Chapter 13 Solutions

13.1 $x = (4.00 \text{ m}) \cos (3.00\pi t + \pi)$

Compare this with $x = A \cos (\omega t + \phi)$ to find

(a) $\omega = 2\pi f = 3.00\pi$

or $\boxed{f = 1.50 \text{ Hz}}$ $\quad T = \frac{1}{f} = \boxed{0.667 \text{ s}}$

(b) $A = \boxed{4.00 \text{ m}}$

(c) $\phi = \boxed{\pi \text{ rad}}$

(d) $x(t = 0.250 \text{ s}) = (4.00 \text{ m}) \cos (1.75\pi) = \boxed{2.83 \text{ m}}$

13.2 (a) Since the collision is perfectly elastic, the ball will rebound to the height of 4.00 m and then repeat the motion over and over again. Thus, the $\boxed{\text{motion is periodic}}$.

(b) To determine the period, we use: $x = \frac{1}{2} gt^2$

The time for the ball to hit the ground is

$$t = \sqrt{\frac{2x}{g}} = \sqrt{\frac{2(4.00 \text{ m})}{9.80 \text{ m/s}^2}} = 0.909 \text{ s}$$

This equals one-half the period, so $T = 2(0.909 \text{ s}) = \boxed{1.82 \text{ s}}$

(c) $\boxed{\text{No}}$ The net force acting on the mass is a constant given by $F = -mg$ (except when it is in contact with the ground), which is not in the form of Hooke's law.

13.3 (a) $\boxed{20.0 \text{ cm}}$

(b) $v_{\max} = \omega A = 2\pi f A = \boxed{94.2 \text{ cm/s}}$

This occurs as the particle passes through equilibrium.

(c) $a_{\max} = \omega^2 A = (2\pi f)^2 A = \boxed{17.8 \text{ m/s}^2}$

This occurs at maximum excursion from equilibrium.

***13.4** (a) $x = (5.00\text{ cm})\cos\left(2t + \frac{\pi}{6}\right)$

At $t = 0$, $x = (5.00\text{ cm})\cos\left(\frac{\pi}{6}\right) = \boxed{4.33\text{ cm}}$

(b) $v = \frac{dx}{dt} = -(10.0\text{ cm/s})\sin\left(2t + \frac{\pi}{6}\right)$

At $t = 0$, $v = \boxed{-5.00\text{ cm/s}}$

(c) $a = \frac{dv}{dt} = -(20.0\text{ cm/s}^2)\cos\left(2t + \frac{\pi}{6}\right)$

At $t = 0$, $a = \boxed{-17.3\text{ cm/s}^2}$

(d) $A = \boxed{5.00\text{ cm}}$ and $T = \frac{2\pi}{\omega} = \frac{2\pi}{2} = \boxed{3.14\text{ s}}$

13.5 (a) At $t = 0$, $x = 0$ and v is positive (to the right). Therefore, this situation corresponds to

$$x = A\sin\omega t \quad \text{and} \quad v = v_i\cos\omega t$$

Since $f = 1.50$ Hz, $\omega = 2\pi f = 3.00\pi$

Also, $A = 2.00$ cm, so that $\boxed{x = (2.00\text{ cm})\sin 3.00\pi t}$

(b) $v_{\text{max}} = v_i = A\omega = (2.00)(3.00\pi) = \boxed{6.00\pi\text{ cm/s}}$

The particle has this speed at $t = 0$ and next at $t = \frac{T}{2} = \boxed{\frac{1}{3}\text{ s}}$

(c) $a_{\text{max}} = A\omega^2 = 2(3.00\pi)^2 = \boxed{18.0\pi^2\text{ cm/s}^2}$

The acceleration has this positive value for the first time at

$$t = \frac{3T}{4} = \boxed{0.500\text{ s}}$$

(d) Since $T = \frac{2}{3}$ s and $A = 2.00$ cm, the particle will travel 8.00 cm in this time.

Hence, in $1.00\text{ s}\left(= \frac{3T}{2}\right)$, the particle will travel

$$8.00\text{ cm} + 4.00\text{ cm} = \boxed{12.0\text{ cm}}$$

13.6 The proposed solution $x(t) = x_i \cos \omega t + \left(\frac{v_i}{\omega}\right) \sin \omega t$

implies velocity $v = \frac{dx}{dt} = -x_i\omega \sin \omega t + v_i \cos \omega t$

and acceleration $a = \frac{dv}{dt} = -x_i\omega^2 \cos \omega t - v_i\omega \sin \omega t$

$$= -\omega^2 \left(x_i \cos \omega t + \left(\frac{v_i}{\omega}\right) \sin \omega t \right) = -\omega^2 x$$

(a) The acceleration being a negative constant times position means we do have SHM, and its angular frequency is ω. At $t = 0$ the equations reduce to

$x = x_i$ and $v = v_i$

so they satisfy all the requirements.

(b) $v^2 - ax = (-x_i\omega \sin \omega t + v_i \cos \omega t)^2$

$-(-x_i\omega^2 \cos \omega t - v_i\omega \sin \omega t)\left(x_i \cos \omega t + \left(\frac{v_i}{\omega}\right) \sin \omega t \right)$

$= x_i^2 \omega^2 \sin^2 \omega t - 2x_i v_i\omega \sin \omega t \cos \omega t + v_i^2 \cos^2 \omega t$

$+ x_i^2 \omega^2 \cos^2 \omega t + x_i v_i \omega \cos \omega t \sin \omega t + x_i v_i \omega \sin \omega t \cos \omega t$

$+ v_i^2 \sin^2 \omega t = x_i^2 \omega^2 + v_i^2$

So this expression is constant in time. On one hand, it must keep its original value

$v_i^2 - a_i x_i$

On the other hand, if we evaluate it at a turning point where $v = 0$ and $x = A$, it is

$A^2\omega^2 + 0^2 = A^2\omega^2$

Thus it is proved.

13.7 $k = \frac{F}{x} = \frac{(10.0 \times 10^{-3} \text{ kg})(9.80 \text{ m/s}^2)}{3.90 \times 10^{-2} \text{ m}} = 2.51 \text{ N/m}$ and

$T = 2\pi \sqrt{\frac{m}{k}} = 2\pi \sqrt{\frac{25.0 \times 10^{-3} \text{ kg}}{2.51 \text{ N/m}}} = \boxed{0.627 \text{ s}}$

13.8 (a) $T = \dfrac{12.0\text{ s}}{5} = \boxed{2.40\text{ s}}$

(b) $f = \dfrac{1}{T} = \dfrac{1}{2.40} = \boxed{0.417\text{ Hz}}$

(c) $\omega = 2\pi f = 2\pi(0.417) = \boxed{2.62\text{ rad/s}}$

13.9 (a) $\omega = \sqrt{\dfrac{k}{m}} = \sqrt{\dfrac{8.00\text{ N/m}}{0.500\text{ kg}}} = 4.00\text{ s}^{-1}$

Therefore, position is given by $x = 10.0\sin(4.00t)$ cm

From this we find that

$v = 40.0\cos(4.00t)$ cm/s $\qquad v_{\max} = \boxed{40.0\text{ cm/s}}$

$a = -160\sin(4.00t)\text{ cm/s}^2 \qquad a_{\max} = \boxed{160\text{ cm/s}^2}$

(b) $t = \left(\dfrac{1}{4.00}\right)\sin^{-1}\left(\dfrac{x}{10.0}\right)$

and when $x = 6.00$ cm, $t = 0.161$ s, and we find

$v = 40.0\cos[4.00(0.161)] = \boxed{32.0\text{ cm/s}}$

$a = -160\sin[4.00(0.161)] = \boxed{-96.0\text{ cm/s}^2}$

(c) Using $t = \left(\dfrac{1}{4.00}\right)\sin^{-1}\left(\dfrac{x}{10.0}\right)$

when $x = 0$, $t = 0$ and when $x = 8.00$ cm, $t = 0.232$ s

Therefore, $\Delta t = \boxed{0.232\text{ s}}$

13.10 $m = 1.00$ kg, $k = 25.0$ N/m, and $A = 3.00$ cm

At $t = 0$, $x = -3.00$ cm

(a) $\omega = \sqrt{\dfrac{k}{m}} = \sqrt{\dfrac{25.0}{1.00}} = 5.00$ rad/s

so that, $T = \dfrac{2\pi}{\omega} = \dfrac{2\pi}{5.00} = \boxed{1.26\text{ s}}$

(b) $v_{max} = A\omega = (3.00 \times 10^{-2}\text{ m})(5.00\text{ rad/s}) = \boxed{0.150\text{ m/s}}$

$a_{max} = A\omega^2 = (3.00 \times 10^{-2}\text{ m})(5.00\text{ rad/s})^2 = \boxed{0.750\text{ m/s}^2}$

(c) Because $x = -3.00$ cm and $v = 0$ at $t = 0$, the required solution is

$x = -A\cos\omega t$

or $\boxed{x = -3.00\cos(5.00t)\text{ cm}}$

$v = \dfrac{dx}{dt} = \boxed{15.0\sin(5.00t)\text{ cm/s}}$

$a = \dfrac{dv}{dt} = \boxed{75.0\cos(5.00t)\text{ cm/s}^2}$

13.11 $f = \dfrac{\omega}{2\pi} = \dfrac{1}{2\pi}\sqrt{\dfrac{k}{m}}$ or $T = \dfrac{1}{f} = 2\pi\sqrt{\dfrac{m}{k}}$

Solving for k, $k = \dfrac{4\pi^2 m}{T^2} = \dfrac{(4\pi)^2(7.00\text{ kg})}{(2.60\text{ s})^2} = \boxed{40.9\text{ N/m}}$

13.12 (a) Energy is conserved between the maximum-displacement and the half-maximum points:

$(K + U)_i = (K + U)_f$

$0 + \dfrac{1}{2}kA^2 = \dfrac{1}{2}mv^2 + \dfrac{1}{2}mx^2$

$\dfrac{1}{2}(6.50\text{ N/m})(0.100\text{ m})^2 = \dfrac{1}{2}m(0.300\text{ m/s})^2 + \dfrac{1}{2}(6.50\text{ N/m})(5.00 \times 10^{-2}\text{ m})^2$

$32.5\text{ mJ} = \dfrac{1}{2}m(0.300\text{ m/s})^2 + 8.12\text{ mJ}$

$m = \dfrac{2(24.4\text{ mJ})}{9.00 \times 10^{-2}\text{ m}^2/\text{s}^2} = \boxed{0.542\text{ kg}}$

