Chapter 10 Solutions

10.1 (a) $\alpha = \dfrac{\omega - \omega_i}{t} = \dfrac{12.0\text{ rad/s}}{3.00\text{ s}} = \boxed{4.00\text{ rad/s}^2}$

(b) $\theta = \omega_i t + \dfrac{1}{2}\alpha t^2 = \dfrac{1}{2}(4.00\text{ rad/s}^2)(3.00\text{ s})^2 = \boxed{18.0\text{ rad}}$

10.2 (a) $\omega = \dfrac{2\pi\text{ rad}}{365\text{ days}}\dfrac{1\text{ day}}{24\text{ h}}\dfrac{1\text{ h}}{3600\text{ s}} = \boxed{1.99 \times 10^{-7}\text{ rad/s}}$

(b) $\omega = \dfrac{2\pi\text{ rad}}{27.3\text{ days}}\dfrac{1\text{ day}}{24\text{ h}}\dfrac{1\text{ h}}{3600\text{ s}} = \boxed{2.65 \times 10^{-6}\text{ rad/s}}$

***10.3** $\omega_i = 2000\text{ rad/s}$

$\alpha = -80.0\text{ rad/s}^2$

(a) $\omega = \omega_i + \alpha t = [2000 - (80.0)(10.0)] = \boxed{1200\text{ rad/s}}$

(b) $0 = \omega_i + \alpha t$

$t = \dfrac{\omega_i}{-\alpha} = \dfrac{2000}{80.0} = \boxed{25.0\text{ s}}$

10.4 (a) Let ω_h and ω_m be the angular speeds of the hour hand and minute hand, so that

$$\omega_h = \frac{2\pi\text{ rad}}{12\text{ h}} = \frac{\pi}{6}\text{ rad/h} \quad \text{and} \quad \omega_m = 2\pi\text{ rad/h}$$

Then if θ_h and θ_m are the angular positions of the hour hand and minute hand with respect to the 12 o'clock position, we have

$$\theta_h = \omega_h t \quad \text{and} \quad \theta_m = \omega_m t$$

For the two hands to coincide, we need $\theta_m = \theta_h + 2\pi n$, where n is a positive integer. Therefore, we may write $\omega_m t - \omega_h t = 2\pi n$, or

$$t_n = \frac{2\pi n}{\omega_m - \omega_h} = \frac{2\pi n}{2\pi - \frac{\pi}{6}} = \boxed{\frac{12n}{11}\text{ h}}$$

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Construct the following table:

n	t_n (h)	time (h:min:s)
0	0.00	12:00:00
1	1.09	1:05:27
2	2.18	2:10:55
3	3.27	3:16:22
4	4.36	4:21:49
5	5.45	5:27:16
6	6.55	6:32:44
7	7.64	7:38:11
8	8.73	8:43:38
9	9.82	9:49:05
10	10.91	10:54:33

(b) Let θ_s and ω_s be the angular position and angular speed of the second hand, then

$$\omega_s = 2\pi \text{ rad/min} = 120\pi \text{ rad/h} \quad \text{and} \quad \theta_s = \omega_s t$$

For all three hands to coincide, we need $\theta_s = \theta_m + 2\pi k$ (k is any positive integer) at any of the times given above. That is, we need

$$\omega_s t_n - \omega_m t_n = 2\pi k, \text{ or}$$

$$k = \frac{\omega_s - \omega_m}{2\pi} t_n = \frac{118}{2\pi}\frac{12}{11} n = \frac{(3)(4)(59)n}{11}$$

to be an integer. This is possible only for $n = 0$ or 11. Therefore, all three hands coincide $\boxed{\text{only when straight up at 12 o'clock}}$.

10.5 $\omega_i = \left(\frac{100 \text{ rev}}{1.00 \text{ min}}\right)\left(\frac{1.00 \text{ min}}{60.0 \text{ s}}\right)\left(\frac{2\pi \text{ rad}}{1.00 \text{ rev}}\right) = \frac{10\pi}{3} \text{ rad/s}, \ \omega_f = 0$

(a) $t = \frac{\omega_f - \omega_i}{\alpha} = \frac{0 - 10\pi/3}{-2.00} \text{ s} = \boxed{5.24 \text{ s}}$

(b) $\theta = \bar{\omega} t = \left(\frac{\omega_f + \omega_i}{2}\right) t = \left(\frac{10\pi}{6} \text{ rad/s}\right)\left(\frac{10\pi}{6} \text{ s}\right) = \boxed{27.4 \text{ rad}}$

***10.6** $\omega_i = 3600 \text{ rev/min} = 3.77 \times 10^2 \text{ rad/s}$

$\theta = 50.0 \text{ rev} = 3.14 \times 10^2 \text{ rad}$ and $\omega_f = 0$

$\omega_f^2 = \omega_i^2 + 2\alpha\theta$

$0 = (3.77 \times 10^2 \text{ rad/s})^2 + 2\alpha(3.14 \times 10^2 \text{ rad})$

$\alpha = \boxed{-2.26 \times 10^2 \text{ rad/s}^2}$

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10.7 (a) $\theta\big|_{t=0} = \boxed{5.00\text{ rad}}$

$$\omega\big|_{t=0} = \left.\frac{d\theta}{dt}\right|_{t=0} = 10.0 + 4.00t\big|_{t=0} = \boxed{10.0\text{ rad/s}}$$

$$\alpha_{t=0} = \left.\frac{d\omega}{dt}\right|_{t=0} = \boxed{4.00\text{ rad/s}^2}$$

(b) $\theta\big|_{t=3.00\text{ s}} = 5.00 + 30.0 + 18.0 = \boxed{53.0\text{ rad}}$

$$\omega\big|_{t=3.00\text{ s}} = \left.\frac{d\theta}{dt}\right|_{t=3.00\text{s}} = 10.0 + 4.00t\big|_{t=3.00\text{ s}} = \boxed{22.0\text{ rad/s}}$$

$$\alpha\big|_{t=3.00\text{ s}} = \left.\frac{d\omega}{dt}\right|_{t=3.00\text{s}} = \boxed{4.00\text{ rad/s}^2}$$

***10.8** $\omega = 5.00\text{ rev/s} = 10.0\pi\text{ rad/s}$

We will break the motion into two stages: (1) an acceleration period and (2) a deceleration period.

While speeding up,

$$\theta_1 = \bar{\omega}t = \frac{0 + 10.0\pi\text{ rad/s}}{2}(8.00\text{ s}) = 40.0\pi\text{ rad}$$

While slowing down,

$$\theta_2 = \bar{\omega}t = \frac{10.0\pi\text{ rad/s} + 0}{2}(12.0\text{ s}) = 60.0\pi\text{ rad}$$

So, $\theta_{\text{total}} = \theta_1 + \theta_2 = 100\pi\text{ rad} = \boxed{50.0\text{ rev}}$

10.9 $\theta - \theta_i = \omega_i t + \frac{1}{2}\alpha t^2$ and $\omega = \omega_i + \alpha t$

are two equations in two unknowns ω_i and α.

$$\omega_i = \omega - \alpha t$$

$$\theta - \theta_i = (\omega - \alpha t)t + \frac{1}{2}\alpha t^2 = \omega t - \frac{1}{2}\alpha t^2$$

$$37.0\text{ rev}\left(\frac{2\pi\text{ rad}}{1\text{ rev}}\right) = (98.0\text{ rad/s})(3.00\text{ s}) - \frac{1}{2}\alpha(3.00\text{ s})^2$$

$$232\text{ rad} = 294\text{ rad} - (4.50\text{ s}^2)\alpha$$

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$$\alpha = \frac{61.5 \text{ rad}}{4.50 \text{ s}^2} = \boxed{13.7 \text{ rad/s}^2}$$

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***10.10** (a) $\omega = \dfrac{\Delta\theta}{\Delta t} = \dfrac{1\text{ rev}}{1\text{ day}} = \dfrac{2\pi\text{ rad}}{86400\text{ s}} = \boxed{7.27 \times 10^{-5}\text{ rad/s}}$

(b) $\Delta t = \dfrac{\Delta\theta}{\omega} = \dfrac{107°}{7.27 \times 10^{-5}\text{ rad/s}}\left(\dfrac{2\pi\text{ rad}}{360°}\right) = \boxed{2.57 \times 10^{4}\text{ s}}$ (428 min)

***10.11** Estimate the tire's radius at 0.250 m and miles driven as 10,000 per year.

$$\theta = \frac{s}{r} = \frac{1.00 \times 10^{4}\text{ mi}}{0.250\text{ m}}\left(\frac{1609\text{ m}}{1\text{ mi}}\right) = 6.44 \times 10^{7}\frac{\text{rad}}{\text{yr}}$$

$$\theta = 6.44 \times 10^{7}\frac{\text{rad}}{\text{yr}}\left(\frac{1\text{ rev}}{2\pi\text{ rad}}\right) = 1.02 \times 10^{7}\frac{\text{rev}}{\text{yr}}$$

or $\boxed{\sim 10^{7}\dfrac{\text{rev}}{\text{yr}}}$

***10.12** Main Rotor:

$$v = r\omega = (3.80\text{ m})\left(450\frac{\text{rev}}{\text{min}}\right)\left(\frac{2\pi\text{ rad}}{1\text{ rev}}\right)\left(\frac{1\text{ min}}{60\text{ s}}\right) = \boxed{179\text{ m/s}}$$

$$v = \left(179\frac{\text{m}}{\text{s}}\right)\left(\frac{v_{\text{sound}}}{343\text{ m/s}}\right) = \boxed{0.522 v_{\text{sound}}}$$

Tail Rotor:

$$v = r\omega = (0.510\text{ m})\left(4138\frac{\text{rev}}{\text{min}}\right)\left(\frac{2\pi\text{ rad}}{1\text{ rev}}\right)\left(\frac{1\text{ min}}{60\text{ s}}\right) = \boxed{221\text{ m/s}}$$

$$v = \left(221\frac{\text{m}}{\text{s}}\right)\left(\frac{v_{\text{sound}}}{343\text{ m/s}}\right) = \boxed{0.644 v_{\text{sound}}}$$

10.13 (a) $v = r\omega;\quad \omega = \dfrac{v}{r} = \dfrac{45.0\text{ m/s}}{250\text{ m}} = \boxed{0.180\text{ rad/s}}$

(b) $a_r = \dfrac{v^2}{r} = \dfrac{(45.0\text{ m/s})^2}{250\text{ m}} = \boxed{8.10\text{ m/s}^2\text{ toward the center of track}}$

10.14 $v = 36.0\dfrac{\text{km}}{\text{h}}\left(\dfrac{1\text{ h}}{3600\text{ s}}\right)\left(\dfrac{10^{3}\text{ m}}{1\text{ km}}\right) = 10.0\text{ m/s}$

$$\omega = \frac{v}{r} = \frac{10.0\text{ m/s}}{0.250\text{ m}} = \boxed{40.0\text{ rad/s}}$$

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10.15 Given $r = 1.00$ m, $\alpha = 4.00\ \mathrm{rad/s^2}$, $\omega_i = 0$, and $\theta_i = 57.3° = 1.00$ rad

(a) $\omega = \omega_i + \alpha t = 0 + \alpha t$

At $t = 2.00$ s, $\omega = (4.00\ \mathrm{rad/s^2})(2.00\ \mathrm{s}) = \boxed{8.00\ \mathrm{rad/s}}$

