Chapter 6 Solutions

6.1 (a) Average speed $= \bar{v} = \frac{200 \text{ m}}{25.0 \text{ s}} = \boxed{8.00 \text{ m/s}}$

(b) $F = \frac{mv^2}{r}$ where $r = \frac{200 \text{ m}}{2\pi} = 31.8 \text{ m}$

$$F = \frac{(1.50 \text{ kg})(8.00 \text{ m/s})^2}{31.8 \text{ m}} = \boxed{3.02 \text{ N}}$$

6.2 (a) $\Sigma F_x = ma_x$

$$T = \frac{mv^2}{r} = \frac{55.0 \text{ kg } (4.00 \text{ m/s})^2}{0.800 \text{ m}} = \boxed{1100 \text{ N}}$$

(b) The tension is $\boxed{\text{larger}}$ than her weight by

$$\frac{1100 \text{ N}}{(55.0 \text{ kg})(9.80 \text{ m/s}^2)} = \boxed{2.04 \text{ times}}$$

6.3 $m = 3.00$ kg: $r = 0.800$ m. The string will break if the tension exceeds the weight corresponding to 25.0 kg, so

$$T_{\text{max}} = Mg = 25.0 \times 9.80 = 245 \text{ N}$$

When the 3.00 kg mass rotates in a horizontal circle, the tension provides the centripetal force, so

$$T = \frac{mv^2}{r} = \frac{(3.00)v^2}{0.800}$$

Then $v^2 = \frac{rT}{m} = \frac{0.800T}{3.00} \le \frac{(0.800T_{\text{max}})}{3.00} = \frac{0.800 \times 245}{3.00} = 65.3 \text{ m}^2/\text{s}^2$

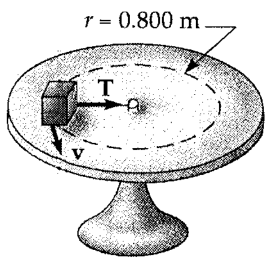
and $0 < v < \sqrt{65.3}$ or $\boxed{0 < v < 8.08 \text{ m/s}}$

Goal Solution

The string will break if the tension T exceeds the test weight it can support,

$$T_{\max} = mg = (25.0\text{ kg})(9.80\text{ m}/\text{s}^2) = 245\text{ N}$$

As the 3.00-kg mass rotates in a horizontal circle, the tension provides the central force.



From $\sum F = ma$, $T = \dfrac{mv^2}{r}$

Then, $v \le \sqrt{\dfrac{rT_{\max}}{m}} = \sqrt{\dfrac{(0.800\text{ m})(245\text{ N})}{(3.00\text{ kg})}} = 8.08\text{ m/s}$

So the mass can have speeds between 0 and 8.08 m/s. ◊

6.4 (a) $F = \dfrac{mv^2}{r} = \dfrac{(9.11\times10^{-31}\text{ kg})(2.20\times10^{6}\text{ m/s})^2}{0.530\times10^{-10}\text{ m}} = \boxed{8.32\times10^{-8}\text{ N}}$ inward

(b) $a = \dfrac{v^2}{r} = \dfrac{(2.20\times10^{6}\text{ m/s})^2}{0.530\times10^{-10}\text{ m}} = \boxed{9.13\times10^{22}\text{ m/s}^2}$ inward

6.5 Neglecting relativistic effects. $F = ma_c = \dfrac{mv^2}{r}$

$$F = (2\times1.661\times10^{-27}\text{ kg})\,\frac{(2.998\times10^{7}\text{ m/s})^2}{(0.480\text{ m})} = \boxed{6.22\times10^{-12}\text{ N}}$$

6.6 (a) We require that $\dfrac{GmM_e}{r^2} = \dfrac{mv^2}{r}$ but $g = \dfrac{M_eG}{R_e^2}$

In this case $r = 2R_e$, therefore, $\dfrac{g}{4} = \dfrac{v^2}{2R_e}$ or $v = \sqrt{\dfrac{gR_e}{2}}$

$$v = \sqrt{\frac{(9.80\text{ m/s}^2)(6.37\times10^{6}\text{ m})}{2}} = \boxed{5.59\times10^{3}\text{ m/s}}$$

(b) $T = \dfrac{2\pi r}{v} = \dfrac{(2\pi)(2)(6.37\times10^{6}\text{ m})}{5.59\times10^{3}\text{ m/s}} = \boxed{239\text{ min}}$

(c) $F = \dfrac{GmM_e}{(2R_e)^2} = \dfrac{mg}{4} = \dfrac{(300\text{ kg})(9.80\text{ m/s}^2)}{4} = \boxed{735\text{ N}}$

6.7 The orbit radius is $r = 1.70\times10^{6}\text{ m} + 100\text{ km} = 1.80\times10^{6}\text{ m}$. Now using the information in Example 6.6,

$$\frac{GM_m m_s}{r^2} = \frac{m_s 2^2\pi^2 r^2}{rT^2} = m_s a$$

(a) $a = \dfrac{GM_m}{r^2} = \dfrac{(6.67\times10^{-11})(7.40\times10^{22})}{(1.80\times10^{6}\text{ m})^2} = \boxed{1.52\text{ m/s}^2}$

(b) $a = \dfrac{v^2}{r}$, $v = \sqrt{(1.52\text{ m/s}^2)(1.80\times10^{6}\text{ m})} = \boxed{1.66\text{ km/s}}$

(c) $v = \dfrac{2\pi r}{T}$, $T = \dfrac{2\pi(1.80\times10^{6})}{1.66\times10^{3}} = \boxed{6820\text{ s}}$

6.8 (a) Speed = distance / time. If the radius of the hand of the clock is r then

$$v = \frac{2\pi r}{T} \Rightarrow vT = 2\pi r$$

$$r_m = r_s \therefore\ T_m v_m = T_s v_s$$

where $v_m = 1.75 \times 10^{-3}$ m / s, $T_m = (60.0 \times 60.0)$s and $T_s = 60.0$ s.

$$v_s = \left(\frac{T_m}{T_s}\right) v_m = \left(\frac{3.60 \times 10^3 \text{ s}}{60.0 \text{ s}}\right)(1.75 \times 10^{-3} \text{ m/s}) = \boxed{0.105 \text{ m/s}}$$

(b) $v = \frac{2\pi r}{T}$ for the second hand, $r = \frac{vT}{2\pi} = \frac{(0.105 \text{ m/s})(60.0 \text{ s})}{2\pi} = 1.00 \text{ m}$

Then $a_r = \frac{v^2}{r} = \frac{(0.105 \text{ m/s})^2}{1.00 \text{ m}} = \boxed{1.10 \times 10^{-2} \text{ m/s}^2}$

6.9 (a) $\boxed{\text{static friction}}$

(b) $ma\,\mathbf{i} = f\,\mathbf{i} + n\,\mathbf{j} + mg(-\mathbf{j})$

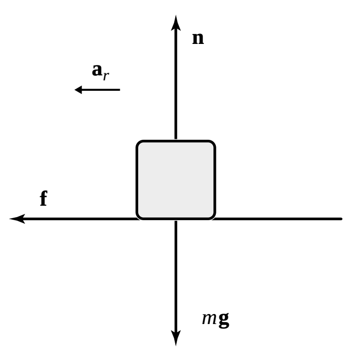
$\Sigma F_y = 0 = n - mg$

thus $n = mg$ and $\Sigma F_r = m\frac{v^2}{r} = f = \mu n = \mu mg$

Then $\mu = \frac{v^2}{rg} = \frac{(50.0 \text{ cm/s})^2}{(30.0 \text{ cm})(980 \text{ cm/s}^2)} = \boxed{0.0850}$

***6.10** $$a = \frac{v^2}{r} = \frac{\left[\left(86.5\,\frac{\text{km}}{\text{h}}\right)\left(\frac{1 \text{ h}}{3600 \text{ s}}\right)\left(\frac{1000 \text{ m}}{1 \text{ km}}\right)\right]^2}{61.0 \text{ m}}\left(\frac{1\,g}{9.80 \text{ m/s}^2}\right) = \boxed{0.966g}$$

6.11 $n = mg$ since $a_y = 0$



The centripetal force is the frictional force f.

