F = \frac{Gm}{r^2}\frac{M}{\pi R^4}\frac{4\pi r^4}{4} = \boxed{\frac{GmMr^2}{R^4}}$$

***14.64** (a) The work must provide the increase in gravitational energy

$$W = \Delta U_g = U_{gf} - U_{gi}$$

$$= -\frac{GM_EM_p}{r_f} + \frac{GM_EM_p}{r_i}$$

$$= -\frac{GM_EM_p}{R_E + y} + \frac{GM_EM_p}{R_E}$$

$$= GM_EM_p\left(\frac{1}{R_E} - \frac{1}{R_E + y}\right)$$

$$= \left(\frac{6.67 \times 10^{-11}\ \text{N}\cdot\text{m}^2}{\text{kg}^2}\right)(5.98 \times 10^{24}\ \text{kg})(100\ \text{kg})\left(\frac{1}{6.37 \times 10^6\ \text{m}} - \frac{1}{7.37 \times 10^6\ \text{m}}\right)$$

$$W = \boxed{850\ \text{MJ}}$$

(b) In a circular orbit, gravity supplies the centripetal force:

$$\frac{GM_EM_p}{(R_E + y)^2} = \frac{M_pv^2}{(R_E + y)}$$

Then, $\frac{1}{2}M_pv^2 = \frac{1}{2}\frac{GM_EM_p}{(R_E + y)}$

So, additional work = kinetic energy required

$$= \frac{1}{2}\frac{(6.67 \times 10^{-11}\ \text{N}\cdot\text{m}^2)(5.98 \times 10^{24}\ \text{kg})(100\ \text{kg})}{(\text{kg}^2)(7.37 \times 10^6\ \text{m})}$$

$$\Delta W = \boxed{2.71 \times 10^9\ \text{J}}$$

**14.65** Centripetal acceleration comes from gravitational acceleration.

$$\frac{v^2}{r} = \frac{M_c G}{r^2} = \frac{4\pi^2 r^2}{T^2 r}$$

$$GM_c T^2 = 4\pi^2 r^3$$

$$(6.67 \times 10^{-11})(20)(1.99 \times 10^{30})(5.00 \times 10^{-3})^2 = 4\pi^2 r^3$$

$$r_{\text{orbit}} = \boxed{119 \text{ km}}$$

**14.66** (a) $T = \frac{2\pi r}{v} = \frac{2\pi(30\,000 \times 9.46 \times 10^{15} \text{ m})}{2.50 \times 10^5 \text{ m/s}} = 7.13 \times 10^{15} \text{ s} = \boxed{2.26 \times 10^8 \text{ yr}}$

(b) $M = \frac{4\pi^2 a^3}{GT^2} = \frac{4\pi^2 (30\,000 \times 9.46 \times 10^{15} \text{ m})^3}{(6.67 \times 10^{-11} \text{ N} \cdot \text{m}^2/\text{kg}^2)(7.13 \times 10^{15} \text{ s})^2} = 2.66 \times 10^{41} \text{ kg}$

$M = 1.34 \times 10^{11}$ solar masses $\boxed{\sim 10^{11} \text{ solar masses}}$

The number of stars is $\boxed{\text{on the order of } 10^{11}}$

***14.67** (a) From the data about perigee, the energy is

$$E = \frac{1}{2} mv_p^2 - \frac{GM_E m}{r_p} = \frac{1}{2}(1.60)(8.23 \times 10^3)^2 - \frac{(6.67 \times 10^{-11})(5.98 \times 10^{24})(1.60)}{7.02 \times 10^6}$$

or $E = \boxed{-3.67 \times 10^7 \text{ J}}$

(b) $L = mvr \sin\theta = mv_p r_p \sin 90.0°$

$= (1.60 \text{ kg})(8.23 \times 10^3 \text{ m/s})(7.02 \times 10^6 \text{ m}) = \boxed{9.24 \times 10^{10} \text{ kg} \cdot \text{m}^2/\text{s}}$