(b) $v = r\omega = (1.00\ \mathrm{m})(8.00\ \mathrm{rad/s}) = \boxed{8.00\ \mathrm{m/s}}$

$a_r = r\omega^2 = (1.00\ \mathrm{m})(8.00\ \mathrm{rad/s})^2 = \boxed{64.0\ \mathrm{m/s^2}}$

$a_t = r\alpha = (1.00\ \mathrm{m})(4.00\ \mathrm{rad/s^2}) = \boxed{4.00\ \mathrm{m/s^2}}$

The magnitude of the total acceleration is:

$$a = \sqrt{a_r^2 + a_t^2} = \sqrt{(64.0\ \mathrm{m/s^2})^2 + (4.00\ \mathrm{m/s^2})^2} = 64.1\ \mathrm{m/s^2}$$

The direction of the total acceleration vector makes an angle ϕ with respect to the radius to point P:

$$\phi = \tan^{-1}\left(\frac{a_t}{a_r}\right) = \tan^{-1}\left(\frac{4.00}{64.0}\right) = 3.58°$$

(c) $\theta = \theta_i + \omega_i t + \frac{1}{2}\alpha t^2 = (1.00\ \mathrm{rad}) + \frac{1}{2}(4.00\ \mathrm{rad/s^2})(2.00\ \mathrm{s})^2 = \boxed{9.00\ \mathrm{rad}}$

10.16 (a) $\omega = \frac{v}{r} = \frac{25.0\ \mathrm{m/s}}{1.00\ \mathrm{m}} = \boxed{25.0\ \mathrm{rad/s}}$

(b) $\omega_f^2 = \omega_i^2 + 2\alpha(\Delta\theta)$

$$\alpha = \frac{\omega_f^2 - \omega_i^2}{2(\Delta\theta)} = \frac{(25.0\ \mathrm{rad/s})^2 - 0}{2[(1.25\ \mathrm{rev})(2\pi\ \mathrm{rad/rev})]} = \boxed{39.8\ \mathrm{rad/s^2}}$$

(c) $\Delta t = \frac{\Delta\omega}{\alpha} = \frac{25.0\ \mathrm{rad/s}}{39.8\ \mathrm{rad/s^2}} = \boxed{0.628\ \mathrm{s}}$

10.17 (a) $s = \bar{v}t = (11.0\ \mathrm{m/s})(9.00\ \mathrm{s}) = 99.0$ m

$$\theta = \frac{s}{r} = \frac{99.0\ \mathrm{m}}{0.290\ \mathrm{m}} = 341\ \mathrm{rad} = \boxed{54.3\ \mathrm{rev}}$$

(b) $\omega = \frac{v}{r} = \frac{22.0\ \mathrm{m/s}}{0.290\ \mathrm{m}} = 75.9\ \mathrm{rad/s} = \boxed{12.1\ \mathrm{rev/s}}$

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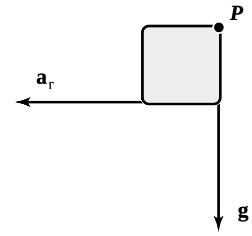
10.18 $K_A + U_A = K_p + U_p$



$$6.00 \times 9.80 \times 5.00 = \frac{1}{2}(6.00)\, v_p^2 + 6.00 \times 9.80 \times 2.00$$

$$v_p^2 = 58.8\ \text{m}^2/\text{s}^2$$

Radial acceleration at P,

$$a_r = \frac{v_p^2}{R} = \boxed{29.4\ \text{m/s}^2}$$

Tangential acceleration at P,

$$a_t = g = \boxed{9.80\ \text{m/s}^2}$$

10.19 (a) $\omega = 2\pi f = \frac{2\pi\ \text{rad}}{\text{rev}}\frac{1200}{60.0}\ \text{rev/s} = \boxed{126\ \text{rad/s}}$

(b) $v = \omega r = (126\ \text{rad/s})(3.00 \times 10^{-2}\ \text{m}) = \boxed{3.77\ \text{m/s}}$

(c) $a_r = \omega^2 r = (126)^2(8.00 \times 10^{-2}) = \boxed{1260\ \text{m/s}^2} = \boxed{1.26\ \text{km/s}^2}$

(d) $s = \theta r = \omega\, t\, r = (126\ \text{rad/s})(2.00\ \text{s})(8.00 \times 10^{-2}\ \text{m}) = \boxed{20.1\ \text{m}}$

10.20 Just before it starts to skid,

$$\sum F_r = m a_r$$

$$f = \frac{mv^2}{r} = \mu_s n = \mu_s mg$$

$$\mu_s = \frac{v^2}{rg} = \frac{\omega^2 r}{g} = \frac{(\omega^2 - \omega_i^2)r}{g} = \frac{2\alpha\theta r}{g} = \frac{2a_t\theta}{g}$$

$$\mu_s = \frac{2(1.70\ \text{m/s}^2)(\pi/2)}{9.80\ \text{m/s}^2} = \boxed{0.545}$$

***10.21** (a) $x = r\cos\theta = (3.00\ \text{m})\cos(9.00\ \text{rad}) = (3.00\ \text{m})\cos 516° = -2.73\ \text{m}$

$y = r\sin\theta = (3.00\ \text{m})\sin(9.00\ \text{rad}) = (3.00\ \text{m})\sin 516° = 1.24\ \text{m}$

$\mathbf{r} = x\mathbf{i} + y\mathbf{j} = \boxed{(-2.73\mathbf{i} + 1.24\mathbf{j})\ \text{m}}$

(b) $516° - 360° = 156°$. This is between 90.0° and 180°, so the object is in the $\boxed{\text{second quadrant}}$.

The vector **r** makes an angle of $\boxed{156°}$ with the positive x-axis or 24.3° with the negative x-axis.

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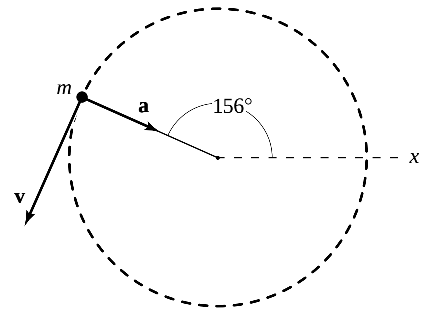
(d) The direction of motion (i.e., the direction of the velocity vector is at $156° + 90.0° = \boxed{246°}$ from the positive x axis. The direction of the acceleration vector is at $156° + 180° = 336°$ from the positive x axis.



(c) $\mathbf{v} = [(4.50\text{ m/s})\cos 246°]\mathbf{i} + [(4.50\text{ m/s})\sin 246°]\mathbf{j}$

$$= \boxed{(-1.85\mathbf{i} - 4.10\mathbf{j})\text{ m/s}}$$

(e) $a = \dfrac{v^2}{r} = \dfrac{(4.50\text{ m/s})^2}{3.00\text{ m}} = 6.75\text{ m/s}^2$ directed toward the center or at 336°

$$\mathbf{a} = (6.75\text{ m/s}^2)(\mathbf{i}\cos 336° + \mathbf{j}\sin 336°) = \boxed{(6.15\mathbf{i} - 2.78\mathbf{j})\text{ m/s}^2}$$

(f) $\sum \mathbf{F} = m\mathbf{a} = (4.00\text{ kg})[(6.15\mathbf{i} - 2.78\mathbf{j})\text{ m/s}^2] = \boxed{(24.6\mathbf{i} - 11.1\mathbf{j})\text{ N}}$

***10.22** When completely rewound, the tape is a hollow cylinder with a difference between the inner and outer radii of ~1 cm. Let N represent the number of revolutions through which the driving spindle turns in 30 minutes (and hence the number of layers of tape on the spool). We can determine N from:

$$N = \frac{\Delta\theta}{2\pi} = \frac{\omega(\Delta t)}{2\pi} = \frac{(1\text{ rad/s})(30\text{ min})(60\text{ s/min})}{2\pi\text{ rad/rev}} = 286\text{ rev}$$

Then, *thickness* $\sim \dfrac{1\text{ cm}}{N}$ $\boxed{\sim 10^{-2}\text{ cm}}$

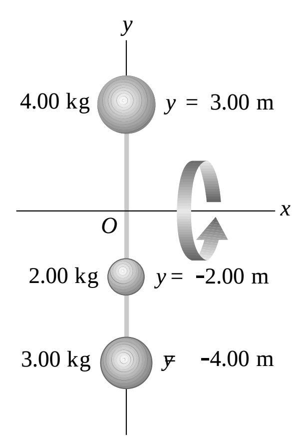
10.23 $m_1 = 4.00$ kg, $r_1 = y_1 = 3.00$ m; $m_2 = 2.00$ kg, $r_2 = |y_2| = 2.00$ m;

$m_3 = 3.00$ kg, $r_3 = |y_3| = 4.00$ m; $\omega = 2.00$ rad/s about the x-axis

(a) $I_x = m_1 r_1^2 + m_2 r_2^2 + m_1 r_2^2 = (4.00)(3.00)^2 + (2.00)(2.00)^2 + (3.00)(4.00)^2$

$$I_x = \boxed{92.0\text{ kg}\cdot\text{m}^2}$$

$$K_R = \frac{1}{2} I_x \omega^2 = \frac{1}{2}(92.0)(2.00)^2 = \boxed{184\text{ J}}$$



(b) $v_1 = r_1\omega = (3.00)(2.00) = \boxed{6.00\text{ m/s}}$

$$v_2 = r_2\omega = (2.00)(2.00) = \boxed{4.00\text{ m/s}}$$

$$v_3 = r_3\omega = (4.00)(2.00) = \boxed{8.00\text{ m/s}}$$

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$$K_1 = \frac{1}{2}m_1 v_1^2 = \frac{1}{2}(4.00)(6.00)^2 = 72.0 \text{ J}$$

$$K_2 = \frac{1}{2}m_2 v_2^2 = \frac{1}{2}(2.00)(4.00)^2 = 16.0 \text{ J}$$

$$K_3 = \frac{1}{2}m_3 v_3^2 = \frac{1}{2}(3.00)(8.00)^2 = 96.0 \text{ J}$$

$$K = K_1 + K_2 + K_3 = 72.0 + 16.0 + 96.0 = \boxed{184 \text{ J}} = \frac{1}{2}I_x\omega^2$$

10.24 $v = 38.0 \text{ m/s}$ $\omega = 125 \text{ rad/s}$

$$\text{RATIO} = \frac{\frac{1}{2}I\omega^2}{\frac{1}{2}mv^2} = \frac{\frac{1}{2}\left(\frac{2}{5}mr^2\right)\omega^2}{\frac{1}{2}mv^2}$$

$$\text{RATIO} = \frac{\frac{2}{5}(3.80\times 10^{-2})^2(125)^2}{(38.0)^2} = \boxed{\frac{1}{160}}$$