From Newton's second law

$$f = ma_r = \frac{mv^2}{r}$$

But the friction condition is

$$f \le \mu_s n$$

i.e., $\dfrac{mv^2}{r} \le \mu_s mg$

$$v \le \sqrt{\mu_s rg} = \sqrt{(0.600)(35.0\text{ m})(9.80\text{ m/s}^2)}$$

$$v \le \boxed{14.3\text{ m/s}}$$

6.12 (b) $v = \dfrac{235\text{ m}}{36.0\text{ s}} = \boxed{6.53\text{ m/s}}$

The radius is given by $\frac{1}{4}\,2\pi r = 235\text{ m}$

$$r = 150\text{ m}$$

(a) $\mathbf{a}_r = \left(\dfrac{v^2}{r}\right)$ toward center

$$= \frac{(6.53\text{ m/s})^2}{150\text{ m}} \text{ at } 35.0° \text{ north of west}$$

$$= (0.285\text{ m/s}^2)(\cos 35.0°(-\mathbf{i}) + \sin 35.0°\,\mathbf{j})$$

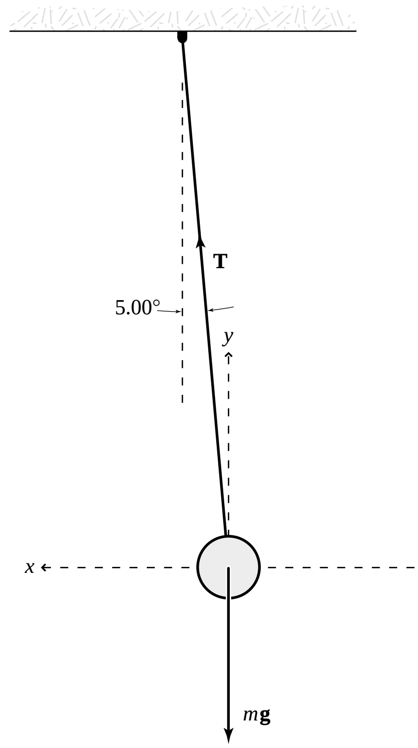
$$= \boxed{-0.233\text{ m/s}^2\,\mathbf{i} + 0.163\text{ m/s}^2\,\mathbf{j}}$$

(c) $\bar{\mathbf{a}} = \dfrac{(\mathbf{v}_f - \mathbf{v}_i)}{t}$

$$= \frac{(6.53\text{ m/s}\,\mathbf{j} - 6.53\text{ m/s}\,\mathbf{i})}{36.0\text{ s}}$$

$$= \boxed{-0.181\text{ m/s}^2\,\mathbf{i} + 0.181\text{ m/s}^2\,\mathbf{j}}$$

6.13 $T\cos 5.00° = mg = (80.0 \text{ kg})(9.80 \text{ m/s}^2)$



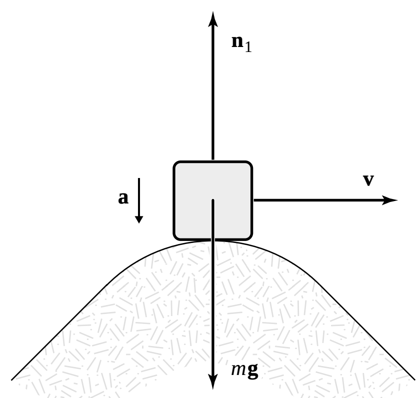
(a) $T = 787$ N

$T = \boxed{(68.6 \text{ N})\mathbf{i} + (784 \text{ N})\mathbf{j}}$

(b) $T\sin 5.00° = ma_r$

$\boxed{a_r = 0.857 \text{ m/s}^2}$

6.14 (a) The reaction force n_1 represents the apparent weight of the woman



$F = ma$

i.e., $mg - n_1 = \dfrac{mv^2}{r}$, so $n_1 = mg - \dfrac{mv^2}{r}$

$$n_1 = 600 - \left(\frac{600}{9.80}\right)\frac{(9.00)^2}{11.0} = \boxed{149 \text{ N}}$$

(b) If $n_1 = 0$, $mg = \dfrac{mv^2}{r}$

This gives $v = \sqrt{rg} = \sqrt{(11.0 \text{ m})(9.80 \text{ m/s}^2)} = \boxed{10.4 \text{ m/s}}$

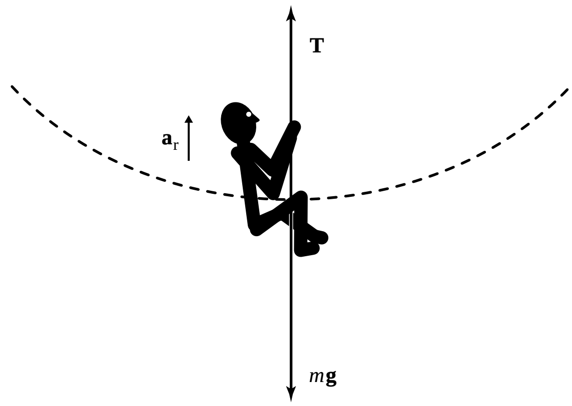
6.15 Let the tension at the lowest point be T.

$$F = ma$$

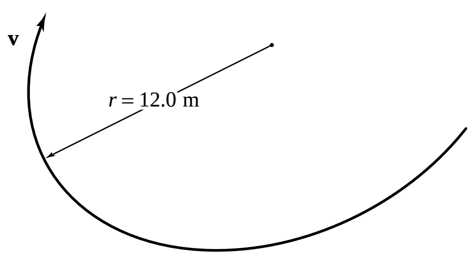
$$T - mg = ma_r = \frac{mv^2}{r}$$

$$T = m\left(g + \frac{v^2}{r}\right) = (85.0 \text{ kg})\left[9.80 \text{ m/s}^2 + \frac{(8.00 \text{ m/s})^2}{10.0 \text{ m}}\right] = 1.38 \text{ kN} > 1000 \text{ N}$$

He doesn't make it across the river because the vine breaks.