(c) At apogee, we must have $\frac{1}{2} mv_a^2 - \frac{GMm}{r_a} = E$, and $mv_a r_a \sin 90.0° = L$ since both energy and angular momentum are conserved. Thus,

$$\frac{1}{2}(1.60)\, v_a^2 - \frac{(6.67 \times 10^{-11})(5.98 \times 10^{24})(1.60)}{r_a} = -3.67 \times 10^7 \text{ J}$$

and $(1.60 \text{ kg})v_a r_a = 9.24 \times 10^{10} \text{ kg} \cdot \text{m}^2/\text{s}$

Solving simultaneously,

$$\frac{1}{2}(1.60)\, v_a^2 - \frac{(6.67 \times 10^{-11})(5.98 \times 10^{24})(1.60)(1.60)v_a}{9.24 \times 10^{10}} = -3.67 \times 10^7$$

which reduces to $0.800v_a^2 - 11046v_a + 3.6723 \times 10^7 = 0$

so $$v_a = \frac{11046 \pm \sqrt{(11046)^2 - 4(0.800)(3.6723 \times 10^7)}}{2(0.800)}$$

This gives $v_a = 8230$ m/s or $\boxed{5580\ \text{m/s}}$ . The smaller answer refers to the velocity at the apogee while the larger refers to perigee.

Thus, $r_a = \dfrac{L}{mv_a} = \dfrac{9.24 \times 10^{10}\ \text{kg} \cdot \text{m}^2/\text{s}}{(1.60\ \text{kg})(5.58 \times 10^3\ \text{m/s})} = \boxed{1.04 \times 10^7\ \text{m}}$

(d) The major axis is $2a = r_p + r_a$, so the semi-major axis is

$$a = \frac{1}{2}(7.02 \times 10^6\ \text{m} + 1.04 \times 10^7\ \text{m}) = \boxed{8.69 \times 10^6\ \text{m}}$$

(e) $T = \sqrt{\dfrac{4\pi^2 a^3}{GM_E}} = \sqrt{\dfrac{4\pi^2(8.69 \times 10^6\ \text{m})^3}{(6.67 \times 10^{-11}\ \text{N} \cdot \text{m}^2/\text{kg}^2)(5.98 \times 10^{24}\ \text{kg})}}$

$T = 8060\ \text{s} = \boxed{134\ \text{min}}$

***14.68** $v_i = 2\sqrt{Rg}$ $\qquad g = \dfrac{MG}{R^2}$

Utilizing conservation of energy,

$$\frac{mv^2}{2} - \frac{mGM}{r} = \frac{mv_i^2}{2} - \frac{mGM}{R}$$

$$\frac{mv^2}{2} = \frac{mv_i^2}{2} - \frac{mGM}{R} + \frac{mGM}{r}$$

$$v^2 = v_i^2 - 2MG\left(\frac{1}{R} - \frac{1}{r}\right)$$

$$v = \sqrt{v_i^2 + 2MG\left(\frac{1}{r} - \frac{1}{R}\right)}$$

$$v = \sqrt{4Rg + 2MG\left(\frac{1}{r} - \frac{1}{R}\right)}$$

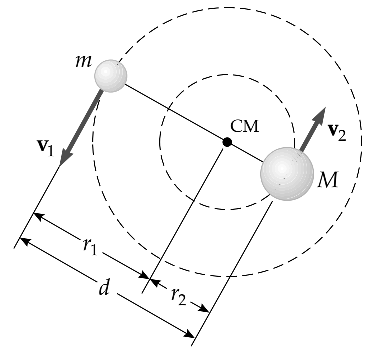
$$v = \sqrt{\frac{4MG}{R} - \frac{2MG}{R} + \frac{2MG}{r}}$$

$$v = \boxed{\sqrt{2MG\left(\frac{1}{R} + \frac{1}{r}\right)} = \sqrt{2R^2 g\left(\frac{1}{R} + \frac{1}{r}\right)}}$$