10.25 (a) $I = \sum_j m_j r_j^2$

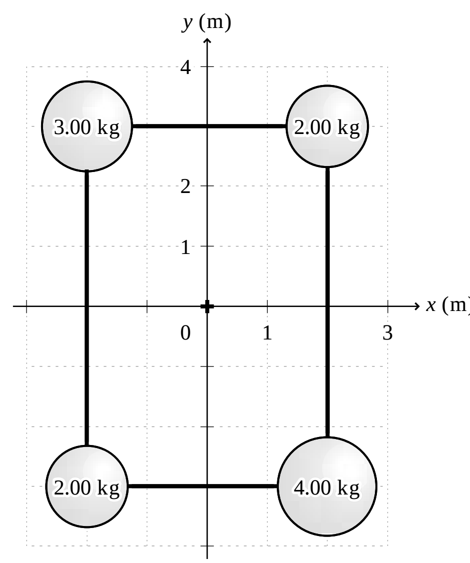
In this case,

$$r_1 = r_2 = r_3 = r_4$$

$$r = \sqrt{(3.00 \text{ m})^2 + (2.00 \text{ m})^2} = \sqrt{13.0} \text{ m}$$

$$I = [\sqrt{13.0} \text{ m}]^2[3.00 + 2.00 + 2.00 + 4.00]\text{kg}$$

$$= \boxed{143 \text{ kg}\cdot\text{m}^2}$$



(b) $K_R = \frac{1}{2}I\omega^2 = \frac{1}{2}(143 \text{ kg}\cdot\text{m}^2)(6.00 \text{ rad/s})^2$

$$= \boxed{2.57\times 10^3 \text{ J}}$$

10.26 The moment of inertia of a thin rod about an axis through one end is $I = \frac{1}{3}ML^2$. The total rotational kinetic energy is given as

$$K_R = \frac{1}{2}I_h\omega_h^2 + \frac{1}{2}I_m\omega_m^2, \text{ with}$$

$$I_h = \frac{m_h L_h^2}{3} = \frac{(60.0 \text{ kg})(2.70 \text{ m})^2}{3} = 146 \text{ kg m}^2, \text{ and}$$

$$I_m = \frac{m_m L_m^2}{3} = \frac{(100 \text{ kg})(4.50 \text{ m})^2}{3} = 675 \text{ kg m}^2$$

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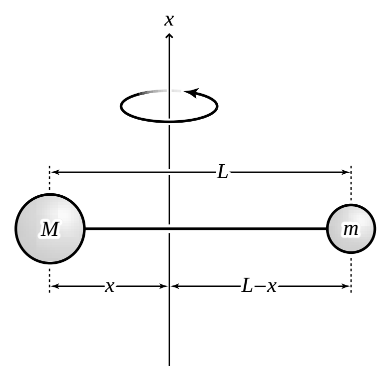
In addition, $\omega_h = \frac{(2\pi \text{ rad})}{(12 \text{ h})}\frac{1 \text{ h}}{3600 \text{ s}} = 1.45 \times 10^{-4} \text{ rad/s}$, while

$$\omega_m = \frac{(2\pi \text{ rad})}{(1 \text{ h})}\frac{1 \text{ h}}{3600 \text{ s}} = 1.75 \times 10^{-3} \text{ rad/s}. \text{ Therefore,}$$

$$K_R = \frac{1}{2}(146)(1.45 \times 10^{-4})^2 + \frac{1}{2}(675)(1.75 \times 10^{-3})^2 = \boxed{1.04 \times 10^{-3} \text{ J}}$$

10.27 $I = Mx^2 + m(L - x)^2$



$$\frac{dI}{dx} = 2Mx - 2m(L - x) = 0 \text{ (for an extremum)}$$

$$\therefore x = \frac{mL}{M + m}$$

$\frac{d^2I}{dx^2} = 2m + 2M$; therefore I is minimum when the axis of rotation passes through $x = \frac{mL}{M + m}$

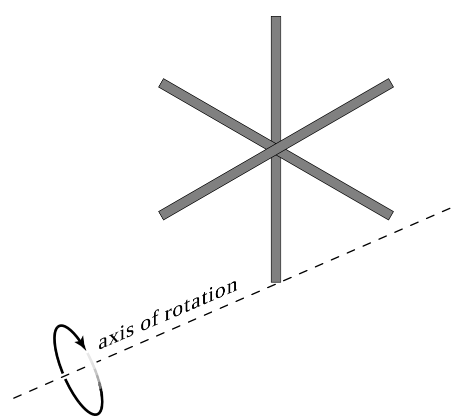
which is also the center of mass of the system. The moment of inertia about an axis passing through x is

$$I_{\text{CM}} = M\left[\frac{mL}{M + m}\right]^2 + m\left[1 - \frac{m}{M + m}\right]^2 L^2$$

$$= \frac{Mm}{M + m} L^2 = \mu L^2$$

where $\mu = \frac{Mm}{M + m}$

10.28 We assume the rods are thin, with radius much less than L. Call the junction of the rods the origin of coordinates, and the axis of rotation the z-axis.



For the rod along the y-axis, $I = \frac{1}{3} mL^2$ from the table.

For the rod parallel to the z-axis, the parallel-axis theorem gives

$$I = \frac{1}{2} mr^2 + m\left(\frac{L}{2}\right)^2 \cong \frac{1}{4} mL^2$$

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In the rod along the x-axis, the bit of material between x and $x + dx$ has mass $(m/L)dx$ and is at distance $r = \sqrt{x^2 + (L/2)^2}$ from the axis of rotation. The total rotational inertia is:

$$I_{total} = \frac{1}{3} mL^2 + \frac{1}{4} mL^2 + \int_{-L/2}^{L/2} (x^2 + L^2/4)(m/L)dx$$

$$= \frac{7}{12} mL^2 + \left(\frac{m}{L}\right) \frac{x^3}{3}\bigg|_{-L/2}^{L/2} + \frac{mL}{4} x\bigg|_{-L/2}^{L/2}$$

$$= \frac{7}{12} mL^2 + \frac{mL^2}{12} + \frac{mL^2}{4} = \boxed{\frac{11\, mL^2}{12}}$$

***10.29** Treat the tire as consisting of three parts. The two sidewalls are each treated as a hollow cylinder of inner radius 16.5 cm, outer radius 30.5 cm, and height 0.635 cm. The tread region is treated as a hollow cylinder of inner radius 30.5 cm, outer radius 33.0 cm, and height 20.0 cm.

Use $I = \frac{1}{2} m(R_1^2 + R_2^2)$ for the moment of inertia of a hollow cylinder.

Sidewall:

$$m = \pi[(0.305 \text{ m})^2 - (0.165 \text{ m})^2](6.35 \times 10^{-3} \text{ m})(1.10 \times 10^3 \text{ kg/m}^3) = 1.44 \text{ kg}$$

$$I_{side} = \frac{1}{2}(1.44 \text{ kg})\ [(0.165 \text{ m})^2 + (0.305 \text{ m})^2] = 8.68 \times 10^{-2} \text{ kg} \cdot \text{m}^2$$

Tread:

$$m = \pi[(0.330 \text{ m})^2 - (0.305 \text{ m})^2](0.200 \text{ m})(1.10 \times 10^3 \text{ kg/m}^3) = 11.0 \text{ kg}$$

$$I_{tread} = \frac{1}{2}(11.0 \text{ kg})\ [(0.330 \text{ m})^2 + (0.305 \text{ m})^2] = 1.11 \text{ kg} \cdot \text{m}^2$$

Entire Tire:

$$I_{total} = 2I_{side} + I_{tread} = 2(8.68 \times 10^{-2} \text{ kg} \cdot \text{m}^2) + 1.11 \text{ kg} \cdot \text{m}^2 = \boxed{1.28 \text{ kg} \cdot \text{m}^2}$$

10.30 (a) $I = I_{CM} + MD^2 = \frac{1}{2} MR^2 + MR^2 = \boxed{\frac{3}{2} MR^2}$

(b) $I = I_{CM} + MD^2 = \frac{2}{5} MR^2 + MR^2 = \boxed{\frac{7}{5} MR^2}$

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***10.31** Model your body as a cylinder of mass 60.0 kg and circumference 75.0 cm. Then its radius is

$$\frac{0.750\text{ m}}{2\pi} = 0.120\text{ m}$$

and its moment of inertia is

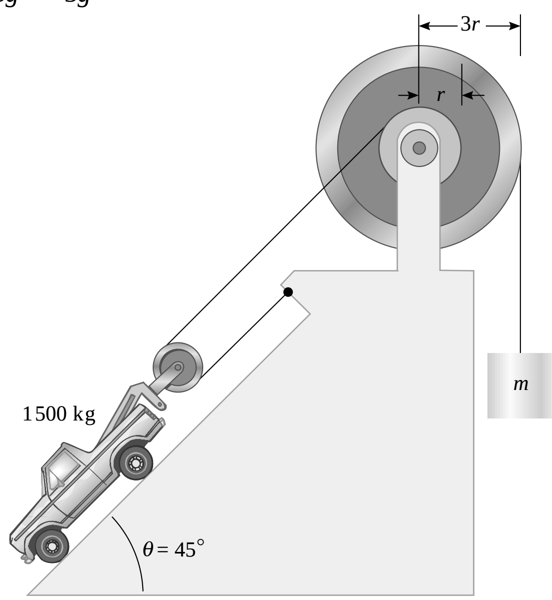
$$\frac{1}{2}MR^2 = \frac{1}{2}(60.0\text{ kg})(0.120\text{ m})^2 = 0.432\text{ kg m}^2 \sim \boxed{10^0\text{ kg}\cdot\text{m}^2 = 1\text{ kg}\cdot\text{m}^2}$$

10.32 $\sum\tau = 0 = mg(3r) - Tr$

$2T - Mg\sin 45.0° = 0$

$$T = \frac{Mg\sin 45.0°}{2} = \frac{1500\text{ kg}(g)\sin 45.0°}{2} = (530)(9.80)\text{ N}$$

$$m = \frac{T}{3g} = \frac{530g}{3g} = \boxed{177\text{ kg}}$$



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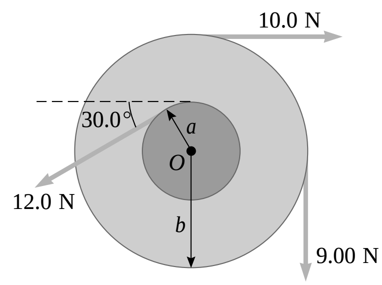
10.33 $\sum \tau$ = (0.100 m)(12.0 N) – (0.250 m)(9.00 N) – (0.250 m)(10.0 N)

= $\boxed{-3.55\ \text{N} \cdot \text{m}}$

The thirty-degree angle is unnecessary information.



Goal Solution

G: By simply examining the magnitudes of the forces and their respective lever arms, it appears that the wheel will rotate clockwise, and the net torque appears to be about 5 Nm.

O: To find the net torque, we simply add the individual torques, remembering to apply the convention that a torque producing clockwise rotation is negative and a counterclockwise torque is positive.

A: $\sum \tau = \sum Fd$

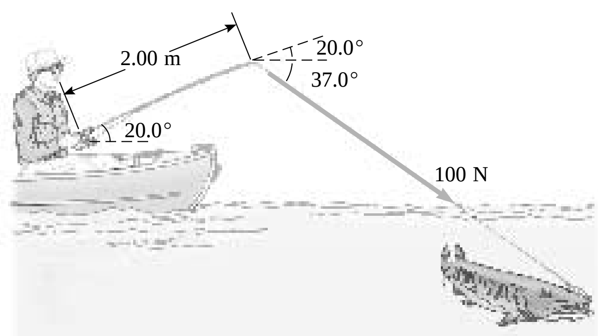
$\sum \tau$ = (12.0 N)(0.100 m) – (10.0 N)(0.250 m) – (9.00 N)(0.250 m)

$\sum \tau$ = –3.55 N · m

The minus sign means perpendicularly into the plane of the paper, or it means clockwise.