6.16 (a) $a_r = \frac{v^2}{r} = \frac{(4.00 \text{ m/s})^2}{12.0 \text{ m}} = \boxed{1.33 \text{ m/s}^2}$



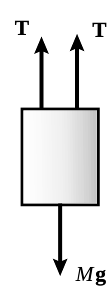
(b) $a = \sqrt{a_r^2 + a_T^2}$

$$a = \sqrt{(1.33)^2 + (1.20)^2} = \boxed{1.79 \text{ m/s}^2}$$

at an angle $\theta = \tan^{-1}\left(\frac{a_r}{a_T}\right) = \boxed{47.9° \text{ inward}}$

6.17 $M = 40.0$ kg, $R = 3.00$ m, $T = 350$ N

(a) $\Sigma F = 2T - Mg = \frac{Mv^2}{R}$



child + seat

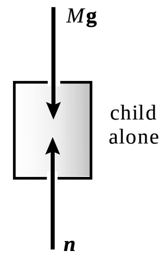
$$v^2 = (2T - Mg)\left(\frac{R}{M}\right)$$

$$v^2 = [700 - (40.0)(9.80)]\left(\frac{3.00}{40.0}\right) = 23.1(\text{m}^2/\text{s}^2)$$

$\boxed{v = 4.81 \text{ m/s}}$

(b) $n - Mg = F = \frac{Mv^2}{R}$

$$n = Mg + \frac{Mv^2}{R} = 40.0\left(9.80 + \frac{23.1}{3.00}\right) = \boxed{700\text{ N}}$$



Goal Solution

G: If the tension in each chain is 350 N at the lowest point, then the force of the seat on the child should just be twice this force or 700 N. The child's speed is not as easy to determine, but somewhere between 0 and 10 m/s would be reasonable for the situation described.

O: We should first draw a free body diagram that shows the forces acting on the seat and apply Newton's laws to solve the problem.

A: We can see from the diagram that the only forces acting on the system of child+seat are the tension in the two chains and the weight of the boy:

$\sum F = 2T - mg = ma$ where $a = \frac{v^2}{r}$ is the centripetal acceleration

$F = F_{\text{net}} = 2(350\text{ N}) - (40.0\text{ kg})(9.80\text{ m/s}^2) = 308\text{ N}$ upwards

$$v = \sqrt{\frac{F_{\max} r}{m}} = \sqrt{\frac{(308\text{ N})(3.00\text{ m})}{40.0\text{ kg}}} = 4.81\text{ m/s} \;\Diamond$$

The child feels a normal force exerted by the seat equal to the total tension in the chains.
$\mathbf{n} = 2(350\text{ N}) = 700\text{ N}$ (upwards) ◊

L: Our answers agree with our predictions. It may seem strange that there is a net upward force on the boy yet he does not move upwards. We must remember that a net force causes an acceleration, but not necessarily a motion in the direction of the force. In this case, the acceleration is due to a change in the direction of the motion. It is also interesting to note that the boy feels about twice as heavy as normal, so he is experiencing an acceleration of about 2g's.

6.18 (a) Consider the forces acting on the system consisting of the child <u>and</u> the seat:

$$\sum F_y = ma_y \Rightarrow 2T - mg = m\frac{v^2}{R}$$

$$v^2 = R\left(\frac{2T}{m} - g\right)$$

$$v = \boxed{\sqrt{R\left(\frac{2T}{m} - g\right)}}$$

(b) Consider the forces acting on the child alone:

$$\Sigma F_y = ma_y \Rightarrow n = m\left(g + \frac{v^2}{R}\right)$$

and from above, $v^2 = R\left(\frac{2T}{m} - g\right)$, so

$$n = m\left(g + \frac{2T}{m} - g\right) = \boxed{2T}$$

6.19 $\Sigma F_y = \frac{mv^2}{r} = mg + n$

But $n = 0$ at this minimum speed condition, so

$$\frac{mv^2}{r} = mg \Rightarrow v = \sqrt{gr} = \sqrt{(9.80 \text{ m/s}^2)(1.00 \text{ m})} = \boxed{3.13 \text{ m/s}}$$

6.20 At the top of the vertical circle,

$$T = m\frac{v^2}{R} - mg$$

or $T = (0.400)\frac{(4.00)^2}{0.500} - (0.400)(9.80) = \boxed{8.88 \text{ N}}$

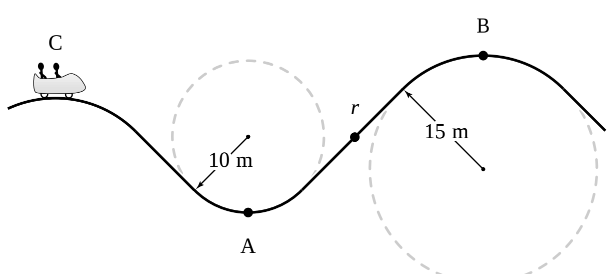
6.21 (a) $v = 20.0$ m/s, n = force of track on roller coaster, and $R = 10.0$ m.

$$\Sigma F = \frac{Mv^2}{R} = n - Mg$$

From this we find

$$n = Mg + \frac{Mv^2}{R} = (500 \text{ kg})(9.80 \text{ m/s}^2) + \frac{(500 \text{ kg})(20.0 \text{ m/s}^2)}{10.0 \text{ m}}$$

$$n = 4900 \text{ N} + 20{,}000 \text{ N} = \boxed{2.49 \times 10^4 \text{ N}}$$



(b) At B, $n - Mg = -\dfrac{Mv^2}{R}$

The max speed at B corresponds to

$$n = 0$$

$$-Mg = -\frac{Mv_{\max}^2}{R} \Rightarrow v_{\max} = \sqrt{Rg} = \sqrt{15.0(9.80)} = \boxed{12.1\ \text{m/s}}$$

6.22 (a) $a_r = \dfrac{v^2}{r}$

$$r = \frac{v^2}{a_r} = \frac{(13.0\ \text{m/s})^2}{2(9.80\ \text{m/s}^2)} = \boxed{8.62\ \text{m}}$$

(b) Let n be the force exerted by rail.

Newton's law gives $Mg + n = \dfrac{Mv^2}{r}$

$$n = M\left(\frac{v^2}{r} - g\right) = M(2g - g) = \boxed{Mg\text{, downward}}$$

(c) $a_r = \dfrac{v^2}{r} = \dfrac{(13.0\ \text{m/s})^2}{20.0\ \text{m}} = \boxed{8.45\ \text{m/s}^2}$

If the force by the rail is n_1, then

$$n_1 + Mg = \frac{Mv^2}{r} = Ma_r$$

$n_1 = M(a_r - g)$ which is < 0,

since $a_r = 8.45\ \text{m/s}^2$

Thus, the normal force would have to point away from the center of the curve. Unless they have belts, the riders will fall from the cars. To be safe we must require n_1 to be positive. Then $a_r > g$. We need

$$\frac{v^2}{r} > g \quad \text{or} \quad v > \sqrt{rg} = \sqrt{(20.0\ \text{m})(9.80\ \text{m/s}^2)}$$

$$v > 14.0\ \text{m/s}$$

6.23 $v = \dfrac{2\pi r}{T} = \dfrac{2\pi(3.00\ \text{m})}{(12.0\ \text{s})} = 1.57\ \text{m/s}$

(a) $a = \dfrac{v^2}{r} = \dfrac{(1.57\ \text{m/s})^2}{(3.00\ \text{m})} = \boxed{0.822\ \text{m/s}^2}$

(b) For no sliding motion,

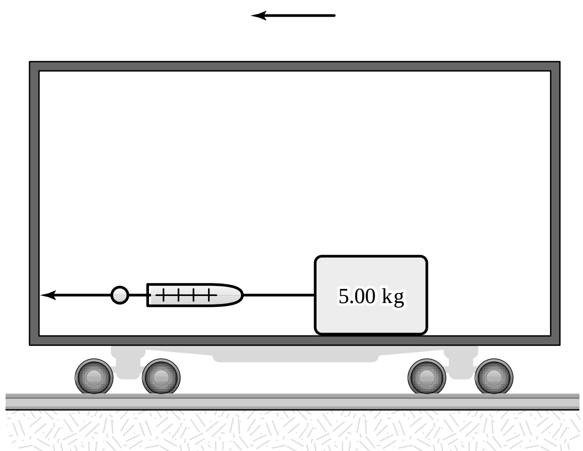
$$f_f = ma = (45.0\ \text{kg})(0.822\ \text{m/s}^2) = \boxed{37.0\ \text{N}}$$

(c) $f_f = \mu mg,\ \mu = \dfrac{37.0\ \text{N}}{(45.0\ \text{kg})(9.80\ \text{m/s}^2)} = \boxed{0.0839}$

6.24 (a) $\Sigma F_x = Ma,\ a = \dfrac{T}{M} = \dfrac{18.0\ \text{N}}{5.00\ \text{kg}} = \boxed{3.60\ \text{m/s}^2}$ to the right.