(b) $\omega = \sqrt{\dfrac{k}{m}} = \sqrt{\dfrac{6.50\text{ N/m}}{0.542\text{ kg}}} = 3.46\text{ rad/s}$

$\therefore T = \dfrac{2\pi}{\omega} = \dfrac{2\pi}{3.46\text{/s}} = \boxed{1.81\text{ s}}$

(c) $a_{max} = \omega^2 A = (3.46\text{/s})^2(0.100\text{ m}) = \boxed{1.20\text{ m/s}^2}$

13.13 (a) $v_{\max} = \omega A$

$$A = \frac{v_{\max}}{\omega} = \frac{1.50 \text{ m/s}}{2.00 \text{ rad/s}} = \boxed{0.750 \text{ m}}$$

(b) $x = \boxed{-(0.750 \text{ m}) \sin 2.00t}$

13.14 (a) $v_{\max} = \omega A$

$$A = \frac{v_{\max}}{\omega} = \boxed{\frac{v}{\omega}}$$

(b) $x = -A \sin \omega t = \boxed{-\left(\frac{v}{\omega}\right) \sin \omega t}$

13.15 The 0.500 s must elapse between one turning point and the other. Thus the period is 1.00 s.

$$\omega = \frac{2\pi}{T} = 6.28\text{/s}$$

and $v_{\max} = \omega A = (6.28\text{/s})(0.100 \text{ m}) = \boxed{0.628 \text{ m/s}}$

***13.16** $m = 200$ g, $T = 0.250$ s, $E = 2.00$ J; $\omega = \frac{2\pi}{T} = \frac{2\pi}{0.250} = 25.1$ rad/s

(a) $k = m\omega^2 = (0.200 \text{ kg})(25.1 \text{ rad/s})^2 = \boxed{126 \text{ N/m}}$

(b) $E = \frac{kA^2}{2} \Rightarrow A = \sqrt{\frac{2E}{k}} = \sqrt{\frac{2(2.00)}{126}} = \boxed{0.178 \text{ m}}$

13.17 By conservation of energy, $\frac{1}{2} mv^2 = \frac{1}{2} kx^2$

$$v = \sqrt{\frac{k}{m}}\, x = \sqrt{\frac{5.00 \times 10^6}{10^3}}\,(3.16 \times 10^{-2} \text{ m}) = \boxed{2.23 \text{ m/s}}$$

Goal Solution

G: If the bumper is only compressed 3 cm, the car is probably not permanently damaged, so v is most likely less than 10 mph (< 5 m/s).

O: Assuming no energy is lost during impact with the wall, the initial energy (kinetic) equals the final energy (elastic potential):

A: $K_i = U_f$ or $\frac{1}{2}mv^2 = \frac{1}{2}kx^2$

$$v = x\sqrt{\frac{k}{m}} = (3.16 \times 10^{-2}\text{ m})\sqrt{\frac{5.00 \times 10^6\text{ N/m}}{1000\text{ kg}}}$$

$$v = 2.23\text{ m/s}$$

L: The speed is less than 5 m/s as predicted, so the answer seems reasonable. If the speed of the car were sufficient to compress the bumper beyond its elastic limit, then some of the initial kinetic energy would be lost to deforming the front of the car. In this case, some other procedure would have to be used to estimate the car's initial speed.

13.18 (a) $E = \frac{kA^2}{2} = \frac{(250\text{ N/m})(3.50 \times 10^{-2}\text{ m})^2}{2} = \boxed{0.153\text{ J}}$

(b) $v_{\max} = A\omega$

where $\omega = \sqrt{\frac{k}{m}} = \sqrt{\frac{250}{0.500}} = 22.4\text{ s}^{-1}$

$$v_{\max} = \boxed{0.784\text{ m/s}}$$

(c) $a_{\max} = A\omega^2 = (3.50 \times 10^{-2}\text{ m})(22.4\text{ s}^{-1})^2 = \boxed{17.5\text{ m/s}^2}$

13.19 (a) $E = \frac{1}{2}kA^2 = \frac{1}{2}(35.0\text{ N/m})(4.00 \times 10^{-2}\text{ m})^2 = \boxed{28.0\text{ mJ}}$

(b) $|v| = \omega\sqrt{A^2 - x^2} = \sqrt{\frac{k}{m}}\sqrt{A^2 - x^2}$

$$|v| = \sqrt{\frac{35.0}{50.0 \times 10^{-3}}}\sqrt{(4.00 \times 10^{-2})^2 - (1.00 \times 10^{-2})^2} = \boxed{1.02\text{ m/s}}$$

(c) $\frac{1}{2}mv^2 = \frac{1}{2}kA^2 - \frac{1}{2}kx^2 = \frac{1}{2}(35.0)\left[(4.00 \times 10^{-2})^2 - (3.00 \times 10^{-2})^2\right] = \boxed{12.2\text{ mJ}}$

(d) $\frac{1}{2}kx^2 = E - \frac{1}{2}mv^2 = \boxed{15.8\text{ mJ}}$

13.20 (a) $k = \frac{F}{x} = \frac{20.0\text{ N}}{0.200\text{ m}} = \boxed{100\text{ N/m}}$

(b) $\omega = \sqrt{\frac{k}{m}} = \sqrt{50.0}\text{ rad/s}$

$f = \frac{\omega}{2\pi} = \boxed{1.13\text{ Hz}}$

(c) $v_{\text{max}} = \omega A = \sqrt{50.0}\,(0.200) = \boxed{1.41\text{ m/s}}$ at $x = 0$

(d) $a_{\text{max}} = \omega^2 A = 50.0(0.200) = \boxed{10.0\text{ m/s}^2}$ at $x = \pm A$

(e) $E = \frac{1}{2}kA^2 = \frac{1}{2}(100)(0.200)^2 = \boxed{2.00\text{ J}}$

(f) $v = \omega\sqrt{A^2 - x^2} = \sqrt{50.0}\sqrt{\frac{8}{9}(0.200)^2} = \boxed{1.33\text{ m/s}}$

(g) $a = \omega^2 x = (50.0)\left(\frac{0.200}{3}\right) = \boxed{3.33\text{ m/s}^2}$

13.21 (a) In the presence of non-conservative forces, we use

$$\Delta E = \frac{1}{2}mv_f^2 - \frac{1}{2}mv_i^2 + mgy_f - mgy_i + \frac{1}{2}kx_f^2 - \frac{1}{2}kx_i^2$$

$$(20.0\text{ N})(0.300\text{ m}) = \frac{1}{2}(1.50\text{ kg})\,v_f^2 - 0 + 0 - 0 + \frac{1}{2}(19.6\text{ N/m})(0.300\text{ m})^2 - 0$$

$$v_f = \boxed{2.61\text{ m/s}}$$

(b) $f_k = \mu_k n = 0.200(14.7\text{ N}) = 2.94\text{ N}$

$(K + U)_i + \Delta E = (K + U)_f$

$$0 + 0 + Fd\cos 0° + fd\cos 180° = \frac{1}{2}mv_f^2 + \frac{1}{2}kx^2$$

$$6.00\text{ J} + (2.94\text{ N})(0.300\text{ m})\cos 180° = \frac{1}{2}(1.50\text{ kg})\,v_f^2 + 0.882\text{ J}$$

$$v_f = \sqrt{2(6.00\text{ J} - 0.882\text{ J} - 0.882\text{ J})/1.50\text{ kg}} = \boxed{2.38\text{ m/s}}$$

13.22 (a) $E = \frac{1}{2}kA^2$, so if $A' = 2A$, $E' = \frac{1}{2}k(A')^2 = \frac{1}{2}k(2A)^2 = 4E$

Therefore $\boxed{E \text{ increases by factor of } 4}$.

(b) $v_{\max} = \sqrt{\frac{k}{m}}\,A$, so if A is doubled, $\boxed{v_{\max} \text{ is doubled}}$.

(c) $a_{\max} = \frac{k}{m}A$, so if A is doubled, $\boxed{a_{\max} \text{ also doubles}}$.

(d) $T = 2\pi\sqrt{\frac{m}{k}}$ is independent of A, so $\boxed{\text{the period is unchanged}}$.

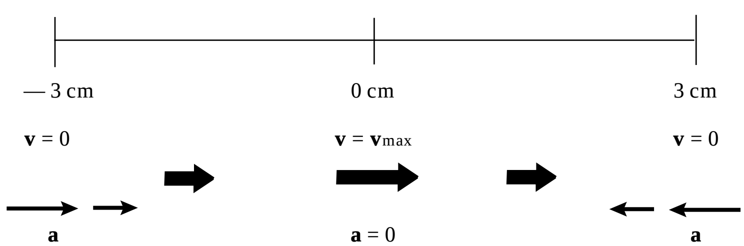
13.23 From energy considerations, $v^2 + \omega^2 x^2 = \omega^2 A^2$

$v_{\max} = \omega A$ and $v = \frac{\omega A}{2}$ so $\left(\frac{w\,A}{2}\right)^2 + \omega^2 x^2 = \omega^2 A^2$

From this we find $x^2 = \frac{3A^2}{4}$ and $x = \frac{A\sqrt{3}}{2} = \boxed{\pm 2.60 \text{ cm}}$ where $A = 3.00$ cm

Goal Solution

G: If we consider the speed of the particle along its path as shown in the sketch, we can see that the particle is at rest momentarily at one endpoint while being accelerated toward the middle by an elastic force that decreases as the particle approaches the equilibrium position. When it reaches the midpoint, the direction of acceleration changes so that the particle slows down until it stops momentarily at the opposite endpoint. From this analysis, we can estimate that $v = v_{\max}/2$ somewhere in the outer half of the travel: $1.5 < x < 3$.



O: We can analyze this problem in more detail by examining the energy of the system, which should be constant since we are told that the motion is SHM (no damping).

A: From energy considerations (Eq. 13.23), $v^2 + \omega^2 x^2 = \omega^2 A^2$. The speed v will be maximum when x is zero. Thus, $v_{max} = \omega A$ and

$$v_{1/2} = \frac{v_{max}}{2} = \frac{\omega A}{2}$$

Substituting $v_{1/2}$ in for v, $\frac{1}{4}\omega^2 A^2 + \omega^2 x^2 = \omega^2 A^2$

Solving for x we find that $x^2 = \frac{3A^2}{4}$

Given that $A = 3.00$ cm, $x = \pm\frac{A\sqrt{3}}{2} = \pm\frac{(3.00\text{ cm})\sqrt{3}}{2} = \pm 2.60\text{ cm}$ ◊

L: The calculated position is in the outer half of the travel as predicted, and is in fact very close to the endpoints. This means that the speed of the particle is mostly constant until it reaches the ends of its travel, where it experiences the maximum restoring force of the spring, which is proportional to x.