L: The resulting torque has a reasonable magnitude and produces clockwise rotation as expected. Note that the 30° angle was not required for the solution since each force acted perpendicular to its lever arm. The 10-N force is to the right, but its torque is negative – that is, clockwise, just like the torque of the downward 9-N force.

10.34 Resolve the 100 N force into components perpendicular to and parallel to the rod, as



F_{par} = (100 N) cos 57.0° = 54.5 N

and

F_{perp} = (100 N) sin 57.0° = 83.9 N

Torque of F_{par} = 0 since its line of action passes through the pivot point.

Torque of F_{perp} is

τ = (83.9 N)(2.00 m) = $\boxed{168\ \text{N} \cdot \text{m}}$ (clockwise)

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***10.35** The normal force exerted by the ground on each wheel is

$$n = \frac{mg}{4} = \frac{(1500\text{ kg})(9.80\text{ m/s}^2)}{4} = 3680\text{ N}$$

The torque of friction can be as large as

$$\tau_{\text{max}} = f_{\text{max}}r = (\mu_s n)r = (0.800)(3680\text{ N})(0.300\text{ m}) = \boxed{882\text{ N}\cdot\text{m}}$$

The torque of the axle on the wheel can be equally as large as the light wheel starts to turn without slipping.

***10.36** We calculated the maximum torque that can be applied without skidding in Problem 35 to be 882 N · m. This same torque is to be applied by the frictional force, f, between the brake pad and the rotor for this wheel. Since the wheel is slipping against the brake pad, we use the coefficient of kinetic friction to calculate the normal force.

$$\tau = fr = (\mu_k n)r,\text{ so } n = \frac{\tau}{\mu_k r} = \frac{882\text{ N}\cdot\text{m}}{(0.500)(0.220\text{ m})} = 8.02\times 10^3\text{ N} = \boxed{8.02\text{ kN}}$$

10.37 $m = 0.750$ kg $\quad F = 0.800$ N

(a) $\tau = rF = (30.0\text{ m})(0.800\text{ N}) = \boxed{24.0\text{ N}\cdot\text{m}}$

(b) $\alpha = \dfrac{\tau}{I} = \dfrac{rF}{mr^2} = \dfrac{24.0}{(0.750)(30.0)^2} = \boxed{0.0356\text{ rad/s}^2}$

(c) $a_T = \alpha r = (0.0356)(30.0) = \boxed{1.07\text{ m/s}^2}$

***10.38** $\tau = 36.0\text{ N}\cdot\text{m} = I\alpha \qquad \omega_f = \omega_i + \alpha t$

$10.0\text{ rad/s} = 0 + \alpha(6.00\text{ s})$

$\alpha = \dfrac{10.00}{6.00}\text{ rad/s}^2 = 1.67\text{ rad/s}^2$

(a) $I = \dfrac{\tau}{\alpha} = \dfrac{36.0\text{ N}\cdot\text{m}}{1.67\text{ rad/s}^2} = \boxed{21.6\text{ kg}\cdot\text{m}^2}$

(b) $\omega_f = \omega_i + \alpha t$

$0 = 10.0 + \alpha(60.0)$

$\alpha = -0.167\text{ rad/s}^2$

$\tau = I\alpha = (21.6\text{ kg}\cdot\text{m})(0.167\text{ rad/s}^2) = \boxed{3.60\text{ N}\cdot\text{m}}$

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(c) Number of revolutions

$$\theta = \theta_i + \omega_i t + \frac{1}{2}\alpha t^2$$

During first 6.00 s

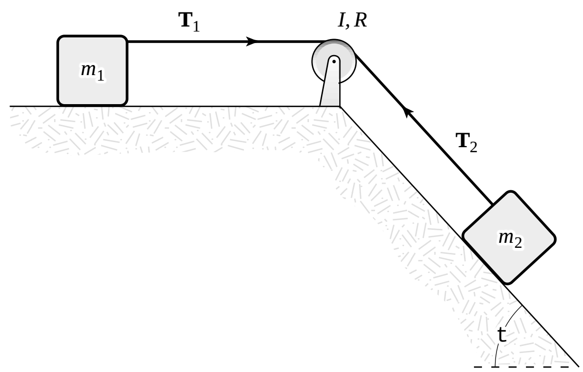
$$\theta = \frac{1}{2}(1.67)(6.00)^2 = 30.1 \text{ rad}$$

During next 60.0 s

$$\theta = 10.0(60.0) - \frac{1}{2}(0.167)(60.0)^2 = 299 \text{ rad}$$

$$\theta_{\text{total}} = (329 \text{ rad})\frac{\text{rev}}{2\pi \text{ rad}} = \boxed{52.4 \text{ rev}}$$

10.39 For m_1: $\sum F_y = ma_y$ $\quad +n - m_1 g = 0$



$n = m_1 g = 19.6$ N

$f_k = \mu_k n = 7.06$ N

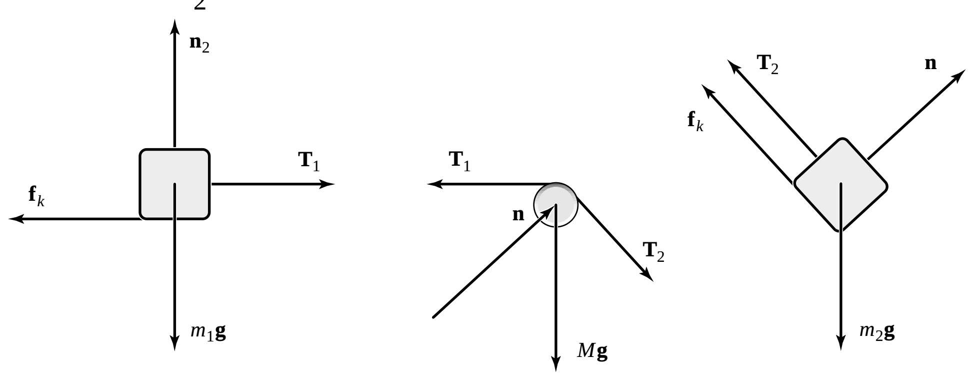
$\sum F_x = ma_x \qquad -7.06 \text{ N} + T_1 = (2.00 \text{ kg})a \quad (1)$

For the pulley

$\sum \tau = I\alpha$

$$-T_1 R + T_2 R = \frac{1}{2} MR^2 \left(\frac{a}{R}\right)$$

$$-T_1 + T_2 = \frac{1}{2}(10.0 \text{ kg})\, a \qquad -T_1 + T_2 = (5.00 \text{ kg})a \quad (2)$$



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For m_2: $+n - m_2 g \cos\theta = 0$

$n = 6.00\text{ kg}(9.80\text{ m/s}^2)\cos 30.0° = 50.9\text{ N}$

$f_k = \mu_k n = 18.3\text{ N}$

$-18.3\text{ N} - T_2 + m_2 g \sin\theta = m_2 a$

$-18.3\text{ N} - T_2 + 29.4\text{ N} = (6.00\text{ kg})a \quad (3)$

(a) Add equations (1) (2) and (3):

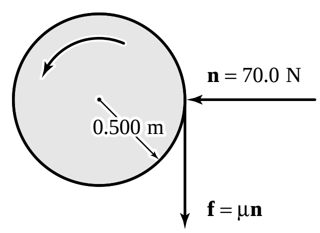
$-7.06\text{ N} - 18.3\text{ N} + 29.4\text{ N} = (13.0\text{ kg})a$

$a = \dfrac{4.01\text{ N}}{13.0\text{ kg}} = \boxed{0.309\text{ m/s}^2}$

(b) $T_1 = 2.00\text{ kg}(0.309\text{ m/s}^2) + 7.06\text{ N} = \boxed{7.67\text{ N}}$

$T_2 = 7.67\text{ N} + 5.00\text{ kg}(0.309\text{ m/s}^2) = \boxed{9.22\text{ N}}$

10.40 $I = \dfrac{1}{2}mR^2 = \dfrac{1}{2}(100\text{ kg})(0.500\text{ m})^2 = 12.5\text{ kg}\cdot\text{m}^2$



$\omega_i = 50.0\text{ rev/min} = 5.24\text{ rad/s}$

$\alpha = \dfrac{\omega_f - \omega_i}{t} = \dfrac{0 - 5.24\text{ rad/s}}{6.00\text{ s}} = -0.873\text{ rad/s}^2$

$\tau = I\alpha = (12.5\text{ kg}\cdot\text{m}^2)(-0.873\text{ rad/s}^2) = -10.9\text{ N}\cdot\text{m}$

The magnitude of the torque is given by $fR = 10.9\text{ N}\cdot\text{m}$, where f is the force of friction.

Therefore, $f = \dfrac{10.9\text{ N}\cdot\text{m}}{0.500\text{ m}}$, and

$f = \mu_k n$ yields $\mu_k = \dfrac{f}{n} = \dfrac{21.8\text{ N}}{70.0\text{ N}} = \boxed{0.312}$

10.41 $I = MR^2 = 1.80\text{ kg}(0.320\text{ m})^2 = 0.184\text{ kg}\cdot\text{m}^2$

$\sum\tau = I\alpha$

(a) $F_a(4.50\times10^{-2}\text{ m}) - 120\text{ N}(0.320\text{ m}) = 0.184\text{ kg}\cdot\text{m}^2(4.50\text{ rad/s}^2)$

$F_a = \dfrac{(0.829\text{ N}\cdot\text{m} + 38.4\text{ N}\cdot\text{m})}{4.50\times10^{-2}\text{ m}} = \boxed{872\text{ N}}$

(b) $F_b(2.80\times10^{-2}\text{ m}) - 38.4\text{ N}\cdot\text{m} = 0.829\text{ N}\cdot\text{m}$

$F_b = \boxed{1.40\text{ kN}}$

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10.42 We assume the rod is thin. For the compound object

$$I = \frac{1}{3}\ M_{\text{rod}}L^2 + \left[\frac{2}{5}M_{\text{ball}}R^2 + M_{\text{ball}}D^2\right]$$

$$I = \frac{1}{3}\ 1.20\text{ kg }(0.240\text{ m})^2 + \frac{2}{5}\ 20.0\text{ kg}(4.00 \times 10^{-2}\text{ m})^2 + 20.0\text{ kg}(0.280\text{ m})^2$$

$$I = 1.60\text{ kg}\cdot\text{m}^2$$

(a) $K_f + U_f = K_i + U_i + \Delta E$

$$\frac{1}{2}\ I\omega^2 + 0 = 0 + M_{\text{rod}}g(L/2) + M_{\text{ball}}g(L+R) + 0$$

$$\frac{1}{2}(1.60\text{ kg}\cdot\text{m}^2)\ \omega^2 = 1.20\text{ kg}(9.80\text{ m/s}^2)(0.120\text{ m}) + 20.0\text{ kg}(9.80\text{ m/s}^2)(0.280\text{ m})$$

$$\frac{1}{2}(1.60\text{ kg}\cdot\text{m}^2)\ \omega^2 = \boxed{56.3\text{ J}}$$

(b) $\omega = \boxed{8.38\text{ rad/s}}$

(c) $v = r\omega = (0.280\text{ m})8.38\text{ rad/s} = \boxed{2.35\text{ m/s}}$

(d) $v^2 = v_i^2 + 2a(y - y_i)$

$$v = \sqrt{0 + 2(9.80\text{ m/s}^2)(0.280\text{ m})} = 2.34\text{ m/s}$$

The speed it attains in swinging is greater by $2.35/2.34 = \boxed{1.00140\text{ times}}$