**14.69** If we choose the coordinate of the center of mass at the origin, then

$$0 = \frac{(Mr_2 - mr_1)}{M + m} \quad \text{and} \quad Mr_2 = mr_1$$

(Note: this is equivalent to saying that the net torque must be zero and the two experience no angular acceleration.) For each mass $F = ma$ and




$$mr_1\omega_1^2 = \frac{MGm}{d^2}$$

and $$Mr_2\omega_2^2 = \frac{MGm}{d^2}$$

Combining these two equations and using $d = r_1 + r_2$ gives

$$(r_1 + r_2)\omega^2 = \frac{(M + m)G}{d^2}$$

with $\omega_1 = \omega_2 = \omega$ and $T = \frac{2\pi}{\omega}$, we find $\boxed{T^2 = \frac{4\pi^2 d^3}{G(M + m)}}$

**Goal Solution**

For the star of mass $M$ and orbital radius $r_2$,

$\Sigma F = ma$ gives

$$\frac{GMm}{d^2} = \frac{Mv_2^2}{r_2} = \frac{M}{r_2}\left(\frac{2\pi r_2}{T}\right)^2$$

For the star of mass $m$, $\Sigma F = ma$ gives

$$\frac{GMm}{d^2} = \frac{mv_1^2}{r_1} = \frac{m}{r_1}\left(\frac{2\pi r_1}{T}\right)^2$$

Cross-multiplying, we then obtain simultaneous equations:

$$GmT^2 = 4\pi^2 d^2 r_2$$

$$GMT^2 = 4\pi^2 d^2 r_1$$

Adding, we find

$$G(M + m)T^2 = 4\pi^2 d^2 (r_1 + r_2) = 4\pi^2 d^3$$

$$T^2 = \frac{4\pi^2 d^3}{G(M + m)}$$

In a visual binary star system $T$, $d$, $r_1$, and $r_2$ can be measured, so the mass of each component can be computed.

***14.70** (a) The gravitational force exerted on $m_2$ by the Earth (mass $m_1$) accelerates $m_2$ according to: $m_2 g_2 = \frac{G m_1 m_2}{r^2}$. The equal magnitude force exerted on the Earth by $m_2$ produces negligible acceleration of the Earth. The acceleration of relative approach is then

$$g_2 = \frac{G m_1}{r^2} = \frac{(6.67 \times 10^{-11}\ \text{N} \cdot \text{m}^2/\text{kg}^2)(5.98 \times 10^{24}\ \text{kg})}{(1.20 \times 10^7\ \text{m})^2} = \boxed{2.77\ \text{m/s}^2}$$

(b) Again, $m_2$ accelerates toward the center of mass with $g_2 = 2.77\ \text{m/s}^2$. Now the Earth accelerates toward $m_2$ with an acceleration given as

$$m_1 g_1 = \frac{G m_1 m_2}{r^2}$$

$$g_1 = \frac{G m_2}{r^2} = \frac{(6.67 \times 10^{-11}\ \text{N} \cdot \text{m}^2/\text{kg}^2)(2.00 \times 10^{24}\ \text{kg})}{(1.20 \times 10^7\ \text{m})^2} = 0.926\ \text{m/s}^2$$