(b) If v = const, $a = 0$, so $\boxed{T = 0}$ (This is also an equilibrium situation.)

(c) Someone in the car (noninertial observer) claims that the forces on the mass along x are T and a fictitious force ($-Ma$). Someone at rest outside the car (inertial observer) claims that T is the only force on M in the x-direction.



6.25 $\sum F_x = T\sin\theta = ma_x$ (1)

$\sum F_x = T\cos\theta - mg = 0$

or $T\cos\theta = mg$ (2)

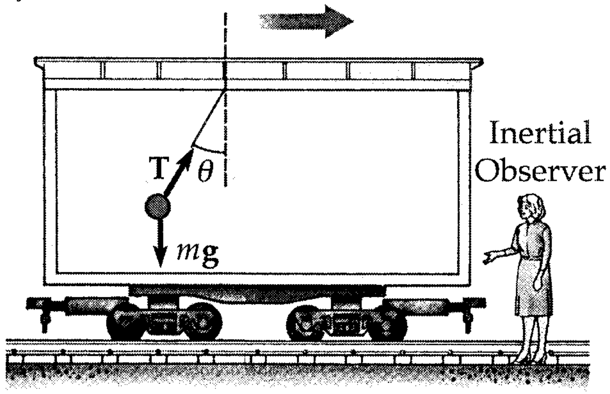
(a) Dividing (1) by (2) gives $\tan\theta = \dfrac{a_x}{g}$

$$\theta = \tan^{-1}\left(\frac{a_x}{g}\right) = \tan^{-1}\left(\frac{3.00}{9.80}\right) = \boxed{17.0°}$$

(b) From (1), $T = \dfrac{ma_x}{\sin\theta} = \dfrac{(0.500\ \text{kg})(3.00\ \text{m/s}^2)}{\sin 17.0°} = \boxed{5.12\ \text{N}}$

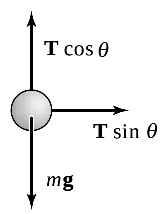
Goal Solution

G: If the horizontal acceleration were zero, then the angle would be 0, and if $a = g$, then the angle would be 45°, but since the acceleration is 3.00 $\mathrm{m/s^2}$, a reasonable estimate of the angle is about 20°. Similarly, the tension in the string should be slightly more than the weight of the object, which is about 5 N.



O: We will apply Newton's second law to solve the problem.

A: The only forces acting on the suspended object are the force of gravity $m\mathbf{g}$ and the force of tension **T**, as shown in the free-body diagram. Applying Newton's second law in the x and y directions,



$$\sum F_x = T \sin\theta = ma \quad (1)$$

$$\sum F_y = T\cos\theta - mg = 0$$

or $T\cos\theta = mg$ (2)

(a) Dividing equation (1) by (2) gives

$$\tan\theta = \frac{a}{g} = \frac{3.00\ \mathrm{m/s^2}}{9.80\ \mathrm{m/s^2}} = 0.306$$

Solving for θ, $\theta = 17.0°$

(b) From Equation (1),

$$T = \frac{ma}{\sin\theta} = \frac{(0.500\ \mathrm{kg})(3.00\ \mathrm{m/s^2})}{\sin(17.0°)} = 5.12\ \mathrm{N}$$

L: Our answers agree with our original estimates. This problem is very similar to Prob. 5.30, so the same concept seems to apply to various situations.

6.26 (a) $\sum F_r = ma_r$

$$mg = \frac{mv^2}{R} = \frac{m}{R}\left(\frac{2\pi R}{T}\right)^2$$

$$g = \frac{4\pi^2 R}{T^2}$$

$$T = \sqrt{\frac{4\pi^2 R}{g}} = 2\pi\sqrt{\frac{6.37\times 10^6\text{ m}}{9.80\text{ m/s}^2}} = 5.07\times 10^3 = \boxed{1.41\text{ h}}$$

(b) $$\text{speed increase factor} = \frac{v_{\text{new}}}{v_{\text{current}}} = \frac{\dfrac{2\pi R}{T_{\text{new}}}}{\dfrac{2\pi R}{T_{\text{current}}}} = \frac{T_{\text{current}}}{T_{\text{new}}} = \frac{24.0\text{ h}}{1.41\text{ h}} = \boxed{17.1}$$

6.27 $F_{\text{max}} = F_g + ma = 591\text{ N}$

$F_{\text{min}} = F_g - ma = 391\text{ N}$

(a) Adding, $2F_g = 982\text{ N}$, $F_g = \boxed{491\text{ N}}$

(b) Since $F_g = mg$, $m = \dfrac{491\text{ N}}{9.80\text{ m/s}^2} = \boxed{50.1\text{ kg}}$

(c) Subtracting the above equations,

$2ma = 200\text{ N}$ $\therefore\ a = \boxed{2.00\text{ m/s}^2}$

***6.28** In an inertial reference frame, the girl is accelerating horizontally inward at

$$\frac{v^2}{r} = \frac{(5.70\text{ m/s})^2}{2.40\text{ m}} = 13.5\text{ m/s}^2$$

In her own non-inertial frame, her head feels a horizontally outward fictitious force equal to its mass times this acceleration. Together this force and the weight of her head add to have a magnitude equal to the mass of her head times an acceleration of

$$\sqrt{g^2 + (v^2/r)^2} = \sqrt{(9.80)^2 + (13.5)^2}\text{ m/s}^2 = 16.7\text{ m/s}^2$$

This is larger than g by a factor of $\dfrac{16.7}{9.80} = 1.71$.

Thus, the force required to lift her head is larger by this factor, or the required force is

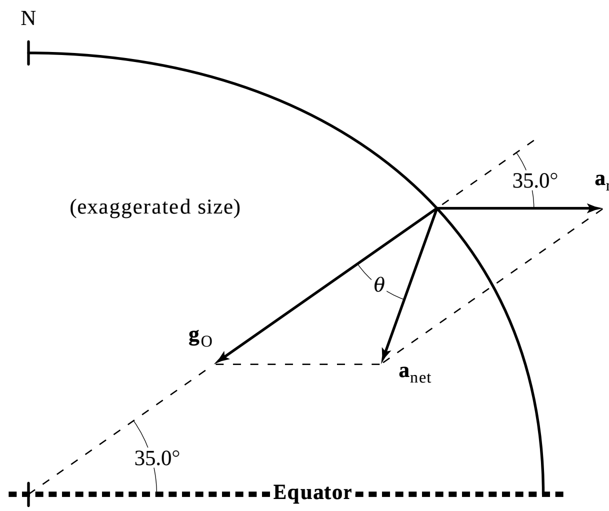
$$F = 1.71(55.0\text{ N}) = \boxed{93.8\text{ N}}$$