10.43 Choose the zero gravitational potential energy at the level where the masses pass.

$$K_f + U_{gf} = K_i + U_{gi} + \Delta E$$

$$\frac{1}{2}\ m_1v^2 + \frac{1}{2}\ m_2v^2 + \frac{1}{2}\ I\omega^2 = 0 + m_1gh_{1i} + m_2gh_{2i} + 0$$

$$\frac{1}{2}(15.0 + 10.0)\ v^2 + \frac{1}{2}\left[\frac{1}{2}(3.00)R^2\right]\left(\frac{v}{R}\right)^2 = 15.0(9.80)(1.50) + 10.0(9.80)(-1.50)$$

$$\frac{1}{2}(26.5\text{ kg})\ v^2 = 73.5\text{ J} \Rightarrow v = \boxed{2.36\text{ m/s}}$$

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10.44 Choose the zero gravitational potential energy at the level where the masses pass.

$$K_f + U_{gf} = K_i + U_{gi} + \Delta E$$

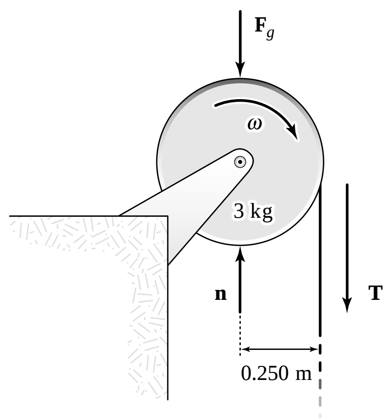
$$\frac{1}{2} m_1 v^2 + \frac{1}{2} m_2 v^2 + \frac{1}{2} I\omega^2 = 0 + m_1 g h_{1i} + m_2 g h_{2i} + 0$$

$$\frac{1}{2}(m_1 + m_2) v^2 + \frac{1}{2}\left[\frac{1}{2} MR^2\right]\left(\frac{v}{R}\right)^2 = m_1 g\left(\frac{d}{2}\right) + m_2 g\left(-\frac{d}{2}\right)$$

$$\frac{1}{2}\left(m_1 + m_2 + \frac{1}{2} M\right) v^2 = (m_1 - m_2) g\left(\frac{d}{2}\right)$$

$$v = \boxed{\sqrt{\frac{(m_1 - m_2) g d}{m_1 + m_2 + \frac{1}{2} M}}}$$

10.45 (a) $50.0 - T = \frac{50.0}{9.80}\, a$



$$TR = I\alpha = I\frac{a}{R}$$

$$I = \frac{1}{2} MR^2 = 0.0938 \text{ kg} \cdot \text{m}^2$$

$$50.0 - T = 5.10\left(\frac{TR^2}{I}\right)$$

$$T = \boxed{11.4 \text{ N}}$$

$$a = \frac{50.0 - 11.4}{5.10} = \boxed{7.57 \text{ m/s}^2}$$

$$v = \sqrt{2a(y_i - 0)} = \boxed{9.53 \text{ m/s}}$$

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(b) Use conservation of energy:

$$(K + U)_i = (K + U)_f$$

$$mgh = \frac{1}{2} mv^2 + \frac{1}{2} I\omega^2$$

$$2mgh = mv^2 + I\left(\frac{v^2}{R^2}\right)$$

$$= v^2\left(m + \frac{I}{R^2}\right)$$

$$v = \sqrt{\frac{2mgh}{m + \frac{I}{R^2}}} = \sqrt{\frac{2(50.0\text{ N})(6.00\text{ m})}{5.10\text{ kg} + \frac{0.0938}{(0.250)^2}}} = \boxed{9.53\text{ m/s}}$$

Goal Solution

G: Since the rotational inertia of the reel will slow the fall of the weight, we should expect the downward acceleration to be less than g. If the reel did not rotate, the tension in the string would be equal to the weight of the object; and if the reel disappeared, the tension would be zero. Therefore, $T < mg$ for the given problem. With similar reasoning, the final speed must be less than if the weight were to fall freely: $v_f < \sqrt{2gy} \approx 11\text{ m/s}$

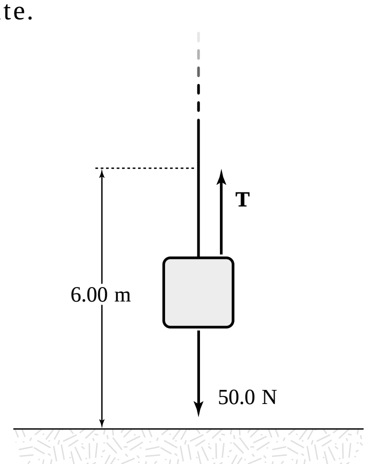
O: We can find the acceleration and tension using the rotational form of Newton's second law. The final speed can be found from the kinematics equation stated above and from conservation of energy. Free-body diagrams will greatly assist in analyzing the forces.

A: (a) Use $\sum\tau = I\alpha$ to find T and a.

First find I for the reel, which we assume to be a uniform disk:

$$I = \frac{1}{2} MR^2 = \frac{1}{2}\, 3.00\text{ kg }(0.250\text{ m})^2 = 0.0938\text{ kg}\cdot\text{m}^2$$

The forces on the reel are shown, including a normal force exerted by its axle. From the diagram, we can see that the tension is the only unbalanced force causing the reel to rotate.



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$\sum\tau = I\alpha$ becomes

$$n(0) + F_g(0) + T(0.250\text{ m}) = (0.0938\text{ kg}\cdot\text{m}^2)(a/0.250\text{ m}) \qquad (1)$$

where we have applied $a_t = r\alpha$ to the point of contact between string and reel.

The falling weight has mass

$$m = \frac{F_g}{g} = \frac{50.0\text{ N}}{9.80\text{ m/s}^2} = 5.10\text{ kg}$$

For this mass, $\sum F_y = ma_y$ becomes

$$+T - 50.0\text{ N} = (5.10\text{ kg})(-a) \qquad (2)$$

Note that since we have defined upwards to be positive, the minus sign shows that its acceleration is downward. We now have our two equations in the unknowns T and a for the two linked objects. Substituting T from equation (2) into equation (1), we have:

$$[50.0\text{ N} - (5.10\text{ kg})a](0.250\text{ m}) = 0.0938\text{ kg}\cdot\text{m}^2\,\frac{a}{0.250\text{ m}}$$

$$12.5\text{ N}\cdot\text{m} - (1.28\text{ kg}\cdot\text{m})a = (0.375\text{ kg}\cdot\text{m})a$$

$$12.5\text{ N}\cdot\text{m} = a(1.65\text{ kg}\cdot\text{m}) \quad \text{or} \quad a = 7.57\text{ m/s}^2$$

and $T = 50.0\text{ N} - 5.10\text{ kg}(7.57\text{ m/s}^2) = 11.4\text{ N}$

For the motion of the weight,

$$v_f^2 = v_i^2 + 2a(y_f - y_i) = 0^2 + 2(7.57\text{ m/s}^2)(6.00\text{ m})$$

$$v_f = 9.53\text{ m/s}$$

(b) The work-energy theorem can take account of multiple objects more easily than Newton's second law. Like your bratty cousins, the work-energy theorem keeps growing between visits. Now it reads:

$$(K_1 + K_{2,\text{rot}} + U_{g1} + U_{g2})_\text{i} = (K_1 + K_{2,\text{rot}} + U_{g1} + U_{g2})_f$$

$$0 + 0 + m_1 g y_{1i} + 0 = \frac{1}{2}\,m_1 v_{1f}^2 + \frac{1}{2}\,I_2\omega_{2f}^2 + 0 + 0$$

Now note that $\omega = \frac{v}{r}$ as the string unwinds from the reel. Making substitutions:

$$50.0\text{ N}(6.00\text{ m}) = \frac{1}{2}(5.10\text{ kg})\,v_f^2 + \frac{1}{2}(0.0938\text{ kg}\cdot\text{m}^2)\left(\frac{v_f}{0.250\text{ m}}\right)^2$$

$$300\text{ N}\cdot\text{m} = \frac{1}{2}(5.10\text{ kg})\,v_f^2 + \frac{1}{2}(1.50\text{ kg})\,v_f^2$$

$$v_f = \sqrt{\frac{2(300\text{ N}\cdot\text{m})}{6.60\text{ kg}}} = 9.53\text{ m/s}$$

L: As we should expect, both methods give the same final speed for the falling object. The acceleration is less than g, and the tension is less than the object's weight as we predicted. Now that we understand the effect of the reel's moment of inertia, this problem solution could be applied to solve other real-world pulley systems with masses that should not be ignored.

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10.46 $\tau \cdot \theta = \frac{1}{2} I\omega^2$

$(25.0 \text{ N} \cdot \text{m})(15.0 \cdot 2\pi) = \frac{1}{2}(0.130 \text{ kg} \cdot \text{m}^2)\,\omega^2$

$\omega = 190 \text{ rad/s} = \boxed{30.3 \text{ rev/s}}$

10.47 From conservation of energy,

$$\frac{1}{2} I\left(\frac{v}{r}\right)^2 + \frac{1}{2} mv^2 = mgh$$

$$I\frac{v^2}{r^2} = 2mgh - mv^2$$

$$I = \boxed{mr^2\left(\frac{2gh}{v^2} - 1\right)}$$

10.48 $E = \frac{1}{2} I\omega^2 = \frac{1}{2}\left(\frac{1}{2} MR^2\right)\left(\frac{3000 \times 2\pi}{60.0}\right)^2 = 6.17 \times 10^6 \text{ J}$

$P = \frac{\Delta E}{\Delta t} = 1.00 \times 10^4 \text{ J/s}$

$\Delta t = \frac{\Delta E}{P} = \frac{6.17 \times 10^6 \text{ J}}{1.00 \times 10^4 \text{ J/s}} = 617 \text{ s} = \boxed{10.3 \text{ min}}$