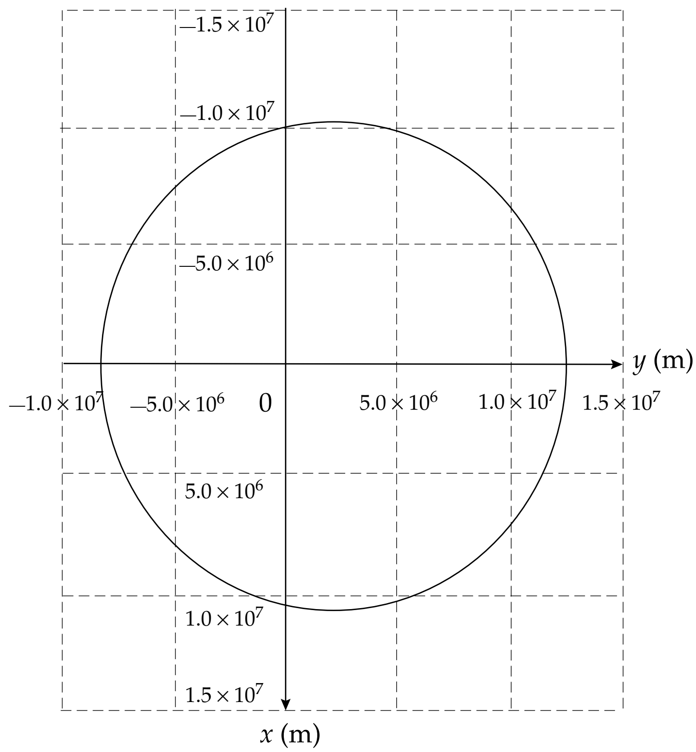
The distance between the masses closes with relative acceleration of

$$g_{\text{rel}} = g_1 + g_2 = 0.926\ \text{m/s}^2 + 2.77\ \text{m/s}^2 = \boxed{3.70\ \text{m/s}^2}$$

**14.71** Initial Conditions and Constants:

Mass of planet: $5.98 \times 10^{24}$ kg
Radius of planet: $6.37 \times 10^6$ m
Initial $x$: 0.0 planet radii
Initial $y$: 2.0 planet radii
Initial $v_x$: +5000 m/s
Initial $v_y$: 0.0 m/s
Time interval: 10.9 s

| $t$ (s) | $x$ (m) | $y$ (m) | $r$ (m) | $v_x$ (m/s) | $v_y$ (m/s) | $a_x$ (m/s) | $a_y$ (m/s$^2$) |
|---|---|---|---|---|---|---|---|
| 0.0 | 0.0 | 12,740,000.0 | 12,740,000.0 | 5,000.0 | 0.0 | 0.0000 | –2.4575 |
| 10.9 | 54,315.3 | 12,740,000.0 | 12,740,115.8 | 4,999.9 | –26.7 | –0.0100 | –2.4574 |
| 21.7 | 108,629.4 | 12,739,710.0 | 12,740,173.1 | 4,999.7 | –53.4 | –0.0210 | –2.4573 |
| 32.6 | 162,941.1 | 12,739,130.0 | 12,740,172.1 | 4,999.3 | –80.1 | –0.0310 | –2.4572 |
| … | | | | | | | |
| 5,431.6 | 112,843.8 | –8,466,816.0 | 8,467,567.9 | –7,523.0 | –39.9 | –0.0740 | 5.5625 |
| 5,442.4 | 31,121.4 | –8,467,249.7 | 8,467,306.9 | –7,523.2 | 20.5 | –0.0200 | 5.5633 |
| 5,453.3 | –50,603.4 | –8,467,026.9 | 8,467,178.2 | –7,522.8 | 80.9 | 0.0330 | 5.5634 |
| 5,464.1 | –132,324.3 | –8,466,147.7 | 8,467,181.7 | –7,521.9 | 141.4 | 0.0870 | 5.5628 |
| … | | | | | | | |
| 10,841.3 | –108,629.0 | 12,739,134.4 | 12,739,597.5 | 4,999.9 | 53.3 | 0.0210 | –2.4575 |
| 10,852.2 | –54,314.9 | 12,739,713.4 | 12,739,829.2 | 5,000.0 | 26.6 | 0.0100 | –2.4575 |
| 10,863.1 | 0.4 | 12,740,002.4 | 12,740,002.4 | 5,000.0 | –0.1 | 0.0000 | –2.4575 |




The $\boxed{\text{object does not hit the Earth}}$; its minimum radius is $\boxed{1.33R_E}$.

Its period is $\boxed{1.09 \times 10^4 \text{ s}}$. A circular orbit would require a speed of $\boxed{5.60 \text{ km/s}}$.