***13.24** The potential energy is

$$U_s = \frac{1}{2}kx^2 = \frac{1}{2}kA^2\cos^2(\omega t)$$

The rate of change of potential energy is

$$\frac{dU_s}{dt} = \frac{1}{2}kA^2\, 2\cos(\omega t)[-\omega\sin(\omega t)] = -\frac{1}{2}kA^2\,\omega\sin 2\omega t$$

(a) This rate of change is maximal and negative at

$$2\omega t = \frac{\pi}{2},\ 2\omega t = 2\pi + \frac{\pi}{2},\ \text{or in general, } 2\omega t = 2n\pi + \frac{\pi}{2} \text{ for integer } n.$$

Then, $t = \frac{\pi}{4\omega}(4n+1) = \frac{\pi(4n+1)}{4(3.60\text{ s}^{-1})}$

For $n = 0$, this gives $t = \boxed{0.218\text{ s}}$ while $n = 1$ gives $t = \boxed{1.09\text{ s}}$.

All other values of n yield times outside the specified range.

(b) $$\left|\frac{dU_s}{dt}\right|_{max} = \frac{1}{2}kA^2\omega = \frac{1}{2}(3.24\text{ N/m})(5.00\times 10^{-2}\text{ m})^2(3.60\text{ s}^{-2}) = \boxed{14.6\text{ mW}}$$

***13.25** (a) $T = 2\pi\sqrt{\dfrac{L}{g}}$

$$L = \frac{gT^2}{4\pi^2} = \frac{(9.80 \text{ m/s}^2)(12.0 \text{ s})^2}{4\pi^2} = \boxed{35.7 \text{ m}}$$

(b) $T_{\text{moon}} = 2\pi\sqrt{\dfrac{L}{g_{\text{moon}}}} = 2\pi\sqrt{\dfrac{35.7 \text{ m}}{1.67 \text{ m/s}^2}} = \boxed{29.1 \text{ s}}$

***13.26** The period in Tokyo is $T_t = 2\pi\sqrt{\dfrac{L_t}{g_t}}$

and the period in Cambridge is $T_c = 2\pi\sqrt{\dfrac{L_c}{g_c}}$

We know $T_t = T_c = 2.00$ s

For which, we see $\dfrac{L_t}{g_t} = \dfrac{L_c}{g_c}$

or $\dfrac{g_c}{g_t} = \dfrac{L_c}{L_t} = \dfrac{0.9942}{0.9927} = \boxed{1.0015}$

***13.27** The swinging box is a physical pendulum with period $T = 2\pi\sqrt{\dfrac{I}{mgd}}$.
The moment of inertia is given approximately by

$I = \dfrac{1}{3}mL^2$ (treating the box as a rod suspended from one end).

Then, with $L \approx 1.0$ m and $d \approx L/2$,

$$T \approx 2\pi\sqrt{\frac{(1/3)mL^2}{mg(L/2)}} = 2\pi\sqrt{\frac{2L}{3g}} = 2\pi\sqrt{\frac{2(1.0 \text{ m})}{3(9.8 \text{ m/s}^2)}} = 1.6 \text{ s} \quad \text{or} \quad T \sim \boxed{10^0 \text{ s}}$$

13.28 $\omega = \dfrac{2\pi}{T}$ $\qquad T = \dfrac{2\pi}{\omega} = \dfrac{2\pi}{4.43} = \boxed{1.42 \text{ s}}$

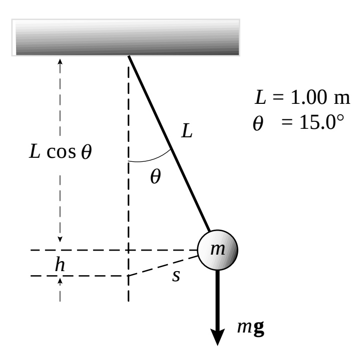
$\omega = \sqrt{\dfrac{g}{L}}$ $\qquad L = \dfrac{g}{\omega^2} = \dfrac{9.80}{(4.43)^2} = \boxed{0.499 \text{ m}}$

13.29 (a) $mgh = \dfrac{1}{2}mv^2$

$h = L(1 - \cos\theta)$

$\therefore v_{\max} = \sqrt{2gL(1 - \cos\theta)}$

$v_{\max} = \boxed{0.817 \text{ m/s}}$



(b) $I\alpha = mgL \sin\theta$

$$\alpha_{\max} = \frac{mgL \sin\theta}{mL^2} = \frac{g}{L}\sin\theta_i = \boxed{2.54 \text{ rad/s}^2}$$

(c) $F_{\max} = mg\sin\theta_i = (0.250)(9.80)(\sin 15.0°) = \boxed{0.634 \text{ N}}$

13.30 (a) The string tension must support the weight of the bob, accelerate it upward, and also provide the restoring force, just as if the elevator were at rest in a gravity field (9.80 + 5.00) m/s^2

$$T = 2\pi\sqrt{\frac{L}{g}} = 2\pi\sqrt{\frac{5.00 \text{ m}}{14.8 \text{ m/s}^2}}$$

$$T = \boxed{3.65 \text{ s}}$$

(b) $T = 2\pi\sqrt{\dfrac{5.00 \text{ m}}{(9.80 \text{ m/s}^2 - 5.00 \text{ m/s}^2)}} = \boxed{6.41 \text{ s}}$

(c) $g_{\text{eff}} = \sqrt{(9.80 \text{ m/s}^2)^2 + (5.00 \text{ m/s}^2)^2} = 11.0 \text{ m/s}^2$

$$T = 2\pi = \sqrt{\frac{5.00 \text{ m}}{11.0 \text{ m/s}^2}} = \boxed{4.24 \text{ s}}$$

13.31 Referring to the sketch we have

$$F = -mg\sin\theta$$

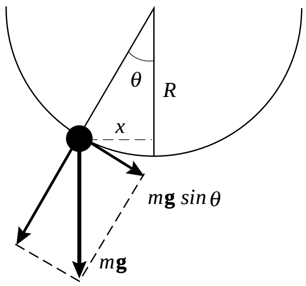
and $\tan\theta \approx \dfrac{x}{R}$

For small displacements,

$$\tan\theta \approx \sin\theta$$

and $F = -\dfrac{mg}{R}x = -kx$

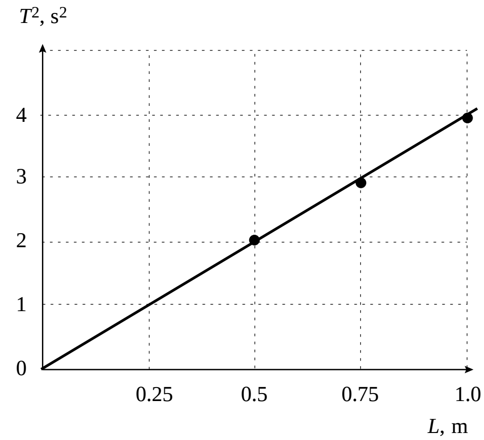
and $\boxed{\omega = \sqrt{\dfrac{k}{m}} = \sqrt{\dfrac{g}{R}}}$



13.32 (a) $T = \dfrac{\text{total measured time}}{50}$

The measured periods are:

Length, L (m)	1.000	0.750	0.500
Period, T (s)	1.996	1.732	1.422



(b) $T = 2\pi\sqrt{\dfrac{L}{g}}$ so $g = \dfrac{4\pi^2 L}{T^2}$

The calculated values for g are:

Period, T (s)	1.996	1.732	1.422
g (m/s^2)	9.91	9.87	9.76

Thus, $g_{\text{ave}} = \boxed{9.85\ \text{m/s}^2}$ this agrees with the accepted value of $g = 9.80\ \text{m/s}^2$ within 0.5%.

(c) Slope of T^2 versus L graph $= 4\pi^2/g = 4.01\ \text{s}^2/\text{m}$

Thus, $g = \dfrac{4\pi^2}{\text{slope}} = 9.85\ \text{m/s}^2$. You should find that $\boxed{\%\ \text{difference} \approx 0.5\%}$.

13.33 $f = 0.450$ Hz, $d = 0.350$ m, and $m = 2.20$ kg

$$T = \frac{1}{f}; \quad T = 2\pi\sqrt{\frac{I}{mgd}}; \quad T^2 = 4\pi^2\left(\frac{I}{mgd}\right)$$

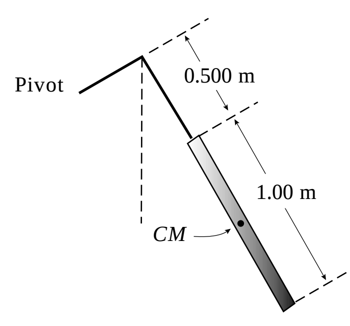
$$I = T^2\frac{mgd}{4\pi^2} = \left(\frac{1}{f}\right)^2\frac{mgd}{4\pi^2} = \frac{(2.20)(9.80)(0.350)}{(0.450\ \text{s}^{-1})^2(4\pi^2)}$$

$$I = \boxed{0.944\ \text{kg}\cdot\text{m}^2}$$

13.34 (a) The parallel-axis theorem:

$$I = I_{\text{CM}} + Md^2 = \frac{1}{12}ML^2 + Md^2$$

$$= \frac{1}{12}M(1.00\ \text{m})^2 + M(1.00\ \text{m})^2 = M\left(\frac{13}{12}\text{m}^2\right)$$



$$T = 2\pi\sqrt{\frac{I}{Mgd}} = 2\pi\sqrt{\frac{M(13\ \text{m}^2)}{12Mg(1.00\ \text{m})}}$$

$$T = 2\pi\sqrt{\frac{13\ \text{m}}{12(9.80\ \text{m/s}^2)}} = \boxed{2.09\ \text{s}}$$

(b) For the simple pendulum

$$T = 2\pi\sqrt{\frac{1.00 \text{ m}}{9.80 \text{ m/s}^2}} = 2.01 \text{ s}$$

$$\text{difference} = \frac{2.09 \text{ s} - 2.01 \text{ s}}{2.01 \text{ s}} = \boxed{4.08\%}$$

13.35 (a) The parallel axis theorem says directly $I = I_{CM} + md^2$

$$\text{so} \quad T = 2\pi\sqrt{\frac{I}{mgd}} = \boxed{2\pi\sqrt{\frac{(I_{CM} + md^2)}{mgd}}}$$

(b) When d is very large $T \to 2\pi\sqrt{\frac{d}{g}}$ gets large.