6.29 $a_r = \left(\dfrac{4\pi^2 R_e}{T^2}\right)\cos 35.0° = 0.0276 \text{ m/s}^2$

$(a_{\text{net}})_y = 9.80 - (a_r)_y = 9.78 \text{ m/s}^2$

$(a_{\text{net}})_x = 0.0158 \text{ m/s}^2$

$\theta = \arctan \dfrac{a_x}{a_y} = \boxed{0.0927°}$



***6.30** $m = 80.0$ kg, $v_T = 50.0$ m/s, $mg = \dfrac{D\rho A v_T^2}{2}$ $\therefore \dfrac{D\rho A}{2} = \dfrac{mg}{v_T^2} = 0.314$

(a) At $v = 30.0$ m/s

$$a = g - \frac{D\rho A v^2/2}{m} = 9.80 - \frac{(0.314)(30.0)^2}{80.0} = \boxed{6.27 \text{ m/s}^2 \text{ downward}}$$

(b) At $v = 50.0$ m/s, terminal velocity has been reached.

$$\Sigma F_y = 0 = mg - R$$

$$\Rightarrow R = mg = (80.0 \text{ kg})(9.80 \text{ m/s}^2) = \boxed{784 \text{ N directed up}}$$

(c) At $v = 30.0$ m/s

$$\frac{D\rho A v^2}{2} = (0.314)(30.0)^2 = \boxed{283 \text{ N}} \text{ upward}$$

6.31 (a) $a = g - bv$

When $v = v_T$, $a = 0$ and $g = bv_T$.

$$b = \frac{g}{v_T}$$

The Styrofoam falls 1.50 m at constant speed v_T in 5.00 s.

Thus, $v_T = \dfrac{y}{t} = \dfrac{1.50 \text{ m}}{5.00 \text{ s}} = 0.300$ m/s

Then $b = \dfrac{9.80 \text{ m/s}^2}{0.300 \text{ m/s}} = \boxed{32.7 \text{ s}^{-1}}$

(b) At $t = 0$, $v = 0$ and $a = g = \boxed{9.80 \text{ m/s}^2}$ down

(c) When $v = 0.150$ m/s,

$$a = g - bv = 9.80 \text{ m/s}^2 - (32.7 \text{ s}^{-1})(0.150 \text{ m/s}) = \boxed{4.90 \text{ m/s}^2} \text{ down}$$

***6.32** (a) $\rho = \frac{m}{V}$; $A = 0.0201 \text{ m}^2$; $R = \frac{1}{2}\rho A D v_t^2 = mg$

$$m = \rho V = (0.830 \text{ g/cm}^3)\left[\frac{4}{3}\pi(8.00 \text{ cm})^3\right] = 1.78 \text{ kg}$$

Assuming a drag coefficient of $D = 0.500$ for this spherical object,

$$v_t = \sqrt{\frac{2(1.78 \text{ kg})(9.80 \text{ m/s}^2)}{0.500(1.20 \text{ kg/m}^3)(0.0201 \text{ m}^2)}} = \boxed{53.8 \text{ m/s}}$$

(b) $v_f^2 = v_i^2 + 2gh = 0 + 2gh$

$$h = \frac{v_f^2}{2g} = \frac{(53.8 \text{ m/s})^2}{2(9.80 \text{ m/s}^2)} = \boxed{148 \text{ m}}$$

***6.33** Since the upward velocity is constant, the resultant force on the ball is zero. Thus, the upward applied force equals the sum of the gravitational and drag forces (both downward): $F = mg + bv$.

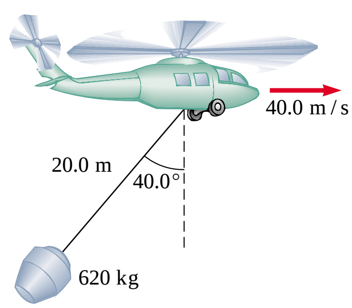
The mass of the copper ball is

$$m = \frac{4\pi\rho r^3}{3} = \left(\frac{4}{3}\right)\pi\left(8.92 \times 10^3 \frac{\text{kg}}{\text{m}^3}\right)(2.00 \times 10^{-2} \text{ m})^3 = 0.299 \text{ kg}$$

The applied force is then

$$F = mg + bv = (0.299)(9.80) + (0.950)(9.00 \times 10^{-2}) = \boxed{3.01 \text{ N}}$$

6.34 $\Sigma F_y = ma_y$



$+T\cos 40.0° - mg = 0$

$$T = \frac{(620 \text{ kg})(9.80 \text{ m/s}^2)}{\cos 40.0°} = 7.93 \times 10^3 \text{ N}$$

$\Sigma F_x = ma_x$

$-R + T\sin 40.0° = 0$

$$R = (7.93 \times 10^3 \text{ N})\sin 40.0° = 5.10 \times 10^3 \text{ N} = \frac{1}{2} D\rho A v^2$$

$$D = \frac{2R}{\rho A v^2} = \frac{2(5.10 \times 10^3 \text{ N})(\text{kg m/s}^2/\text{N})}{(1.20 \text{ kg/m}^2)3.80 \text{ m}^2 (40.0 \text{ m/s})^2} = \boxed{1.40}$$

6.35 (a) At terminal velocity,

$R = v_t b = mg$

$$\therefore b = \frac{mg}{v_t} = \frac{(3.00 \times 10^{-3}\ \text{kg})(9.80\ \text{m/s}^2)}{(2.00 \times 10^{-2}\ \text{m/s})} = \boxed{1.47\ \text{N} \cdot \text{s/m}}$$

(b) From Equation 6.5, the velocity on the bead is

$v = v_t\,(1 - e^{-bt/m})$

$v = 0.630\, v_t$ when $e^{-bt/m} = 0.370$

or at time $t = -\left(\frac{m}{b}\right)\ln(0.370) = \boxed{2.04 \times 10^{-3}\ \text{s}}$

(c) At terminal velocity,

$R = v_t b = mg = \boxed{2.94 \times 10^{-2}\ \text{N}}$

***6.36** The resistive force is

$$R = \frac{1}{2} D\rho A v^2 = \frac{1}{2}(0.250)(1.20\ \text{kg/m}^3)(2.20\ \text{m}^2)(27.8\ \text{m/s})^2$$

$R = 255$ N

$\boldsymbol{a} = -R/m = -(255\ \text{N})/(1200\ \text{kg}) = \boxed{-0.212\ \text{m/s}^2}$

6.37 (a) $v(t) = v_i e^{-ct}$

$v(20.0\ \text{s}) = 5.00 = v_i e^{-20.0c}$, $v_i = 10.0$ m/s

So $5.00 = 10.0 e^{-20.0c}$

and $-20.0c = \ln\left(\frac{1}{2}\right)$

$$c = -\frac{\ln\left(\frac{1}{2}\right)}{20.0} = \boxed{3.47 \times 10^{-2}\ \text{s}^{-1}}$$

(b) At $t = 40.0$ s

$$v = (10.0\text{ m/s})e^{-40.0c} = (10.0\text{ m/s})(0.250) = \boxed{2.50\text{ m/s}}$$