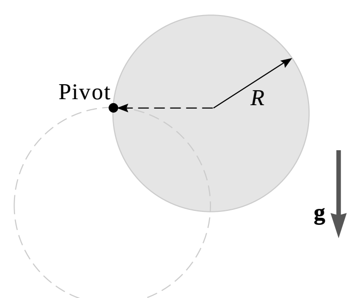
10.49 (a) Find the velocity of the CM

$$(K + U)_i = (K + U)_f$$

$$0 + mgR = \frac{1}{2} I\omega^2$$

$$\omega = \sqrt{\frac{2mgR}{I}} = \sqrt{\frac{2mgR}{\frac{3}{2}mR^2}}$$

$$v_{\text{CM}} = R\sqrt{\frac{4g}{3R}} = \boxed{2\sqrt{\frac{Rg}{3}}}$$



(b) $v_L = 2v_{\text{CM}} = \boxed{4\sqrt{\frac{Rg}{3}}}$

(c) $v_{\text{CM}} = \sqrt{\frac{2mgR}{2m}} = \boxed{\sqrt{Rg}}$

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***10.50** The moment of inertia of the cylinder is

$$I = \frac{1}{2}mr^2 = \frac{1}{2}(81.6\text{ kg})(1.50\text{ m})^2 = 91.8\text{ kg}\cdot\text{m}^2$$

and the angular acceleration of the merry-go-round is found as

$$\alpha = \frac{\tau}{I} = \frac{(Fr)}{I} = \frac{(50.0\text{ N})(1.50\text{ m})}{(91.8\text{ kg}\cdot\text{m}^2)} = 0.817\text{ rad/s}^2$$

At $t = 3.00$ s, we find the angular velocity

$$\omega = \omega_i + \alpha t$$

$$\omega = 0 + (0.817\text{ rad/s}^2)(3.00\text{ s}) = 2.45\text{ rad/s}$$

and $K = \frac{1}{2}I\omega^2 = \frac{1}{2}(91.8\text{ kg}\cdot\text{m}^2)(2.45\text{ rad/s})^2 = \boxed{276\text{ J}}$

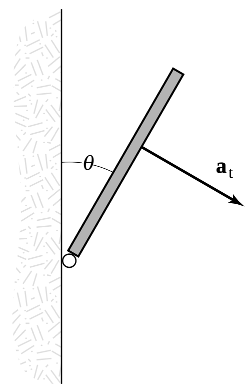
10.51 $mg\frac{\ell}{2}\sin\theta = \frac{1}{3}m\ell^2\alpha$

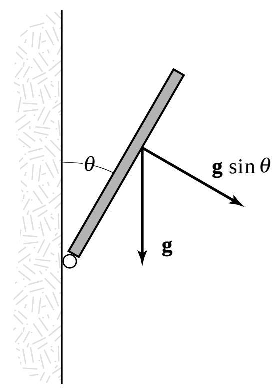
$$\alpha = \frac{3}{2}\frac{g}{\ell}\sin\theta$$

$$a_t = \left(\frac{3}{2}\frac{g}{\ell}\sin\theta\right)r$$

Then $\left(\frac{3}{2}\frac{g}{\ell}\right)r > g\sin\theta$

for $r > \frac{2}{3}\ell$





$\therefore$ About $\boxed{\text{1/3 the length of the chimney}}$ will have a tangential acceleration greater than $g\sin\theta$.

***10.52** The resistive force on each ball is $R = D\rho Av^2$. Here $v = r\omega$, where r is the radius of each ball's path. The resistive torque on each ball is $\tau = rR$, so the total resistive torque on the three ball system is $\tau_{\text{total}} = 3rR$. The power required to maintain a constant rotation rate is $P = \tau_{\text{total}}\omega = 3rR\omega$. This required power may be written as

$$P = \tau_{\text{total}}\omega = 3r\,[D\rho A(r\omega)^2]\omega = (3r^3DA\omega^3)\rho$$

With $\omega = \left(\frac{2\pi\text{ rad}}{1\text{ rev}}\right)\left(10^3\frac{\text{rev}}{\text{min}}\right)\left(\frac{1\text{ min}}{60.0\text{ s}}\right) = \left(\frac{1000\pi}{30.0}\right)$ rad/s,

$$P = 3(0.100\text{ m})^3(0.600)(4.00\times10^{-4}\text{ m}^2)(1000\pi/30.0\text{ s})^3\rho$$

or $P = (0.827\text{ m}^5/\text{s}^3)\rho$ where ρ is the density of the resisting medium.

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(a) In air, $\rho = 1.20 \text{ kg/m}^3$, and

$$P = (0.827 \text{ m}^5/\text{s}^3)(1.20 \text{ kg/m}^3) = 0.992 \text{ N}\cdot\text{m/s} = \boxed{0.992 \text{ W}}$$

(b) In water, $\rho = 1000 \text{ kg/m}^3$ and $P = \boxed{827 \text{ W}}$

10.53 (a) $I = \frac{1}{2} MR^2 = \frac{1}{2}(2.00 \text{ kg})(7.00 \times 10^{-2} \text{ m})^2 = 4.90 \times 10^{-3} \text{ kg}\cdot\text{m}^2$

$$\alpha = \frac{\tau}{I} = \frac{0.600}{4.90 \times 10^{-3}} = 122 \text{ rad/s}^2$$

$$\alpha = \frac{\Delta\omega}{\Delta t}$$

$$\Delta t = \frac{\Delta\omega}{\alpha} = \frac{1200\left(\frac{2\pi}{60}\right)}{122} = \boxed{1.03 \text{ s}}$$

(b) $\theta = \frac{1}{2}\alpha t^2 = \frac{1}{2}(122 \text{ rad/s})(1.03 \text{ s})^2 = 64.7 \text{ rad} = \boxed{10.3 \text{ rev}}$

10.54 For a spherical shell $\frac{2}{3}\, dm\, r^2 = \frac{2}{3}\,[(4\pi r^2 dr)\rho]r^2$

$$I \int dI = \int \frac{2}{3}(4\pi r^2)\, r^2 \rho(r) dr$$

$$I = \int_0^R \frac{2}{3}(4\pi r^4)\left(14.2 - 11.6\,\frac{r}{R}\right)\left(10^3\,\frac{\text{kg}}{\text{m}^3}\right) dr$$

$$= \left(\frac{2}{3}\right) 4\pi(14.2 \times 10^3)\frac{R^5}{5} - \left(\frac{2}{3}\right) 4\pi(11.6 \times 10^3)\frac{R^5}{6}$$

$$I = \frac{8\pi}{3}(10^3)\, R^5\left(\frac{14.2}{5} - \frac{11.6}{6}\right)$$

$$M = \int dm = \int_0^R 4\pi r^2\left(14.2 - 11.6\,\frac{r}{R}\right) 10^3\, dr$$

$$= 4\pi \times 10^3\left(\frac{14.2}{3} - \frac{11.6}{4}\right) R^3$$

$$\frac{I}{MR^2} = \frac{\frac{8\pi}{3}(10^3)\, R^5\left(\frac{14.2}{5} - \frac{11.6}{6}\right)}{4\pi \times 10^3 R^3 R^2\left(\frac{14.2}{3} - \frac{11.6}{4}\right)} = \frac{2}{3}\left(\frac{.907}{1.83}\right) = 0.330$$

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$$\therefore I = \boxed{0.330MR^2}$$

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10.55 (a) $W = \Delta K = \frac{1}{2} I\omega^2 - \frac{1}{2} I\omega_i^2$

$= \frac{1}{2} I(\omega^2 - \omega_i^2)$ where $I = \frac{1}{2} mR^2$

$= \left(\frac{1}{2}\right)\left(\frac{1}{2}\right)(1.00\text{ kg})(0.500\text{ m})^2\left[\left(8.00\frac{\text{rad}}{\text{s}}\right)^2 - 0\right] = \boxed{4.00\text{ J}}$

(b) $t = \frac{\omega - 0}{\alpha} = \frac{\omega r}{a} = \frac{(8.00\text{ rad/s})(0.500\text{ m})}{2.50\text{ m/s}^2} = \boxed{1.60\text{ s}}$

(c) $\theta = \theta_i + \omega_i t + \frac{1}{2}\alpha t^2$; $\theta_i = 0$; $\omega_i = 0$

$\theta = \frac{1}{2}\alpha t^2 = \frac{1}{2}\left(\frac{2.50\text{ m/s}^2}{0.500\text{ m}}\right)(1.60\text{ s})^2 = 6.40\text{ rad}$

$s = r\theta = (0.500\text{ m})(6.40\text{ rad}) = \boxed{3.20\text{ m} < 4.00\text{ m Yes}}$

10.56 (a) $I = \frac{1}{2} mr^2 = \frac{1}{2}(200\text{ kg})(0.300\text{ m})^2 = \boxed{9.00\text{ kg}\cdot\text{m}^2}$

(b) $\omega = (1000\text{ rev/min})(1\text{ min/60 s})(2\pi\text{ rad})/1\text{ rev} = 105\text{ rad/s}$

$W = K = \frac{1}{2} I\omega^2 = \frac{1}{2}(9.00)(104.7)^2 = \boxed{49.3\text{ kJ}}$

(c) $\omega_f = 500\text{ rev/min} \times 1\text{ min/60 s}(2\pi\text{ rad/1 rev}) = 52.4\text{ rad/s}$

$K_f = \frac{1}{2} I\omega^2 = 12.3\text{ kJ}$

$W = \Delta K = 12.3\text{ kJ} - 49.3\text{ kJ} = \boxed{-37.0\text{ kJ}}$

***10.57** $\alpha = -10.0\text{ rad/s}^2 - (5.00\text{ rad/s}^3)t = d\omega/dt$

$\int_{65.0}^{\omega} d\omega = \int_0^t [-10.0 - 5.00t]dt = -10.0t - 2.50t^2 = \omega - 65.0\text{ rad/s}$

$\omega = \frac{d\theta}{dt} = 65.0\text{ rad/s} - (10.0\text{ rad/s}^2)t - (2.50\text{ rad/s}^3)t^2$

(a) At $t = 3.00$ s,

$\omega = 65.0\text{ rad/s} - (10.0\text{ rad/s}^2)(3.00\text{ s}) - (2.50\text{ rad/s}^3)(9.00\text{ s}^2) = \boxed{12.5\text{ rad/s}}$

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(b) $\int_0^\theta d\theta = \int_0^t \omega\, dt = \int_0^t [65.0\text{ rad/s} - (10.0\text{ rad/s}^2)t - (2.50\text{ rad/s}^3)t^2]dt$

$\theta = (65.0\text{ rad/s})t - (5.00\text{ rad/s}^2)t^2 - (0.833\text{ rad/s}^3)t^3$

At $t = 3.00$ s,

$\theta = (65.0\text{ rad/s})(3.00\text{ s}) - (5.00\text{ rad/s}^2)9.00\text{ s}^2 - (0.833\text{ rad/s}^3)27.0\text{ s}^3$

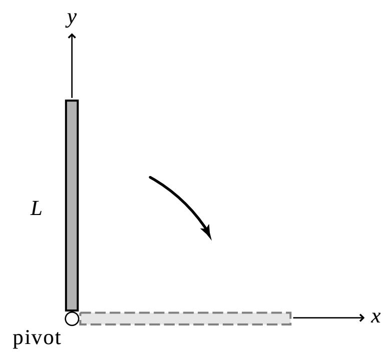
$\theta = \boxed{128\text{ rad}}$

10.58 (a) $MK^2 = \frac{MR^2}{2}$, $K = \frac{R}{\sqrt{2}}$

(b) $MK^2 = \frac{ML^2}{12}$, $K = \frac{L\sqrt{3}}{6}$

(c) $MK^2 = \frac{2}{5}MR^2$, $K = R\sqrt{\frac{2}{5}}$

10.59 (a) Since only conservative forces act, $\Delta E = 0$, so



$K_f + U_f = K_i + U_i$

$\frac{1}{2}I\omega^2 + 0 = 0 + mg\left(\frac{L}{2}\right)$, where $I = \frac{1}{3}mL^2$