When d is very small $T \to 2\pi\sqrt{\frac{I_{CM}}{mgd}}$ gets large.

So there must be a minimum, found by

$$\frac{dT}{dd} = 0 = \frac{d}{dd}\, 2\pi\,(I_{CM} + md^2)^{1/2}(mgd)^{-1/2}$$

$$= 2\pi\,(I_{CM} + md^2)^{1/2}\left(-\frac{1}{2}\right)(mgd)^{-3/2}\, mg$$

$$+ 2\pi(mgd)^{-1/2}\left(\frac{1}{2}\right)(I_{CM} + md^2)^{-1/2}\, 2md$$

= **Error!** $mg,(I_{CM} + md^2)^{1/2}\,(mgd)^{3/2})$ + **Error!** $= 0$

(b) This requires

$$-I_{CM} - md^2 + 2md^2 = 0$$

$$\text{or} \quad \boxed{I_{CM} = md^2}$$

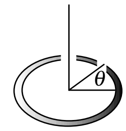
13.36 We suppose the stick moves in a horizontal plane. Then,

$$I = \frac{1}{12}\, mL^2 = \frac{1}{12}(2.00 \text{ kg})(1.00 \text{ m})^2 = 0.167 \text{ kg} \cdot \text{m}^2$$

$$T = 2\pi\sqrt{I/\kappa}$$

$$\kappa = \frac{4\pi^2 I}{T^2} = \frac{4\pi^2(0.167 \text{ kg} \cdot \text{m}^2)}{(180 \text{ s})^2} = \boxed{203\ \mu\text{N} \cdot \text{m}}$$

13.37 $T = 0.250$ s; $I = mr^2 = (20.0 \times 10^{-3}\text{ kg})(5.00 \times 10^{-3}\text{ m})^2$



(a) $I = \boxed{5.00 \times 10^{-7}\text{ kg} \cdot \text{m}^2}$

(b) $I\dfrac{d^2\theta}{dt^2} = -\kappa\theta;\quad \sqrt{\dfrac{\kappa}{I}} = \omega = \dfrac{2\pi}{T}$

$$\kappa = I\omega^2 = (5.00 \times 10^{-7})\left(\frac{2\pi}{0.250}\right)^2 = \boxed{3.16 \times 10^{-4}\ \frac{\text{N} \cdot \text{m}}{\text{rad}}}$$

13.38 (a) The motion is simple harmonic because the tire is rotating with constant velocity and you are looking at the motion of the boss projected in a plane perpendicular to the tire.

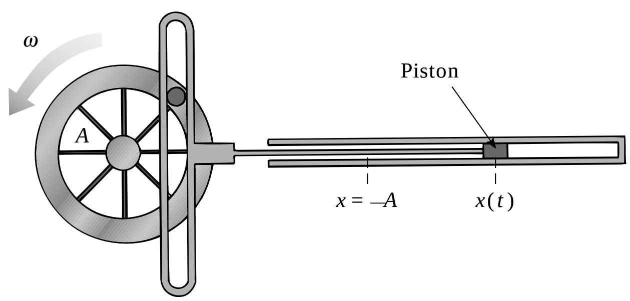
(b) Since the car is moving with speed $v = 3.00$ m/s, and its radius is 0.300 m, we have:

$$\omega = \frac{3.00\text{ m/s}}{0.300\text{ m}} = 10.0\text{ rad/s}$$

Therefore, the period of the motion is:

$$T = \frac{2\pi}{\omega} = \frac{2\pi}{(10.0\text{ rad/s})} = \boxed{0.628\text{ s}}$$

13.39 The angle of the crank pin is $\theta = \omega t$. Its x-coordinate is $\boxed{x = A\cos\theta = A\cos\omega t}$ where A is the distance from the center of the wheel to the crank pin. This is of the form $x = A\cos(\omega t + \phi)$, so the yoke and piston rod move with simple harmonic motion.



13.40 $E = \dfrac{1}{2}mv^2 + \dfrac{1}{2}kx^2$

$$\frac{dE}{dt} = mv\frac{d^2x}{dt^2} + kxv$$

Use Equation 13.32:

$$\frac{md^2x}{dt^2} = -kx - bv$$

$$\frac{dE}{dt} = v(-kx - bv) + kvx$$

$$\boxed{\frac{dE}{dt} = -bv^2 < 0}$$

13.41 $\theta_i = 15.0°$ $\theta(t = 1000) = 5.50°$

$$x = Ae^{-bt/2m}$$

$$\frac{x_{1000}}{x_i} = \frac{Ae^{-bt/2m}}{A} = \frac{5.50}{15.0} = e^{-b(1000)/2m} \approx e^{-1}$$

$$\therefore \frac{b}{2m} = \boxed{1.00 \times 10^{-3}\ \text{s}^{-1}}$$

***13.42** Show that $x = Ae^{-bt/2m}\cos(\omega t + \phi)$ is a solution of

$$-kx - b\frac{dx}{dt} = m\frac{d^2x}{dt^2} \qquad (1)$$

and $$\omega = \sqrt{\frac{k}{m} - \left(\frac{b}{2m}\right)^2} \qquad (2)$$

$$x = Ae^{-bt/2m}\cos(\omega t + \phi) \qquad (3)$$

$$\frac{dx}{dt} = Ae^{-bt/2m}\left(-\frac{b}{2m}\right)\cos(\omega t + \phi) - Ae^{-bt/2m}\,\omega\,(\sin(\omega t + \phi)) \qquad (4)$$

$$\frac{d^2x}{dt^2} = -\frac{b}{2m}\left\{Ae^{-bt/2m}\left(-\frac{b}{2m}\right)\cos(\omega t + \phi) - Ae^{-bt/2m}\,\omega\sin(\omega t + \phi)\right\}$$

$$-\left\{Ae^{-bt/2m}\left(-\frac{b}{2m}\right)\omega\sin(\omega t + \phi) + Ae^{-bt/2m}\,\omega^2\cos(\omega t + \phi)\right\} \qquad (5)$$

Substitute (3), (4) into the left side of (1) and (5) into the right side of (1);

$$-kAe^{-bt/2m}\cos(\omega t + \phi) + \frac{b^2}{2m}\ Ae^{-bt/2m}\cos(\omega t + \phi)$$

$$+\ b\omega Ae^{-bt/2m}\sin(\omega t + \phi)$$

$$= -\frac{b}{2}\left\{Ae^{-bt/2m}\left(-\frac{b}{2m}\right)\cos(\omega t + \phi) - Ae^{-bt/2m}\,\omega\sin(\omega t + \phi)\right\}$$

$$+\ \frac{b}{2}\ Ae^{-bt/2m}\,\omega\sin(\omega t + \phi) - m\omega^2 Ae^{-bt/2m}\cos(\omega t + \phi)$$

Compare the coefficients of $Ae^{-bt/2m}\cos(\omega t+\phi)$ and $Ae^{-bt/2m}\sin(\omega t+\phi)$:

cosine-term: $-k+\dfrac{b^2}{2m} = -\dfrac{b}{2}\left(-\dfrac{b}{2m}\right) - m\omega^2 = \dfrac{b^2}{4m} - (m)\left(\dfrac{k}{m}-\dfrac{b^2}{4m^2}\right)$

$$= -k+\frac{b^2}{2m}$$

sine-term: $b\omega = +\dfrac{b}{2}(\omega) + \dfrac{b}{2}\,\omega = b\omega$

Since the coefficients are equal,

$x = Ae^{-bt/2m}\cos(\omega t+\phi)$ is a solution of the equation.

***13.43** (a) For resonance, her frequency must match

$$f_0 = \frac{\omega_0}{2\pi} = \frac{1}{2\pi}\sqrt{\frac{k}{m}} = \frac{1}{2\pi}\sqrt{\frac{4.30\times10^3\ \text{N/m}}{12.5\ \text{kg}}} = \boxed{2.95\ \text{Hz}}$$

(b) From $x = A\cos\omega t$, $v = dx/dt = -A\omega\sin\omega t$, and $a = dv/dt = -A\omega^2\cos\omega t$, the maximum acceleration is $A\omega^2$. When this becomes equal to the acceleration of gravity, the normal force exerted on her by the mattress will drop to zero at one point in the cycle:

$$A\omega^2 = g \quad \text{or} \quad A = \frac{g}{\omega^2} = \frac{g}{k/m} = \frac{gm}{k}$$

$$A = \frac{(9.80\ \text{m/s}^2)(12.5\ \text{kg})}{4.30\times10^3\ \text{N/m}} = \boxed{2.85\ \text{cm}}$$

13.44 $F = 3.00\cos(2\pi t)$ N and $k = 20.0$ N/m

(a) $\omega = \dfrac{2\pi}{T} = 2\pi\ \text{rad/s}$ so $T = \boxed{1.00\ \text{s}}$

(b) In this case, $\omega_0 = \sqrt{\dfrac{k}{m}} = \sqrt{\dfrac{20.0}{2.00}} = 3.16\ \text{rad/s}$

Taking $b = 0$ in Equation 13.37 gives

$$A = \left(\frac{F_0}{m}\right)(\omega^2-\omega_0^2)^{-1} = \frac{3}{2}\left[4\pi^2 - (3.16)^2\right]^{-1}$$

$$A = 0.0509\ \text{m} = \boxed{5.09\ \text{cm}}$$

***13.45** $F_0 \cos(\omega t) - kx = m\dfrac{d^2x}{dt^2}$ $\omega_0 = \sqrt{\dfrac{k}{m}}$ (1)

$x = A\cos(\omega t + \phi)$ (2)

$\dfrac{dx}{dt} = -A\omega \sin(\omega t + \phi)$ (3)

$\dfrac{d^2x}{dt^2} = -A\omega^2 \cos(\omega t + \phi)$ (4)

Substitute (2) and (4) into (1):

$$F_0 \cos(\omega t) - kA\cos(\omega t + \phi) = m(-A\omega^2)\cos(\omega t + \phi)$$

Solve for the amplitude: $(kA - mA\omega^2)\cos(\omega t + \phi) = F_0 \cos \omega t$