(c) $v = v_i e^{-ct}$

$$a = \frac{dv}{dt} = -cv_i e^{-ct} = \boxed{-cv}$$

6.38 $\Sigma F_x = ma_x$

$$-kmv^2 = ma_x = m\frac{dv}{dt}$$

$$-k\int_0^t dt = \int_{v_f}^{v} v^{-2}\,dv$$

$$-k(t-0) = \left.\frac{v^{-1}}{-1}\right|_{v_f}^{v} = -\frac{1}{v} + \frac{1}{v_f}$$

$$v = \boxed{\frac{v_f}{(1 + ktv_f)}}$$

***6.39** In $R = \frac{1}{2}D\rho Av^2$, we estimate that $D = 1.00$, $\rho = 1.20\text{ kg/m}^3$, $A = (0.100\text{ m})(0.160\text{ m}) = 1.60 \times 10^{-2}\text{ m}^2$ and $v = 27.0$ m/s. The resistance force is then

$$R = \frac{1}{2}(1.00)(1.20\text{ kg/m}^3)(1.60 \times 10^{-2}\text{ m}^2)(27.0\text{ m/s})^2 = 7.00\text{ N}$$

or $R \sim \boxed{10^1\text{ N}}$

***6.40** (a) At $v = v_t$, $a = 0$, $-mg - bv_t = 0$

$$v_t = \frac{-mg}{b} = -\frac{(3.00 \times 10^{-3}\text{ kg})(9.80\text{ m/s}^2)}{3.00 \times 10^{-2}\text{ kg/s}} = \boxed{-0.980\text{ m/s}}$$

(b)

t(s)	x(m)	v(m/s)	F(mN)	a(m/s^2)
0	2	0	–29.4	–9.8
0.005	2	–0.049	–27.93	–9.31
0.01	1.999755	–0.09555	–26.534	–8.8445
0.015	1.9993	–0.13977	–25.2	–8.40
. . . we list the result after each tenth iteration				
0.5	1.990	–0.393	–17.6	–5.87
0.1	1.965	–0.629	–10.5	–3.51
0.15	1.930	–0.770	–6.31	–2.10
0.2	1.889	–0.854	–3.78	–1.26
0.25	1.845	–0.904	–2.26	–0.754
0.3	1.799	–0.935	–1.35	–0.451
0.35	1.752	–0.953	–0.811	–0.270
0.4	1.704	–0.964	–0.486	–0.162
0.45	1.65	–0.970	–0.291	–0.0969
0.5	1.61	–0.974	–0.174	–0.0580
0.55	1.56	–0.977	–0.110	–0.0347
0.6	1.51	–0.978	–0.0624	–0.0208
0.65	1.46	–0.979	–0.0374	–0.0125

Terminal velocity is never reached. The leaf is at 99.9% of v_t after 0.67 s. The fall to the ground takes about 2.14 s. Repeating with Δt = 0.001 s, we find the fall takes 2.14 s.

***6.41** (a) When $v = v_t$, $a = 0$, $\Sigma F = -mg + Cv_t^2 = 0$

$$v_t = -\sqrt{\frac{mg}{C}} = -\sqrt{\frac{(4.80 \times 10^{-4}\text{ kg})(9.80\text{ m/s}^2)}{2.50 \times 10^{-5}\text{ kg/m}}} = \boxed{-13.7\text{ m/s}}$$

(b)

t(s)	x(m)	v(m/s)	F(mN)	a(m/s^2)
0	0	0	– 4.704	–9.8
0.2	0	–1.96	– 4.608	–9.5999
0.4	–0.392	–3.88	– 4.3276	–9.0159
0.6	–1.168	–5.6832	–3.8965	–8.1178
0.8	–2.30	–7.3068	–3.3693	–7.0193
1.0	–3.77	–8.7107	–2.8071	–5.8481
1.2	–5.51	–9.8803	–2.2635	–4.7156
1.4	–7.48	–10.823	–1.7753	–3.6986
1.6	–9.65	–11.563	–1.3616	–2.8366
1.8	–11.96	–12.13	–1.03	–2.14
2	–14.4	–12.56	–0.762	–1.59
... listing results after each fifth step				
3	–27.4	–13.49	–0.154	–0.321
4	–41.0	–13.67	–0.0291	–0.0606
5	–54.7	–13.71	–0.00542	–0.0113

The hailstone reaches 99.95% of v_t after 5.0 s, 99.99% of v_t after 6.0 s, 99.999% of v_t after 7.4 s.

6.42 (a) At terminal velocity, $\sum F = 0 = -mg + Cv_t^2$.

$$C = \frac{mg}{v_t^2} = \frac{(0.142\text{ kg})(9.80\text{ m/s}^2)}{(42.5\text{ m/s})^2} = \boxed{7.70 \times 10^{-4}\text{ kg/m}}$$

(b) $Cv^2 = (7.70 \times 10^{-4}\text{ kg/m})(36.0\text{ m/s})^2 = \boxed{0.998\text{ N}}$

(c)

Elapsed Time (s)	Altitude (m)	Speed (m/s)	Resistance Force (N)	Net Force (N)	Acceleration (m/s²)
0.00000	0.00000	36.00000	-0.99849	–2.39009	–16.83158
0.05000	1.75792	35.15842	–0.95235	–2.34395	–16.50667
…					
2.95000	48.62327	0.82494	–0.00052	–1.39212	–9.80369
3.00000	48.64000	0.33476	–0.00009	–1.39169	–9.80061
3.05000	48.63224	–0.15527	0.00002	–1.39158	–9.79987
…					
6.25000	1.25085	–26.85297	0.55555	–0.83605	–5.88769
6.30000	–0.10652	–27.14736	0.56780	–0.82380	–5.80144

Maximum height is about $\boxed{49\text{ m}}$. It returns to the ground after about $\boxed{6.3\text{ s}}$ with a speed of approximately $\boxed{27\text{ m/s}}$.

6.43 (a) At constant velocity $\Sigma F = 0 = -mg + Cv_t^2$

$$v_t = -\sqrt{\frac{mg}{C}} = -\sqrt{\frac{(50.0\text{ kg})(9.80\text{ m/s}^2)}{0.200\text{ kg/m}}} = \boxed{-49.5\text{ m/s}}$$ with chute closed and

$$v_t = -\sqrt{\frac{(50.0\text{ kg})(9.80\text{ m/s})}{20.0\text{ kg/m}}} = \boxed{-4.95\text{ m/s}}$$ with chute open.

(b)

time(s)	height(m)	velocity(m/s)
0	1000	0
1	995	– 9.7
2	980	–18.6
4	929	– 32.7
7	812	– 43.7
10	674	– 47.7
10.1	671	– 16.7
10.3	669	– 8.02
11	665	– 5.09
12	659	– 4.95
50	471	– 4.95
100	224	– 4.95
145	0	– 4.95

6.44 (a)

time(s)	x(m)	y(m)
0	0	0
0.100	7.81	5.43
0.200	14.9	10.2
0.400	27.1	18.3
1.00	51.9	32.7
1.92	70.0	38.5
2.00	70.9	38.5
4.00	80.4	26.7
5.00	81.4	17.7
6.85	81.8	0

(b) range = $\boxed{81.8\text{ m}}$

(c)

with θ	we find range
30.0°	86.410 m
35.0°	81.8 m
25.0°	90.181 m
20.0°	92.874 m
15.0°	93.812 m
10.0°	90.965 m
17.0°	93.732 m
16.0°	93.8398 m
15.5°	93.829 m
15.8°	93.839 m
16.1°	93.838 m
15.9°	93.8402 m

So we have maximum range at $\theta = \boxed{15.9°}$

***6.45** (a) At terminal speed, $\sum F = -mg + Cv^2 = 0$. Thus,

$$C = \frac{mg}{v^2} = \frac{(0.0460\text{ kg})(9.80\text{ m/s}^2)}{(44.0\text{ m/s})^2} = \boxed{2.33 \times 10^{-4}\text{ kg/m}}$$