$\omega = \boxed{\sqrt{3g/L}}$

(b) $\tau = I\alpha$ so that in the horizontal, $mg\left(\frac{L}{2}\right) = \frac{mL^2}{3}\alpha$ $\alpha = \boxed{\frac{3g}{2L}}$

(c) $a_x = a_r = r\omega^2 = \left(\frac{L}{2}\right)\omega^2 = \boxed{-\frac{3g}{2}}$ $a_y = -a_t = -r\alpha = -\alpha\left(\frac{L}{2}\right) = \boxed{-\frac{3g}{4}}$

(d) Using Newton's second law, we have $R_x = ma_x = \boxed{-\frac{3}{2}mg}$

$R_y - mg = -ma_y$ or $R_y = \boxed{\frac{1}{4}mg}$

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10.60 The first drop has a velocity leaving the wheel given by $\frac{1}{2}mv_i^2 = mgh_1$, so

$$v_1 = \sqrt{2gh_1} = \sqrt{2(9.80\text{ m/s}^2)(0.540\text{ m})} = 3.25\text{ m/s}$$

The second drop has a velocity given by

$$v_2 = \sqrt{2gh_2} = \sqrt{2(9.80\text{ m/s}^2)(0.510\text{ m})} = 3.16\text{ m/s}$$

From $\omega = \frac{v}{r}$, we find

$$\omega_1 = \frac{v_1}{r} = \frac{3.25\text{ m/s}}{0.381\text{ m}} = 8.53\text{ rad/s} \quad \text{and} \quad \omega_2 = \frac{v_2}{r} = \frac{3.16\text{ m/s}}{0.381\text{ m}} = 8.29\text{ rad/s}$$

or $$\alpha = \frac{\omega_2^2 - \omega_1^2}{2\theta} = \frac{(8.29\text{ rad/s})^2 - (8.53\text{ rad/s})^2}{4\pi} = \boxed{-0.322\text{ rad/s}^2}$$

10.61 At the instant it comes off the wheel, the first drop has a velocity v_1, directed upward. The magnitude of this velocity is found from

$$K_i + U_{gi} = K_f + U_{gf}$$

$$\frac{1}{2}mv_1^2 + 0 = 0 + mgh_1 \quad \text{or} \quad v_1 = \sqrt{2gh_1}$$

and the angular velocity of the wheel at the instant the first drop leaves is

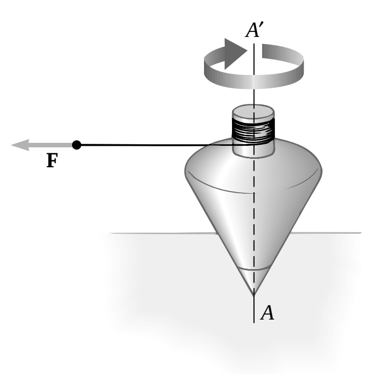
$$\omega_1 = \frac{v_1}{R} = \sqrt{\frac{2gh_1}{R^2}}$$

Similarly for the second drop: $v_2 = \sqrt{2gh_2}$ and $\omega_2 = \frac{v_2}{R} = \sqrt{\frac{2gh_2}{R^2}}$

The angular acceleration of the wheel is then

$$\alpha = \frac{\omega_2^2 - \omega_1^2}{2\theta} = \frac{2gh_2/R^2 - 2gh_1/R^2}{2(2\pi)} = \boxed{\frac{g(h_2 - h_1)}{2\pi R^2}}$$

10.62 Work done $= Fs = (5.57\text{ N})(0.800\text{ m}) = 4.46\text{ J}$



and Work $= \Delta K = \frac{1}{2}I\omega_f^2 - \frac{1}{2}I\omega_i^2$

(The last term is zero because the top starts from rest.)

Thus, $4.46\text{ J} = \frac{1}{2}(4.00 \times 10^{-4}\text{ kg}\cdot\text{m}^2)\,\omega_f^2$

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and from this, $\omega_f = \boxed{149 \text{ rad/s}}$

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10.63 $K_f = \frac{1}{2} Mv_f^2 + \frac{1}{2} I\omega_f^2$: $U_f = Mgh_f = 0$; $K_i = \frac{1}{2} Mv_i^2 + \frac{1}{2} I\omega_i^2 = 0$

$U_i = (Mgh)_i$: $f = \mu N = \mu Mg \cos\theta$; $\omega = \frac{v}{r}$; $h = d \sin\theta$ and $I = \frac{1}{2} mr^2$

(a) $\Delta E = E_f - E_i$ or $-fd = K_f + U_f - K_i - U_i$

$$-fd = \frac{1}{2} Mv_f^2 + \frac{1}{2} I\omega_f^2 - Mgh$$

$$-(\mu Mg \cos\theta)d = \frac{1}{2} Mv^2 + (mr^2/2)(v^2/r^2)/2 - Mgd \sin\theta$$

$$\frac{1}{2}\left[M + \frac{m}{2}\right] v^2 = Mgd \sin\theta - (\mu Mg \cos\theta)d \text{ or}$$

$$v^2 = 2Mgd \frac{(\sin\theta - \mu \cos\theta)}{(m/2) + M}$$

$$v_d = \left[4gd \frac{M}{(m + 2M)}(\sin\theta - \mu \cos\theta)\right]^{1/2}$$

(b) $v^2 = v_i^2 - 2as$, $v_d^2 = 2ad$

$$a = \frac{v_d^2}{2d} = \boxed{2g\left(\frac{M}{m + 2M}\right)(\sin\theta - \mu\cos\theta)}$$

10.64 (a) $E = \frac{1}{2}\left(\frac{2}{5} MR^2\right)(\omega^2)$

$$E = \frac{1}{2} \cdot \frac{2}{5}(5.98 \times 10^{24})(6.37 \times 10^6)^2 \left(\frac{2\pi}{86400}\right)^2 = \boxed{2.57 \times 10^{29}\ \text{J}}$$

(b) $\frac{dE}{dt} = \frac{d}{dt}\left[\frac{1}{2}\left(\frac{2}{5} MR^2\right)\left(\frac{2\pi}{T}\right)^2\right]$

$$= \frac{1}{5} MR^2(2\pi)^2(-2T^{-3}) \frac{dT}{dt}$$

$$= \frac{1}{5} MR^2\left(\frac{2\pi}{T}\right)^2\left(\frac{-2}{T}\right)\frac{dT}{dt}$$

$$= (2.57 \times 10^{29}\ \text{J})\left(\frac{-2}{86400\ \text{s}}\right)\left(\frac{10 \times 10^{-6}\ \text{s}}{3.16 \times 10^7\ \text{s}}\right)(86400\ \text{s/day})$$

$$\frac{dE}{dt} = \boxed{-1.63 \times 10^{17}\ \text{J/day}}$$

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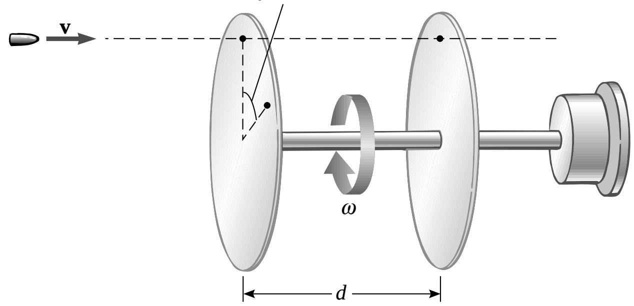
10.65 $\Delta\theta = \omega t$

$$t = \frac{\Delta\theta}{\omega} = \frac{(31.0°/360°)\text{ rev}}{900\text{ rev}/60\text{ s}} = 0.00574\text{ s}$$

$$v = \frac{0.800\text{ m}}{0.00574\text{ s}} = \boxed{139\text{ m/s}}$$



10.66 (a) Each spoke counts as a thin rod pivoted at one end.

$$I = \boxed{MR^2 + n\frac{mR^2}{3}}$$

(b) By the parallel-axis theorem,

$$I = MR^2 + \frac{nmR^2}{3} + (M + nm)R^2$$

$$= \boxed{2MR^2 + \frac{4\,nmR^2}{3}}$$

***10.67** Every particle in the door could be slid straight down into a high-density rod across its bottom, without changing the particle's distance from the rotation axis of the door. Thus, a rod 0.870 m long with mass 23.0 kg, pivoted about one end, has the same rotational inertia as the door:

$$I = \frac{1}{3}ML^2 = \frac{1}{3}(23.0\text{ kg})(0.870\text{ m})^2 = \boxed{5.80\text{ kg}\cdot\text{m}^2}$$

The $\boxed{\text{height of the door is unnecessary}}$ data.

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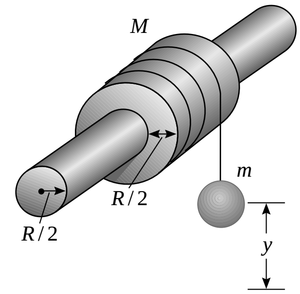
10.68 τ_f will oppose the torque causing the motion:



$$\sum \tau = I\alpha = TR - \tau_f \Rightarrow \tau_f = TR - I\alpha \qquad (1)$$

Now find T, I and α in given or known terms and substitute into equation (1)

$$\sum F_y = T - mg = -ma \text{ then } T = m(g - a) \qquad (2)$$

also $\Delta y = v_i t + \frac{at^2}{2} \Rightarrow a = \frac{2y}{t^2}$ (3)

and $\alpha = \frac{a}{R} = \frac{2y}{Rt^2}$ (4)

$$I = \frac{1}{2} M\left[R^2 + \left(\frac{R}{2}\right)^2\right] = \frac{5}{8} MR^2 \qquad (5)$$

Substituting (2), (3), (4) and (5) into (1) we find

$$\tau_f = m\left(g - \frac{2y}{t^2}\right) R - \frac{5}{8}\frac{MR^2 2y}{(Rt^2)} = \boxed{R\left[m\left(g - \frac{2y}{t^2}\right) - \frac{5}{4}\frac{My}{t^2}\right]}$$

10.69 (a) While decelerating,

$$\tau_f = I\alpha' = (20\,000 \text{ kg} \cdot \text{m}^2)\left(\frac{2.00 \text{ rev/min } (2\pi \text{ rad/rev})(1 \text{ min}/60 \text{ s})}{10.0 \text{ s}}\right)$$

$$\tau_f = 419 \text{ N} \cdot \text{m}$$

While accelerating,

$$\sum \tau = I\alpha \quad \text{or} \quad \tau - \tau_f = I(\Delta\omega/\Delta t)$$

$$\tau = 419 \text{ N} \cdot \text{m} + (20\,000 \text{ kg} \cdot \text{m}^2)\left(\frac{10.00 \text{ rev/min } (2\pi \text{ rad/rev})(1 \text{ min}/60 \text{ s})}{12.0 \text{ s}}\right)$$

$$\tau = \boxed{2.16 \times 10^3 \text{ N} \cdot \text{m}}$$

(b) $P = \tau_f \cdot \omega = (419 \text{ N} \cdot \text{m})\left[10.0 \frac{\text{rev}}{\text{min}}\left(\frac{2\pi \text{ rad}}{1 \text{ rev}}\right)\left(\frac{1 \text{ min}}{60 \text{ s}}\right)\right] = \boxed{439 \text{ W}}$ (≈ 0.6 hp)