These will be equal, provided only that ϕ must be zero and $(kA - mA\omega^2) = F_0$

Thus, $A = \dfrac{F_0 / m}{\dfrac{k}{m} - \omega^2}$

13.46 From Equation 13.37 with no damping,

$$A = \frac{F_0/m}{\sqrt{(\omega^2 - \omega_0^2)^2}}$$

$\omega = 2\pi f = (20.0\pi \text{ s}^{-1})$ $\omega_0^2 = \dfrac{k}{m} = \dfrac{200}{(40.0/9.80)} = 49.0 \text{ s}^{-2}$

$F_0 = mA(\omega^2 - \omega_0^2)$

$F_0 = \left(\dfrac{40.0}{9.80}\right)(2.00 \times 10^{-2})(3950 - 49.0) = \boxed{318 \text{ N}}$

***13.47** $A = \dfrac{F_{ext}/m}{\sqrt{(\omega^2 - \omega_0^2)^2 + (b\omega/m)^2}}$

With $b = 0$, $A = \dfrac{F_{ext}/m}{\sqrt{(\omega^2 - \omega_0^2)^2}} = \dfrac{F_{ext}/m}{\pm(\omega^2 - \omega_0^2)} = \pm\dfrac{F_{ext}/m}{\omega^2 - \omega_0^2}$

Thus, $\omega^2 = \omega_0^2 \pm \dfrac{F_{ext}/m}{A} = \dfrac{k}{m} \pm \dfrac{F_{ext}}{mA} = \dfrac{6.30 \text{ N/m}}{0.150 \text{ kg}} \pm \dfrac{1.70 \text{ N}}{(0.150 \text{ kg})(0.440 \text{ m})}$

This yields $\omega = 8.23$ rad/s or $\omega = 4.03$ rad/s

Then, $f = \dfrac{\omega}{2\pi}$ gives either $f = \boxed{1.31 \text{ Hz}}$ or $f = \boxed{0.641 \text{ Hz}}$

***13.48** The beeper must resonate at the frequency of a simple pendulum of length 8.21 cm:

$$f = \frac{1}{2\pi}\sqrt{\frac{g}{L}} = \frac{1}{2\pi}\sqrt{\frac{9.80 \text{ m/s}^2}{0.0821 \text{ m}}} = \boxed{1.74 \text{ Hz}}$$

13.49 Assume that each spring supports an equal portion of the car's mass, i.e. $\frac{m}{4}$.

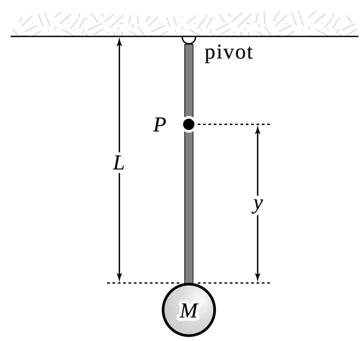
Then $T = 2\pi\sqrt{\frac{m}{4k}}$ and $k = \frac{4\pi^2 m}{4T^2} = \frac{4\pi^2 1500}{(4)(1.50)^2} = \boxed{6580 \text{ N/m}}$

13.50 $$\frac{T_1}{T_0} = \frac{2\pi/\omega_1}{2\pi/\omega_0} = \frac{2\pi/\sqrt{k/m_1}}{2\pi/\sqrt{k/m_0}}$$

$$\frac{T_1}{T_0} = \sqrt{\frac{m_1}{m_0}} = \sqrt{\frac{1650}{1500}} = \sqrt{1.10}$$

$$T_1 = \sqrt{1.10} \times 1.50 = \boxed{1.57 \text{ s}}$$

13.51 Let F represent the tension in the rod.



(a) At the pivot, $F = Mg + Mg = \boxed{2Mg}$

A fraction of the rod's weight $Mg\left(\frac{y}{L}\right)$ as well as the weight of the ball pulls down on point P. Thus, the tension in the rod at point P is

$$F = Mg\left(\frac{y}{L}\right) + Mg = \boxed{Mg\left(1 + \frac{y}{L}\right)}$$

(b) Relative to the pivot, $I = I_{\text{rod}} + I_{\text{ball}} = \frac{1}{3}ML^2 + ML^2 = \frac{4}{3}ML^2$

For the physical pendulum, $T = 2\pi\sqrt{\frac{I}{mgd}}$ where $m = 2M$ and d is the distance from the pivot to the center of mass of the rod and ball combination. Therefore,

$$d = \frac{M(L/2) + ML}{M + M} = \frac{3L}{4} \quad \text{and} \quad T = 2\pi\sqrt{\frac{(4/3)ML^2}{(2M)g(3L/4)}} = \boxed{\frac{4\pi}{3}\sqrt{\frac{2L}{g}}}$$

For $L = 2.00$ m, $T = \frac{4\pi}{3}\sqrt{\frac{2(2.00 \text{ m})}{9.80 \text{ m/s}^2}} = \boxed{2.68 \text{ s}}$

<u>Goal Solution</u>

G: The tension in the rod at the pivot = weight of rod + weight of $M = 2\,Mg$. The tension at point P should be slightly less since the portion of the rod between P and the pivot does not contribute to the tension.

The period should be slightly less than for a simple pendulum since the mass of the rod effectively shortens the length of the simple pendulum (massless rod) by moving the center of mass closer to the pivot, so that $T < 2\pi\sqrt{\frac{L}{g}}$

O: The tension can be found from applying Newton's Second Law to the static case. The period of oscillation can be found by analyzing the components of this physical pendulum and using Equation 13.28.

A: (a) At the pivot, the net downward force is: $T = Mg + Mg = 2Mg$ ◊

At P, a fraction of the rod's mass (y/L) pulls down along with the ball.

Therefore, $T = Mg\left(\frac{y}{L}\right) + Mg = Mg\left(1 + \frac{y}{L}\right)$ ◊

(b) Relative to the pivot, $I_{\text{total}} = I_{\text{rod}} + I_{\text{ball}} = \frac{1}{3}\,ML^2 + ML^2 = \frac{4}{3}\,ML^2$

For a physical pendulum, $T = 2\pi\sqrt{\frac{I}{mgd}}$

In this case, $m = 2M$ and d is the distance from the pivot to the center of mass.

$$d = \frac{\left(\frac{ML}{2} + ML\right)}{(M + M)} = \frac{3L}{4}$$

so we have, $T = 2\pi\sqrt{\frac{I}{mgd}} = 2\pi\sqrt{\frac{(4ML^2)4}{3(2M)g(3L)}} = \frac{4\pi}{3}\sqrt{\frac{2L}{g}}$ ◊

For $L = 2.00$ m, $T = \frac{4\pi}{3}\sqrt{\frac{2(2.00\text{ m})}{9.80\text{ m/s}^2}} = 2.68\text{ s}$ ◊

L: In part (a), the tensions agree with the initial predictions. In part (b) we found that the period is indeed slightly less (by about 6%) than a simple pendulum of length L. It is interesting to note that we were able to calculate a value for the period despite not knowing the mass value. This is because the period of any pendulum depends on the *location* of the center of mass and not on the *size* of the mass.

13.52 (a) Total energy $= \frac{1}{2}kA^2 = \frac{1}{2}(100\text{ N/m})(0.200\text{ m})^2 = 2.00\text{ J}$

At equilibrium, the total energy is:

$$\frac{1}{2}(m_1 + m_2)\,v^2 = \frac{1}{2}(16.0\text{ kg})\,v^2 = (8.00\text{ kg})v^2$$

Therefore, $(8.00\text{ kg})v^2 = 2.00\text{ J}$, and $v = \boxed{0.500\text{ m/s}}$

This is the speed of m_1 and m_2 at the equilibrium point. Beyond this point, the mass m_2 moves with the constant speed of 0.500 m / s while mass m_1 starts to slow down due to the restoring force of the spring.

(b) The energy of the m_1-spring at equilibrium is:

$$\frac{1}{2}m_1v^2 = \frac{1}{2}(9.00\text{ kg})(0.500\text{ m/s})^2 = 1.125\text{ J}$$

This is also equal to $\frac{1}{2}k(A')^2$, where A' is the amplitude of the m_1-spring system.

Therefore, $\frac{1}{2}(100)(A')^2 = 1.125$ or $A' = 0.150\text{ m}$

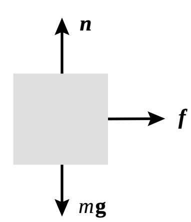
The period of the m_1-spring system is:

$$T = 2\pi\sqrt{\frac{m_1}{k}} = 1.885\text{ s}$$

and it takes $\frac{T}{4} = 0.471\text{ s}$ after it passes the equilibrium point for the spring to become fully stretched the first time. The distance separating m_1 and m_2 at this time is:

$$D = v\left(\frac{T}{4}\right) - A' = (0.500\text{ m/s})(0.471\text{ s}) - 0.150\text{ m} = 0.0856 = \boxed{8.56\text{ cm}}$$

13.53 $\left(\frac{d^2x}{dt^2}\right)_{\text{max}} = A\omega^2$ $\qquad f_{\text{max}} = \mu_s n = \mu_s mg = mA\omega^2$



$$A = \frac{\mu_s g}{\omega^2} = \boxed{6.62\text{ cm}}$$

13.54 The maximum acceleration of the oscillating system is $a_{max} = A\omega^2 = 4\pi^2 Af^2$. The friction force exerted between the two blocks must be capable of accelerating block B at this rate. Thus, if Block B is about to slip,

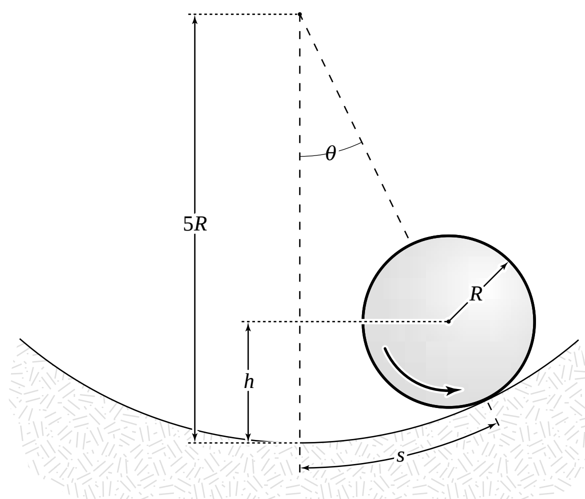
$$f = f_{max} = \mu_s n = \mu_s mg = m(4\pi^2 Af^2) \quad \text{or} \quad A = \boxed{\frac{\mu_s g}{4\pi^2 f^2}}$$