(b) We set up a spreadsheet to calculate the motion, try different initial speeds, and home in on $\boxed{53\text{ m/s}}$ as that required for horizontal range of 155 m, thus:

Time t (s)	x (m)	v_x (m/s)	a_x (m/s²)	y (m)	v_y (m/s)	a_y (m/s²)	$v=\sqrt{v_x^2+v_y^2}$ (m/s)	$\tan^{-1}(v_y/v_x)$ (deg)
0.0000	0.0000	45.6870	–10.5659	0.0000	27.4515	–13.6146	53.3000	31.0000
0.0027	0.1211	45.6590	–10.5529	0.0727	27.4155	–13.6046	53.2574	30.9822
…								
2.5016	90.1946	28.9375	–4.2388	32.5024	0.0235	–9.8000	28.9375	0.0466
2.5043	90.2713	28.9263	–4.2355	32.5024	–0.0024	–9.8000	28.9263	–0.0048
2.5069	90.3480	28.9150	–4.2322	32.5024	–0.0284	–9.8000	28.9151	–0.0563
…								
3.4238	115.2298	25.4926	–3.2896	28.3972	–8.8905	–9.3999	26.9984	–19.2262
3.4265	115.2974	25.4839	–3.2874	28.3736	–8.9154	–9.3977	26.9984	–19.2822
3.4291	115.3649	25.4751	–3.2851	28.3500	–8.9403	–9.3954	26.9984	–19.3382
…								
5.1516	154.9968	20.8438	–2.1992	0.0059	–23.3087	–7.0498	31.2692	–48.1954
5.1543	155.0520	20.8380	–2.1980	–0.0559	–23.3274	–7.0454	31.2792	–48.2262

(c) Similarly, the initial speed is $\boxed{42\text{ m/s}}$. The motion proceeds thus:

Time t (s)	x (m)	v_x (m/s)	a_x (m/s²)	y (m)	v_y (m/s)	a_y (m/s²)	$v=\sqrt{v_x^2+v_y^2}$ (m/s)	$\tan^{-1}(v_y/v_x)$ (deg)
0.0000	0.0000	28.7462	–4.1829	0.0000	30.8266	–14.6103	42.1500	47.0000
0.0035	0.1006	28.7316	–4.1787	0.1079	30.7754	–14.5943	42.1026	46.9671
…								
2.7405	66.3078	20.5484	–2.1374	39.4854	0.0260	–9.8000	20.5485	0.0725
2.7440	66.3797	20.5410	–2.1358	39.4855	–0.0083	–9.8000	20.5410	–0.0231
2.7475	66.4516	20.5335	–2.1343	39.4855	–0.0426	–9.8000	20.5335	–0.1188
…								
3.1465	74.4805	19.7156	–1.9676	38.6963	–3.9423	–9.7213	20.1058	–11.3077
3.1500	74.5495	19.7087	–1.9662	38.6825	–3.9764	–9.7200	20.1058	–11.4067
3.1535	74.6185	19.7018	–1.9649	38.6686	–4.0104	–9.7186	20.1058	–11.5056
…								
5.6770	118.9697	15.7394	–1.2540	0.0465	–25.2600	–6.5701	29.7623	–58.0731
5.6805	119.0248	15.7350	–1.2533	–0.0419	–25.2830	–6.5642	29.7795	–58.1037

The trajectory in (c) reaches maximum height 39 m, as opposed to 33 m in (b). In both, the ball reaches maximum height when it has covered about 57% of its range. Its speed is a minimum somewhat later. The impact speeds are both about 30 m/s.

6.46 (a) $\Sigma F_y = ma_y = \frac{mv^2}{R}$ down

$$+n - 1800\text{ kg }(9.80\text{ m/s}^2) = \frac{-(1800\text{ kg})(16.0\text{ m/s})^2}{42.0\text{ m}} = -1.10 \times 10^4\text{ N}$$

$$n = \boxed{6.67 \times 10^3\text{ N}}$$

(b) $0 - mg = \frac{-mv^2}{r}$

$$v = \sqrt{gr} = \sqrt{9.80\text{ m/s}^2\,(42.0\text{ m})} = \boxed{20.3\text{ m/s}}$$

6.47 (a) $\Sigma F_y = ma_y = \frac{mv^2}{R}$

$$mg - n = \frac{mv^2}{R}$$

$$n = \boxed{mg - \frac{mv^2}{R}}$$

(b) When $n = 0$, $mg = \frac{mv^2}{R}$

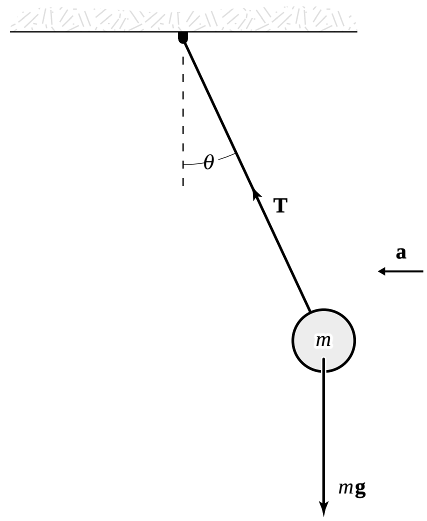
Then, $v = \boxed{\sqrt{gR}}$

6.48 $F = m\frac{v^2}{r}$

$$v = \sqrt{\frac{rF}{m}} = \sqrt{\frac{(5.30 \times 10^{-11}\text{ m})(8.20 \times 10^{-8}\text{ N})}{9.11 \times 10^{-31}\text{ kg}}} = 2.18 \times 10^6\text{ m/s}$$

$$\text{frequency} = (2.18 \times 10^6\text{ m/s})\left[\frac{1\text{ rev}}{2\pi(5.30 \times 10^{-11}\text{ m})}\right] = \boxed{6.56 \times 10^{15}\text{ rev/s}}$$

6.49 (a) While the car negotiates the curve, the accelerometer is at the angle θ.



Horizontally: $T \sin\theta = \dfrac{mv^2}{r}$

Vertically: $T\cos\theta = mg$

where r is the radius of the curve, and v is the speed of the car.

By division $\tan\theta = \dfrac{v^2}{rg}$. Then

$$a_r = \frac{v^2}{r} = g\tan\theta$$

$$a_r = (9.80\ \text{m/s}^2)\tan 15.0°$$

$$a_r = \boxed{2.63\ \text{m/s}^2}$$

(b) $r = \dfrac{v^2}{a_r} = \dfrac{(23.0\ \text{m/s})^2}{2.63\ \text{m/s}^2} = \boxed{201\ \text{m}}$

(c) $v^2 = rg\tan\theta = (201\ \text{m})(9.80\ \text{m/s}^2)\tan 9.00°$

$$v = \boxed{17.7\ \text{m/s}}$$

6.50 Take x-axis up the hill

$$\sum F_x = ma_x$$

$$+T\sin\theta - mg\sin\phi = ma$$

$$a = (T/m)\sin\theta - g\sin\phi$$

$$\sum F_y = ma_y$$

$$+T\cos\theta - mg\cos\phi = 0$$

$$T = mg\cos\phi/\cos\theta$$

$$a = g\cos\phi\sin\theta/\cos\theta - g\sin\phi$$

$$a = \boxed{g(\cos\phi\tan\theta - \sin\phi)}$$

6.51 (a) Since the 1.00 kg mass is in equilibrium, we have for the tension in the string,

$$T = mg = (1.00)(9.80) = \boxed{9.80\ \text{N}}$$

(b) The centripetal force is provided by the tension in the string. Hence,

$$F_r = T = \boxed{9.80\ \text{N}}$$

(c) Using $F_r = \dfrac{m_{\text{puck}} v^2}{r}$, we have $v = \sqrt{\dfrac{rF_r}{m_{\text{puck}}}} = \sqrt{\dfrac{(1.00)(9.80)}{0.250}} = \boxed{6.26\ \text{m/s}}$

6.52 (a) Since the mass m_2 is in equilibrium,

$$\Sigma F_y = T - m_2 g = 0$$

or $T = \boxed{m_2 g}$

(b) The tension in the string provides the required centripetal force for the puck.