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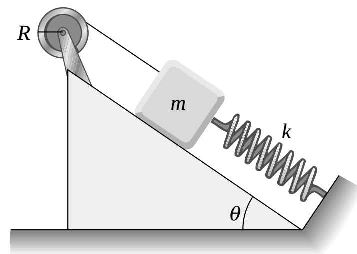
10.70 (a) $W = \Delta K + \Delta U$



$$W = K_f - K_i + U_f - U_i$$

$$0 = \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2 - mgd\sin\theta - \frac{1}{2}kd^2$$

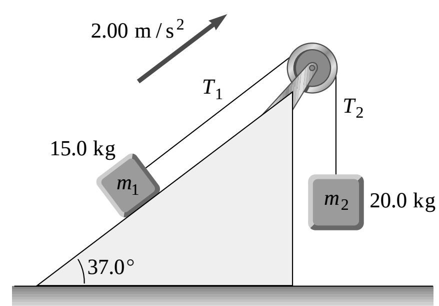
$$\frac{1}{2}\omega^2(I + mR^2) = mgd\sin\theta + \frac{1}{2}kd^2$$

$$\boxed{\omega = \sqrt{\frac{2mgd\sin\theta + kd^2}{I + mR^2}}}$$

(b) $$\omega = \sqrt{\frac{2(0.500\text{ kg})(9.80\text{ m/s}^2)(0.200\text{ m})(\sin 37.0°) + (50.0\text{ N/m})(0.200\text{ m})^2}{1.00\text{ kg}\cdot\text{m}^2 + (0.500\text{ kg})(0.300\text{ m})^2}}$$

$$\omega = \sqrt{\frac{1.18 + 2.00}{1.05}} = \sqrt{3.04} = \boxed{1.74\text{ rad/s}}$$

10.71 (a) $m_2g - T_2 = m_2a$



$$T_2 = m_2(g - a) = (20.0\text{ kg})(9.80 - 2.00)\text{m/s}^2 = \boxed{156\text{ N}}$$

$$T_1 = m_1g\sin 37.0° + m_1a$$

$$T_1 = (15.0\text{ kg})(9.80\sin 37.0° + 2.00)\text{m/s}^2 = \boxed{118\text{ N}}$$

(b) $$(T_2 - T_1)R = I\alpha = I\left(\frac{a}{R}\right)$$

$$I = \frac{(T_2 - T_1)R^2}{a} = \frac{(156 - 118)\text{N}(0.250\text{ m})^2}{2.00\text{ m/s}^2} = \boxed{1.17\text{ kg}\cdot\text{m}^2}$$

Goal Solution

G: In earlier problems, we assumed that the tension in a string was the same on either side of a pulley. Here we see that the moment of inertia changes that assumption, but we should still expect the tensions to be similar in magnitude (about the weight of each mass ~150 N), and $T_2 > T_1$ for the pulley to rotate clockwise as shown.

If we knew the mass of the pulley, we could calculate its moment of inertia, but since we only know the acceleration, it is difficult to estimate I. We at least know that I must have units of kgm^2, and a 50-cm disk probably has a mass less than 10 kg, so I is probably less than 0.3 kgm^2.

O: For each block, we know its mass and acceleration, so we can use Newton's second law to find the net force, and from it the tension. The difference in the two tensions causes the pulley to rotate, so this net torque and the resulting angular acceleration can be used to find the pulley's moment of inertia.

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A: (a) Apply $\sum F = ma$ to each block to find each string tension.

The forces acting on the 15-kg block are its weight, the normal support force from the incline, and T_1. Taking the positive x axis as directed up the incline, $\sum F_x = ma_x$ yields:

$$-(m_1 g)_x + T_1 = m_1(+a)$$

$$-(15.0 \text{ kg})(9.80 \text{ m/s}^2) \sin 37° + T_1 = (15.0 \text{ kg})(2.00 \text{ m/s}^2)$$

$$T_1 = 118 \text{ N}$$

Similarly for the counterweight, we have $\sum F_y = ma_y$, or $T_2 - m_2 g = m_2(-a)$

$$T_2 - (20.0 \text{ kg})(9.80 \text{ m/s}^2) = (20.0 \text{ kg})(-2.00 \text{ m/s}^2)$$

So, $T_2 = 156$ N

(b) Now for the pulley, $\sum \tau = r(T_2 - T_1) = I\alpha$. We may choose to call clockwise positive. The angular acceleration is

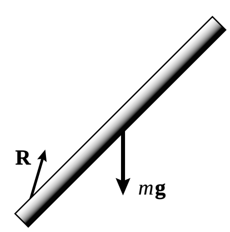
$$\alpha = \frac{a}{r} = \frac{2.00 \text{ m/s}^2}{0.250 \text{ m}} = 8.00 \text{ rad/s}^2$$

$$\sum \tau = I\alpha \quad \text{or} \quad (0.250 \text{ m})(156 \text{ N} - 118 \text{ N}) = I(8.00 \text{ rad/s}^2)$$

$$I = \frac{9.38 \text{ N} \cdot \text{m}}{8.00 \text{ rad/s}^2} = 1.17 \text{ kg} \cdot \text{m}^2$$

L: The tensions are close to the weight of each mass and $T_2 > T_1$ as expected. However, the moment of inertia for the pulley is about 4 times greater than expected. Unless we made a mistake in solving this problem, our result means that the pulley has a mass of 37.4 kg (about 80 lb), which means that the pulley is probably made of a dense material, like steel. This is certainly not a problem where the mass of the pulley can be ignored since the pulley has more mass than the combination of the two blocks!

10.72 For the board just starting to move,



$$\sum \tau = I\alpha$$

$$mg \frac{l}{2} \cos\theta = \left(\frac{1}{3} m l^2\right) \alpha$$

$$\alpha = \frac{3}{2}\left(\frac{g}{l}\right) \cos\theta$$

The tangential acceleration of the end is

$$a_t = \ell\alpha = \frac{3}{2}g\cos\theta$$

Its vertical component is $a_y = a_t \cos\theta = \frac{3}{2}g\cos^2\theta$

If this is greater than g, the board will pull ahead of the ball in falling:

(a) $\frac{3}{2}g\cos^2\theta \geq g \Rightarrow \cos^2\theta \geq \frac{2}{3}$

so $\cos\theta \geq \sqrt{\frac{2}{3}}$ and $\boxed{\theta \leq 35.3°}$

(b) When $\theta = 35.3°$ ($\Rightarrow \cos^2\theta = 2/3$), the cup will land underneath the release-point of the ball if

$$r_c = \ell\cos\theta = \frac{\ell\cos^2\theta}{\cos\theta} = \boxed{\frac{2\ell}{3\cos\theta}}$$

(c) When $\ell = 1.00$ m, and $\theta = 35.3°$

$$r_c = \frac{2(1.00\text{ m})}{3\sqrt{2/3}} = 0.816\text{ m}$$

which is (1.00 m – 0.816 m) = $\boxed{\text{0.184 m from the moving end}}$

10.73 At $t = 0$, $\omega = 3.50$ rad/s $= \omega_0 e^0$. Thus, $\omega_0 = 3.50$ rad/s

At $t = 9.30$ s, $\omega = 2.00$ rad/s $= \omega_0 e^{-\sigma(9.30\text{ s})}$, yielding $\sigma = 6.02 \times 10^{-2}\text{ s}^{-1}$

(a) $\alpha = \frac{d\omega}{dt} = \frac{d(\omega_0 e^{-\sigma t})}{dt} = \omega_0(-\sigma)e^{-\sigma t}$

At $t = 3.00$ s,

$$\alpha = (3.50\text{ rad/s})(-6.02 \times 10^{-2}\text{ s}^{-1})e^{-3.00(6.02 \times 10^{-2})} = \boxed{-0.176\text{ rad/s}^2}$$

(b) $\theta = \int_0^t \omega_0 e^{-\sigma t}\,dt = \frac{\omega_0}{-\sigma}[e^{-\sigma t} - 1] = \frac{\omega_0}{\sigma}[1 - e^{-\sigma t}]$

At $t = 2.50$ s,

$$\theta = \frac{3.50\text{ rad/s}}{(6.02 \times 10^{-2})1/\text{s}}[1 - e^{-(6.02 \times 10^{-2})(2.50)}] = 8.12\text{ rad} = \boxed{1.29\text{ rev}}$$

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(c) As $t \to \infty$, $\theta \to \frac{\omega_0}{\sigma}(1 - e^{-\infty}) = \frac{3.50 \text{ rad/s}}{6.02 \times 10^{-2} \text{ s}^{-1}} = 58.2 \text{ rad} = \boxed{9.26 \text{ rev}}$

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10.74 Consider the total weight of each hand to act at the center of gravity (mid-point) of that hand. Then the total torque (taking CCW as positive) of these hands about the center of the clock is given by

$$\tau = -m_h g\left(\frac{L_h}{2}\right)\sin\theta_h - m_m g\left(\frac{L_m}{2}\right)\sin\theta_m = \frac{-g}{2}(m_h L_h \sin\theta_h + m_m L_m \sin\theta_m)$$

If we take $t = 0$ at 12 o'clock, then the angular positions of the hands at time t are

$$\theta_h = \omega_h t,\ \text{where } \omega_h = \frac{\pi}{6}\ \text{rad/h and } \theta_m = \omega_m t,\ \text{where } \omega_m = 2\pi\ \text{rad/h}$$

Therefore,

$$\tau = -\left(4.90\ \frac{\text{m}}{\text{s}^2}\right)[(60.0\ \text{kg})(2.70\ \text{m})\sin(\pi t/6) + (100\ \text{kg})(4.50\ \text{m})\sin 2\pi t]$$

or $\tau = -794\ \text{N}\cdot\text{m}[\sin(\pi t/6) + 2.78\sin 2\pi t]$, where t is in hours.

(a) (i) At 3:00, $t = 3.00$ h, so

$$\tau = -794\ \text{N}\cdot\text{m}\ [\sin(\pi/2) + 2.78\sin 6\pi] = \boxed{-794\ \text{N}\cdot\text{m}}$$

(ii) At 5:15, $t = 5\ \text{h} + \frac{15}{60}\ \text{h} = 5.25\ \text{h}$, and substitution gives:

$$\tau = \boxed{-2510\ \text{N}\cdot\text{m}}$$

(iii) At 6:00, $\tau = \boxed{0\ \text{N}\cdot\text{m}}$

(iv) At 8:20, $\tau = \boxed{-1160\ \text{N}\cdot\text{m}}$

(v) At 9:45, $\tau = \boxed{-2940\ \text{N}\cdot\text{m}}$

(b) The total torque is zero at those times when

$$\sin(\pi t/6) + 2.78\sin 2\pi t = 0$$

We proceed numerically, to find 0, 0.5152955, ..., corresponding to the times

12:00:00	12:30:55	12:58:19	1:32:31	1:57:01
2:33:25	2:56:29	3:33:22	3:56:55	4:32:24
4:58:14	5:30:52	6:00:00	6:29:08	7:01:46
7:27:36	8:03:05	8:26:38	9:03:31	9:26:35
10:02:59	10:27:29	11:01:41	11:29:05	

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