13.55 $M_{D_2} = 2M_{H_2}$

$$\frac{\omega_D}{\omega_H} = \frac{\sqrt{k/M_D}}{\sqrt{k/M_H}} = \sqrt{\frac{M_H}{M_D}} = \sqrt{\frac{1}{2}}$$

$$f_{D_2} = \frac{f_{H_2}}{\sqrt{2}} = \boxed{0.919 \times 10^{14}\ \text{Hz}}$$

13.56 The kinetic energy of the ball is $K = \frac{1}{2}mv^2 + \frac{1}{2}I\Omega^2$, where Ω is the rotation rate of the ball about its center of mass. Since the center of the ball moves along a circle of radius $4R$, its displacement from equilibrium is $s = (4R)\theta$ and its speed is $v = \frac{ds}{dt} = 4R\left(\frac{d\theta}{dt}\right)$. Also, since the ball rolls without slipping,



$$v = \frac{ds}{dt} = R\Omega \quad \text{so} \quad \Omega = \frac{v}{R} = 4\left(\frac{d\theta}{dt}\right)$$

The kinetic energy is then

$$K = \frac{1}{2}m\left(4R\frac{d\theta}{dt}\right)^2 + \frac{1}{2}\left(\frac{2}{5}mR^2\right)\left(4\frac{d\theta}{dt}\right)^2 = \frac{112mR^2}{10}\left(\frac{d\theta}{dt}\right)^2$$

When the ball has an angular displacement θ, its center is distance $h = 4R(1 - \cos\theta)$ higher than when at the equilibrium position. Thus, the potential energy is $U_g = mgh = 4mgR(1 - \cos\theta)$. For small angles, $(1 - \cos\theta) \approx \frac{\theta^2}{2}$ (see Appendix B) . Hence, $U_g \approx 2mgR\theta^2$, and the total energy is

$$E = K + U_g = \frac{112mR^2}{10}\left(\frac{d\theta}{dt}\right)^2 + 2mgR\theta^2$$

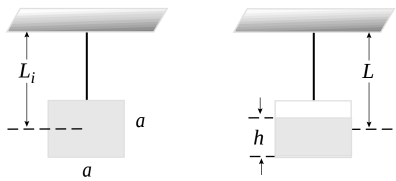
Since E = constant in time, $\frac{dE}{dt} = 0 = \frac{112mR^2}{5}\left(\frac{d\theta}{dt}\right)\frac{d^2\theta}{dt^2} + 4mgR\theta\left(\frac{d\theta}{dt}\right)$

This reduces to $\frac{28R}{5}\frac{d^2\theta}{dt^2} + g\theta = 0$, or $\frac{d^2\theta}{dt} = -\left(\frac{5g}{28R}\right)\theta$

This is in the classical form of a simple harmonic motion equation with $\omega = \sqrt{\frac{5g}{28R}}$.

The period of the simple harmonic motion is then $T = \frac{2\pi}{\omega} = \boxed{2\pi\sqrt{\frac{28R}{5g}}}$

13.57 (a)



(b) $T = 2\pi\sqrt{\frac{L}{g}}$ $\qquad \frac{dT}{dt} = \frac{\pi}{\sqrt{g}}\frac{1}{\sqrt{L}}\frac{dL}{dt}$ (1)

We need to find $L(t)$ and $\frac{dL}{dt}$. From the diagram in (a),

$$L = L_i + \frac{a}{2} - \frac{h}{2}\ ; \quad \frac{dL}{dt} = -\left(\frac{1}{2}\right)\frac{dh}{dt}$$

But $\frac{dM}{dt} = \rho\frac{dV}{dt} = -\rho A\frac{dh}{dt}$. Therefore,

$$\frac{dh}{dt} = -\frac{1}{\rho A}\frac{dM}{dt}\ ; \quad \frac{dL}{dt} = \left(\frac{1}{2\rho A}\right)\frac{dM}{dt} \quad (2)$$

Also, $\int_{L_i}^{L} dL = \left(\frac{1}{2\rho A}\right)\left(\frac{dM}{dt}\right) t = L - L_i$ (3)

Substituting Equation (2) and Equation (3) into Equation (1):

$$\frac{dT}{dt} = \boxed{\frac{\pi}{\sqrt{g}}\left(\frac{1}{2\rho a^2}\right)\left(\frac{dM}{dt}\right)\frac{1}{\sqrt{L_i + \frac{1}{2\rho a^2}\left(\frac{dM}{dt}\right)t}}}$$

(c) Substitute Equation (3) into the equation for the period.

$$T = \boxed{\frac{2\pi}{\sqrt{g}}\sqrt{L_i + \frac{1}{2\rho a^2}\left(\frac{dM}{dt}\right)t}}$$

Or one can obtain T by integrating (b):

$$\int_T^{T_i} dT = \frac{\pi}{\sqrt{g}}\left(\frac{1}{2\rho a^2}\right)\left(\frac{dM}{dt}\right)\int_0^t \frac{dt}{\sqrt{L_i + \frac{1}{2\rho a^2}\left(\frac{dM}{dt}\right)t}}$$

$$T - T_i = \frac{\pi}{\sqrt{g}}\left(\frac{1}{2\rho a^2}\right)\left(\frac{dM}{dt}\right)\left[\frac{2}{\frac{1}{2\rho a^2}\left(\frac{dM}{dt}\right)}\right]\left[\sqrt{L_i + \frac{1}{2\rho a^2}\left(\frac{dM}{dt}\right)t} - \sqrt{L_i}\right]$$

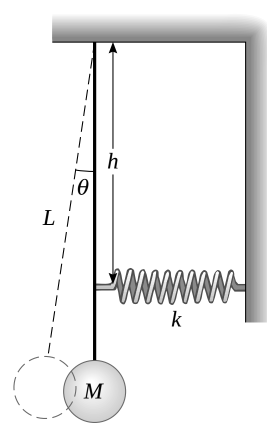
But $T_i = 2\pi\sqrt{\frac{L_i}{g}}$, so $T = \frac{2\pi}{\sqrt{g}}\sqrt{L_i + \frac{1}{2\rho a^2}\left(\frac{dM}{dt}\right)t}$

13.58 $\omega_0 = \sqrt{\frac{k}{m}} = \frac{2\pi}{T}$

(a) $k = \omega_0^2\, m = \boxed{\frac{4\pi^2 m}{T^2}}$

(b) $m' = \frac{k(T')^2}{4\pi^2} = \boxed{m\left(\frac{T'}{T}\right)^2}$

13.59 For the pendulum (see sketch) we have



$\tau = I\alpha$ and $\frac{d^2\theta}{dt^2} = -\alpha$

$$\tau = MgL\sin\theta + kxh\cos\theta = -I\frac{d^2\theta}{dt^2}$$

For small amplitude vibrations, use the approximations:

$\sin\theta \approx \theta$ $\cos\theta \approx 1$

and $x \approx s = h\theta$

Therefore,

$$\frac{d^2\theta}{dt^2} = -\left(\frac{MgL + kh^2}{I}\right)\theta = -\omega^2\theta$$

$$\omega = \sqrt{\frac{MgL + kh^2}{ML^2}} = 2\pi f$$

$$f = \boxed{\frac{1}{2\pi}\sqrt{\frac{MgL + kh^2}{ML^2}}}$$

Goal Solution

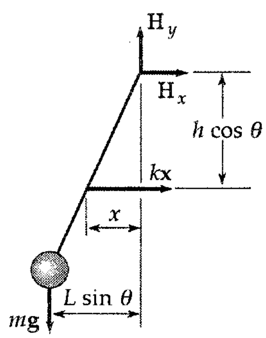
G: The frequency of vibration should be greater than that of a simple pendulum since the spring adds an additional restoring force: $f > \frac{1}{2\pi}\sqrt{\frac{g}{L}}$

O: We can find the frequency of oscillation from the angular frequency, which is found in the equation for angular SHM: $\frac{d^2\theta}{dt^2} = -\omega^2\theta$. The angular acceleration can be found from analyzing the torques acting on the pendulum.

A: For the pendulum (see sketch), we have

$$\sum\tau = I\alpha \quad \text{and} \quad \frac{d^2\theta}{dt^2} = -\alpha$$

The negative sign appears because positive θ is measured clockwise in the picture. We take torque around the point of suspension:



$$\sum\tau = MgL\sin\theta + kxh\cos\theta = I\alpha$$

For small amplitude vibrations, use the approximations:

$\sin\theta \approx \theta$, $\cos\theta \approx 1$, and $x \approx s = h\theta$

Therefore, with $I = mL^2$,

$$\frac{d^2\theta}{dt^2} = -\left[\frac{MgL + kh^2}{I}\right]\theta = -\left[\frac{MgL + kh^2}{ML^2}\right]\theta$$

This is of the form $\frac{d^2\theta}{dt^2} = -\omega^2\theta$ required for SHM,

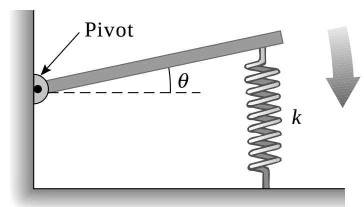
with angular frequency, $\omega = \sqrt{\frac{MgL + kh^2}{ML^2}} = 2\pi f$

The ordinary frequency is $f = \frac{\omega}{2\pi} = \frac{1}{2\pi}\sqrt{\frac{MgL + kh^2}{ML^2}}$

L: The frequency is greater than for a simple pendulum as we expected. In fact, the additional portion resembles the frequency of a mass on a spring scaled by *h/L* since the spring is connected to the rod and not directly to the mass. So we can think of the solution as:

$$f^2 = \frac{1}{4\pi^2}\frac{MgL + kh^2}{ML^2} = \frac{1}{4\pi^2}\frac{g}{L} + \frac{h^2}{L^2}\frac{1}{4\pi^2}\frac{k}{M} = f^2_{\text{pendulum}} + \frac{h^2}{L^2} f^2_{\text{spring}}$$

***13.60** (a) At equilibrium, we have



$$\sum\tau = 0 = -mg\frac{L}{2} + kx_0L$$

where x_0 is the equilibrium compression.

After displacement by a small angle,

$$\sum\tau = -mg\frac{L}{2} + kxL$$

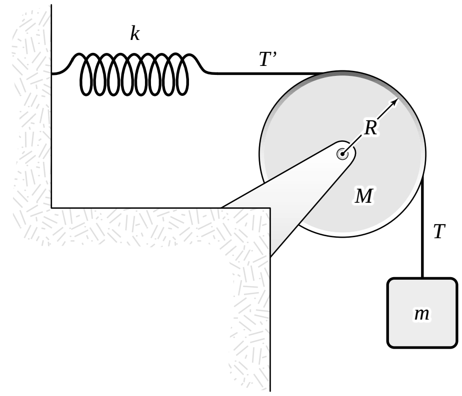
$$= -mg\frac{L}{2} + k(x_0 - L\theta)L$$

$$= -k\theta L^2 = I\alpha = \frac{1}{3}mL^2\frac{d^2\theta}{dt^2}$$

So $\frac{d^2\theta}{dt^2} = -\frac{3k}{m}\theta$

The angular acceleration is opposite in direction and proportional to the displacement, so we have simple harmonic motion with $\boxed{\omega^2 = \frac{3k}{m}}$.

(b) $f = \frac{\omega}{2\pi} = \frac{1}{2\pi}\sqrt{\frac{3k}{m}} = \frac{1}{2\pi}\sqrt{\frac{3(100\ \text{N/m})}{5.00\ \text{kg}}} = \boxed{1.23\ \text{Hz}}$



***13.61** One can write the following equations of motion:

$$mg - T = ma = m\frac{d^2x}{dt^2} \quad \text{(for the mass)}$$

$$T' - kx = 0 \quad \text{(describes the spring)}$$

$$R(T - T') = I\frac{d^2\theta}{dt^2} = \frac{I}{R}\frac{d^2x}{dt^2} \quad \text{(for the pulley)}$$

with $I = \frac{1}{2}MR^2$.

Combining these equations gives the equation of motion

$$\left(m + \frac{1}{2}M\right)\frac{d^2x}{dt^2} + kx = mg$$

The solution is $x(t) = A\sin\omega t +$
Error! , with frequency

$$f = \frac{\omega}{2\pi} = \frac{1}{2\pi}\sqrt{\frac{k}{m + \frac{1}{2}M}} = \frac{1}{2\pi}\sqrt{\frac{100\text{ N/m}}{0.200\text{ kg} + \frac{1}{2}M}}$$

(a) For $M = 0$, $f = \boxed{3.56\text{ Hz}}$

(b) For $M = 0.250$ kg, $f = \boxed{2.79\text{ Hz}}$

(c) For $M = 0.750$ kg, $f = \boxed{2.10\text{ Hz}}$

13.62 (a) $\omega_0 = \sqrt{\frac{k}{m}} = \boxed{15.8\text{ rad/s}}$

(b) $F_s - mg = ma = m\left(\frac{1}{3}g\right)$

$$F_s = \frac{4}{3}mg = 26.1\text{ N}$$

$$x_s = \frac{F_s}{k} = \boxed{5.23\text{ cm}}$$

(c) When the acceleration of the car is zero, the new equilibrium position can be found as follows:

$$F'_s = mg = 19.6\text{ N} = kx'_s \qquad x'_s = 3.92\text{ cm}$$

Thus, $A = \left|x'_s - x_s\right| = \boxed{1.31\text{ cm}}$

The phase constant is $\boxed{\pi\text{ rad}}$

13.63 (a) $T = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{L}{g}} = \boxed{3.00\text{ s}}$

(b) $E = \frac{1}{2}mv^2 = \frac{1}{2}(6.74)(2.06)^2 = \boxed{14.3\text{ J}}$

(c) At maximum angular displacement,

$$mgh = \frac{1}{2}mv^2$$

$$h = \frac{v^2}{2g} = 0.217\text{ m}$$

$$h = L - L\cos\theta = L(1 - \cos\theta)$$

$$\cos\theta = 1 - \frac{h}{L}$$

$$\boxed{\theta = 25.5°}$$

***13.64** Suppose a 100-kg biker compresses the suspension 2.00 cm. Then,

$$k = \frac{F}{x} = \frac{980\text{ N}}{2.00\times10^{-2}\text{ m}} = 4.90\times10^{4}\text{ N/m}$$

If total mass of motorcycle and biker is 500 kg, the frequency of free vibration is

$$f = \frac{1}{2\pi}\sqrt{\frac{k}{m}} = \frac{1}{2\pi}\sqrt{\frac{4.90\times10^{4}\text{ N/m}}{500\text{ kg}}} = 1.58\text{ Hz}$$

If he encounters washboard bumps at the same frequency, resonance will make the motorcycle bounce a lot. Assuming a speed of 20.0 m/s, these ridges are separated by

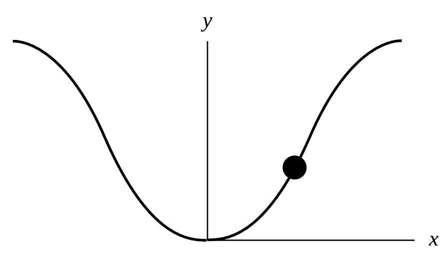
$$\frac{(20.0\text{ m/s})}{1.58/\text{s}} = 12.7\text{ m}\ \boxed{\sim 10^{1}\text{ m}}$$

In addition to this vibration mode of bouncing up and down as one unit, the motorcycle can also vibrate at higher frequencies by rocking back and forth between front and rear wheels, by having just the front wheel bounce inside its fork, or by doing other things. Other spacings of bumps will excite all of these other resonances.

***13.65** $y = (20.0\text{ cm})[1 - \cos(0.160\text{ m}^{-1}x)]$

$$\approx (20.0\text{ cm})\left[1 - 1 + \frac{1}{2}(0.160\text{ m}^{-1}x)^2\right]$$

or $y \approx (10.0\text{ cm})(0.160\text{ m}^{-1}x)^2$



The geometric slope of the wire is

$$\frac{dy}{dx} = (0.100\text{ m})(0.160\text{ m}^{-1})^2(2x) = (5.12\times10^{-3}\text{ m}^{-1})x$$

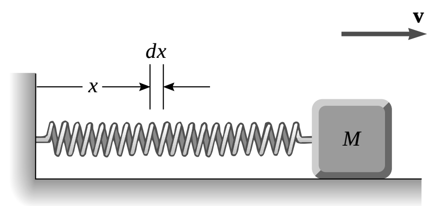
If m is the mass of the bead, the component of the bead's weight that acts as a restoring force is

$$\Sigma F = -mg\frac{dy}{dx} = -m(9.80\text{ m/s}^2)(5.12\times10^{-3}\text{ m}^{-1})x = ma$$

Thus, $a = -(0.0502\text{ s}^{-2})x = -\omega^2 x$. Since the acceleration of the bead is opposite the displacement from equilibrium and is proportional to the displacement, the motion is simple harmonic with $\omega^2 = 0.0502\text{ s}^{-2}$, or $\omega = \boxed{0.224\text{ rad/s}}$.

***13.66** (a) For each segment of the spring

$$dK = \frac{1}{2}(dm)\,v_x^2$$



Also, $v_x = \frac{x}{\ell}v$

and $dm = \frac{m}{\ell}dx$

Therefore, the total kinetic energy is

$$K = \frac{1}{2}Mv^2 + \frac{1}{2}\int_0^\ell \left(\frac{x^2v^2}{\ell^2}\right)\frac{m}{\ell}dx = \boxed{\frac{1}{2}\left(M + \frac{m}{3}\right)v^2}$$

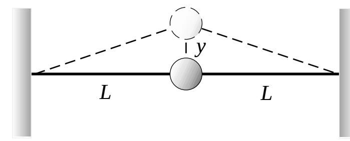
(b) $\omega = \sqrt{\frac{k}{m_{\text{eff}}}}$

and $\frac{1}{2}m_{\text{eff}}v^2 = \frac{1}{2}\left(M + \frac{m}{3}\right)v^2$

Therefore, $T = \frac{2\pi}{\omega} = \boxed{2\pi\sqrt{\frac{M + \frac{m}{3}}{k}}}$

13.67 (a) $\sum \mathbf{F} = -2T \sin \theta \, \mathbf{j}$

where $\theta = \tan^{-1}\left(\frac{y}{L}\right)$



Therefore, for a small displacement

$$\sin \theta \approx \tan \theta = \frac{y}{L}$$

and $\boxed{\sum \mathbf{F} = \frac{-2Ty}{L} \mathbf{j}}$

(b) For a spring system, $\sum \mathbf{F} = -k\mathbf{x}$ becomes $\sum \mathbf{F} = -\frac{2T}{L} \mathbf{y}$

Therefore, $\boxed{\omega = \sqrt{\frac{k}{m}} = \sqrt{\frac{2T}{mL}}}$

13.68 (a) Assuming a Hooke's Law type spring,

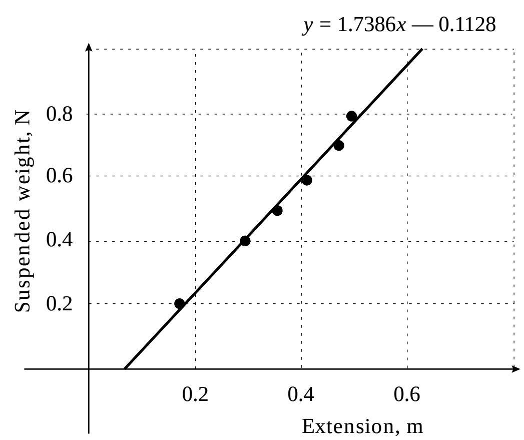
$$F = Mg = kx$$

and empirically $Mg = 1.74x - 0.113$

so $\boxed{k \approx 1.74 \text{ N/m} \pm 6\%}$

M, kg	x, m	Mg, N
0.0200	0.17	0.196
0.0400	0.293	0.392
0.0500	0.353	0.49
0.0600	0.413	0.588
0.0700	0.471	0.686
0.0800	0.493	0.784

Static stretching of a spring



(b) We may write the equation as theoretically

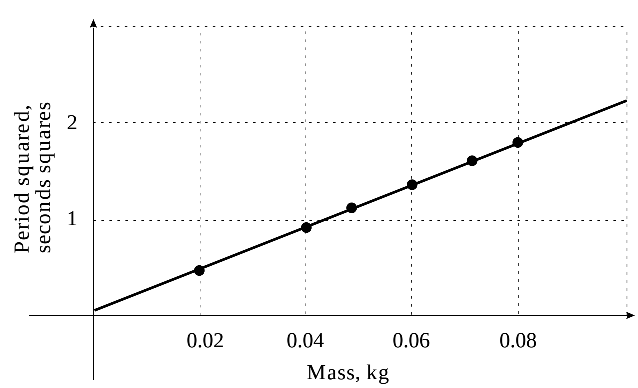
$$T^2 = \frac{4\pi^2}{k} M + \frac{4\pi^2}{3k} m_s$$

and empirically $T^2 = 21.7\,M + 0.0589$

so $k = \frac{4\pi^2}{21.7} \boxed{\approx 1.82 \text{ N/m} \pm 3\%}$

Time, s	T, s	M, kg	T^2, s^2
7.03	0.703	0.0200	0.494
9.62	0.962	0.0400	0.925
10.67	1.067	0.0500	1.138
11.67	1.167	0.0600	1.362
12.52	1.252	0.0700	1.568
13.41	1.341	0.0800	1.798

Squared period as a function of the mass of an object bouncing on a spring



The k values 1.74 N/m ± 6% and 1.82 N/m ± 3% differ by 4%, so they agree.

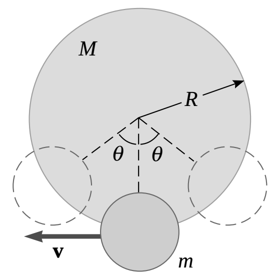
(c) Utilizing the axis-crossing point,

$$m_s = 3\left(\frac{0.0589}{21.7}\right) \text{ kg} \approx \boxed{8 \text{ grams} \pm 12\%}$$

in agreement with 7.4 grams.

13.69 (a) $\Delta K + \Delta U = 0$

Thus, $K_{\text{top}} + U_{\text{top}} = K_{\text{bot}} + U_{\text{bot}}$



where $K_{\text{top}} = U_{\text{bot}} = 0$

Therefore, $mgh = \frac{1}{2} I\omega^2$, but

$$h = R - R\cos\theta = R(1 - \cos\theta)$$

$$\omega = \frac{v}{R}$$

and $I = \frac{MR^2}{2} + \frac{mr^2}{2} + mR^2$

Substituting we find

$$mgR(1 - \cos\theta) = \frac{1}{2}\left(\frac{MR^2}{2} + \frac{mr^2}{2} + mR^2\right)\frac{v^2}{R^2}$$

$$mgR(1 - \cos\theta) = \left[\frac{M}{4} + \frac{mr^2}{4R^2} + \frac{m}{2}\right] v^2$$

and $v^2 = 4gR \dfrac{(1 - \cos\theta)}{\left(\frac{M}{m} + \frac{r^2}{R^2} + 2\right)}$

so $\boxed{v = 2\sqrt{\dfrac{Rg(1 - \cos\theta)}{\frac{M}{m} + \frac{r^2}{R^2} + 2}}}$

(b) $T = 2\pi\sqrt{\dfrac{I}{m_T g d_{\text{CM}}}}$

$m_T = m + M$ $\qquad d_{\text{CM}} = \dfrac{mR + M(0)}{m + M}$

$$T = \boxed{2\pi\sqrt{\frac{\frac{1}{2}MR^2 + \frac{1}{2}mr^2 + mR^2}{mgR}}}$$

13.70 (a) We require

$$Ae^{-bt/2m} = \frac{A}{2}$$

$$e^{+bt/2m} = 2$$

or $\frac{bt}{2m} = \ln 2$ or $\frac{(0.100 \text{ kg/s})}{2(0.375 \text{ kg})} t = 0.693$

$\therefore t = \boxed{5.20 \text{ s}}$ The spring constant is irrelevant.

(b) We can evaluate the energy at successive turning points, where

$\cos(\omega t + \phi) = \pm 1$ and the energy is $\frac{1}{2} kx^2 = \frac{1}{2} kA^2 e^{-bt/m}$

We require $\frac{1}{2} kA^2 e^{-bt/m} = \frac{1}{2}\left(\frac{1}{2} kA^2\right)$

or $e^{+bt/m} = 2$

$$\therefore t = \frac{m \ln 2}{b} = \frac{0.375 \text{ kg}(0.693)}{0.100 \text{ kg/s}} = \boxed{2.60 \text{ s}}$$

(c) From $E = \frac{1}{2} kA^2$,

the fractional rate of change of energy over time is

$$\frac{\frac{dE}{dt}}{E} = \frac{\frac{d}{dt}\frac{1}{2}kA^2}{\frac{1}{2}kA^2} = \frac{\frac{1}{2}k2A\frac{dA}{dt}}{\frac{1}{2}kA^2} = 2\frac{\frac{dA}{dt}}{A}$$

two times faster than the fractional rate of change in amplitude.

13.71 (a) When the mass is displaced a distance x from equilibrium, spring 1 is stretched a distance x_1 and spring 2 is stretched a distance x_2. By Newton's third law, we expect $k_1x_1 = k_2x_2$. When this is combined with the requirement that $x = x_1 + x_2$, we find

$$x_1 = \left[\frac{k_2}{(k_1 + k_2)}\right] x$$

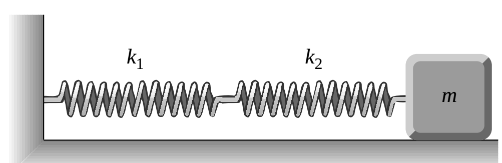
The force on either spring is given by

$$F_1 = \left[\frac{k_1k_2}{(k_1 + k_2)}\right] x = ma$$

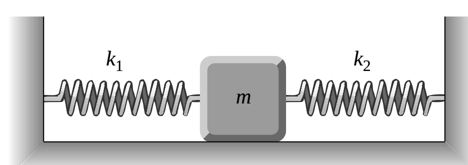
where a is the acceleration of the mass m.

This is in the form $F = k_{\text{eff}}x = ma$

and $T = 2\pi\sqrt{\frac{m}{k_{\text{eff}}}} = \boxed{2\pi\sqrt{\frac{m(k_1 + k_2)}{k_1k_2}}}$



(a)



(b)

(b) In this case each spring is stretched by the distance x which the mass is displaced. Therefore, the restoring force is

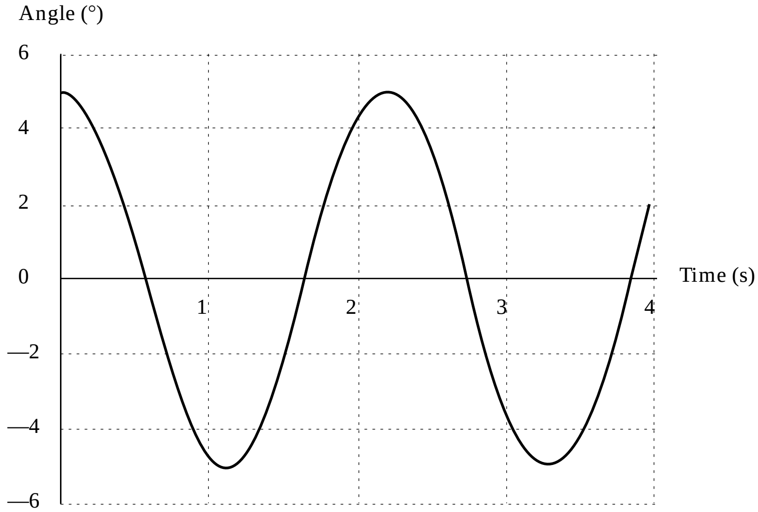
$$F = -(k_1 + k_2)x \quad \text{and} \quad k_{\text{eff}} = (k_1 + k_2)$$

so that $T = \boxed{2\pi\sqrt{\dfrac{m}{(k_1 + k_2)}}}$

13.72 For $\theta_{\max} = 5.00°$, the motion calculated by the Euler method agrees quite precisely with the prediction of $\theta_{\max} \cos \omega t$. The period is $T = 2.20$ s.

Time, t (s)	Angle, θ (°)	Ang. speed (°/s)	Ang. Accel. (°/s^2)	$\theta_{\max} \cos \omega t$
0.000	5.0000	0.0000	–40.7815	5.0000
0.004	4.9993	–0.1631	–40.7762	4.9997
0.008	4.9980	–0.3262	–40.7656	4.9987
…				
0.544	0.0560	–14.2823	–0.4576	0.0810
0.548	–0.0011	–14.2842	0.0090	0.0239
0.552	–0.0582	–14.2841	0.4756	–0.0333
…				
1.092	–4.9994	–0.3199	40.7765	–4.9989
1.096	–5.0000	–0.1568	40.7816	–4.9998
1.100	–5.0000	0.0063	40.7814	–5.0000
1.104	–4.9993	0.1694	40.7759	–4.9996
…				
1.644	–0.0638	14.2824	0.4397	–0.0716
1.648	0.0033	14.2842	–0.0270	–0.0145
1.652	0.0604	14.2841	–0.4936	0.0427
…				
2.192	4.9994	0.3137	–40.7768	4.9991
2.196	5.0000	0.1506	–40.7817	4.9999
2.200	5.0000	–0.0126	–40.7813	5.0000
2.204	4.9993	–0.1757	–40.7756	4.9994

Simple Pendulum, small amplitude



For θ_{max} = 100°, the simple harmonic motion approximation $\theta_{max} \cos \omega t$ diverges greatly from the Euler calculation. The period is T = 2.71 s, larger than the small–angle period by 23%.

Time, t (s)	Angle, θ (˚)	Ang. speed (˚/s)	Ang. Accel. (°/s²)	$\theta_{max} \cos \omega t$
0.000	100.0000	0.0000	–460.6066	100.0000
0.004	99.9926	–1.8432	–460.8173	99.9935
0.008	99.9776	–3.6865	–460.8382	99.9739
…				
1.096	–84.7449	–120.1910	465.9488	–99.9954
1.100	–85.2182	–118.3272	466.2869	–99.9998
1.104	–85.6840	–116.4620	466.5886	–99.9911
…				
1.348	–99.9960	–3.0533	460.8125	–75.7979
1.352	–100.0008	–1.2100	460.8057	–75.0474
1.356	–99.9983	0.6332	460.8093	–74.2870
…				
2.196	40.1509	224.8677	–301.7132	99.9971
2.200	41.0455	223.6609	–307.2607	99.9993
2204	41.9353	222.4318	–312.7035	99.9885
…				
2.704	99.9985	2.4200	–460.8090	12.6422
2.708	100.0008	0.5768	–460.8057	11.5075
2.712	99.9957	–1.2664	–460.8129	10.3712