Thus, $F_r = T = \boxed{m_2 g}$

(c) From $F_r = \dfrac{m_1 v^2}{R}$, we have $v = \sqrt{\dfrac{RF_r}{m_1}} = \boxed{\sqrt{\left(\dfrac{m_2}{m_1}\right) gR}}$

6.53 (a) Since the centripetal acceleration of a person is downward (toward the axis of the earth), it is equivalent to the effect of a falling elevator.

Therefore, $F_g' = F_g - \dfrac{mv^2}{r}$ or $\boxed{F_g > F_g'}$

(b) At the poles $v = 0$, and $F_g' = F_g = mg = (75.0)(9.80) = \boxed{735\ \text{N}}$ down

At the equator, $F_g' = F_g - ma_r = 735\ \text{N} - (75.0)(0.0337)\ \text{N} = \boxed{732\ \text{N}}$ down

Goal Solution

G: Since the centripetal acceleration is a small fraction (~0.3%) of g, we should expect that a person would have an apparent weight that is just slightly less at the equator than at the poles due to the rotation of the Earth.

O: We will apply Newton's second law and the equation for centripetal acceleration.

A: (a) Let **n** represent the force exerted on the person by a scale, which is the "apparent weight." The true weight is $m\mathbf{g}$. Summing up forces on the object in the direction towards the Earth's center gives

$$mg - n = ma_c \qquad (1)$$

where $a_c = \dfrac{v^2}{R_x} = 0.0337\ \text{m/s}^2$

is the centripetal acceleration directed toward the center of the Earth.

Thus, we see that $n = m(g - a_c) < mg$

or $\quad mg = n + ma_c > n \quad ◊ \qquad (2)$

(b) If $m = 75.0$ kg, $a_c = 0.0337\ \text{m/s}^2$, and $g = 9.800\ \text{m/s}^2$,

at the Equator: $n = m(g - a_c) = (75.0\ \text{kg})(9.800\ \text{m/s}^2 - 0.0337\ \text{m/s}^2) = 732.5\ \text{N}$ ◊

at the Poles: $n = mg = (75.0\ \text{kg})(9.800\ \text{m/s}^2) = 735.0\ \text{N}$ ◊ $\quad (a_c = 0)$

L: As we expected, the person does appear to weigh about 0.3% less at the equator than the poles. We might extend this problem to consider the effect of the earth's bulge on a person's weight. Since the earth is fatter at the equator than the poles, g is less than $9.80\ \text{m/s}^2$ at the equator and slightly more at the poles, but the difference is not as significant as from the centripetal acceleration. (Can you prove this?)

6.54 $\sum F_x = ma_x \Rightarrow T_x = m\dfrac{v^2}{r} = m\dfrac{(20.4\ \text{m/s})^2}{(2.50\ \text{m})} = m\,(166\ \text{m/s}^2)$

$\sum F_y = ma_y \Rightarrow T_y - mg = 0$

or $\quad T_y = mg = m(9.80\ \text{m/s}^2)$

The total tension in the string is

$$T = \sqrt{T_x^2 + T_y^2} = m\sqrt{(166)^2 + (9.80)^2} = 50.0\ \text{N}$$

Thus, $m = \dfrac{50.0\ \text{N}}{\sqrt{(166)^2 + (9.80)^2}\ \text{m/s}^2} = 0.300\ \text{kg}$

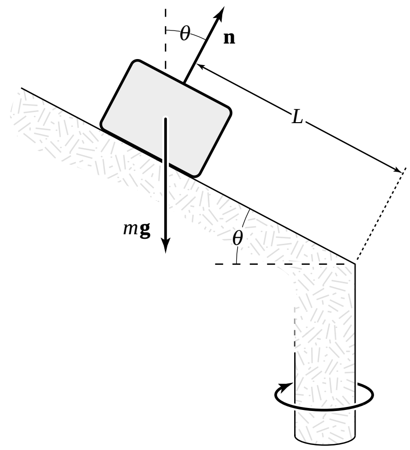
When the string is at the breaking point,

$$T_x = m\frac{v^2}{r} = (0.300\text{ kg})\frac{(51.0\text{ m/s}^2)}{(1.00\text{ m})} = 780\text{ N}$$

and $T_y = mg = (0.300\text{ kg})(9.80\text{ m/s}^2) = 2.94\text{ N}$

Hence, $T = \sqrt{T_x^2 + T_y^2} = \sqrt{(780)^2 + (2.94)^2}\text{ N} = \boxed{780\text{ N}}$

6.55 Let the angle that the wedge makes with the horizontal be θ. The equations for the mass m are



$$mg = n\cos\theta \quad \text{and} \quad n\sin\theta = \frac{mv^2}{r}$$

where $r = L\cos\theta$.

Eliminating n gives $\frac{n\cos\theta}{n\cos\theta} = \tan\theta = \frac{mv^2}{mg\,L\cos\theta}$

Therefore $v^2 = Lg\cos\theta\tan\theta = Lg\sin\theta$

$$\boxed{v = \sqrt{gL\sin\theta}}$$

6.56 (a) $v = 300\text{ mi/h}\left(\frac{88.0\text{ ft/s}}{60.0\text{ mi/h}}\right) = 440\text{ ft/s}$

At the lowest point, his seat exerts an upward force; therefore, his weight seems to increase. His apparent weight is

$$F_g' = mg + m\frac{v^2}{r} = 160 + \left(\frac{160}{32.0}\right)\frac{(440)^2}{1200} = \boxed{967\text{ lb}}$$

(b) At the highest point, the force of the seat on the pilot is directed down and $F_g' = mg - m\frac{v^2}{r} = \boxed{-647\text{ lb}}$

Since the plane is upside down, the seat exerts this downward force.

(c) When $F_g' = 0$, then $mg = \frac{mv^2}{R}$. If we vary the aircraft's R and v such that the above is true, then the pilot feels weightless.

6.57 Call the proportionality constant k:

$$a_r = k/r^2$$

$$v^2/r = k/r^2$$

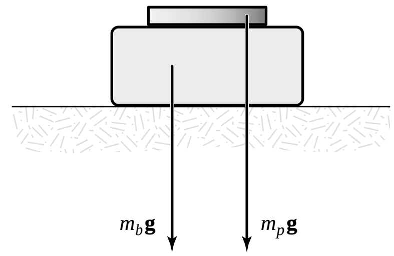
(a) $v = k^{1/2} r^{-1/2}$ so $\boxed{v \propto r^{-1/2}}$

(b) $v = 2\pi r / T = k^{1/2} r^{-1/2}$

$$T = \frac{2\pi r}{(k^{1/2} r^{-1/2})} = \left(\frac{2\pi}{k^{1/2}}\right) r^{3/2}$$

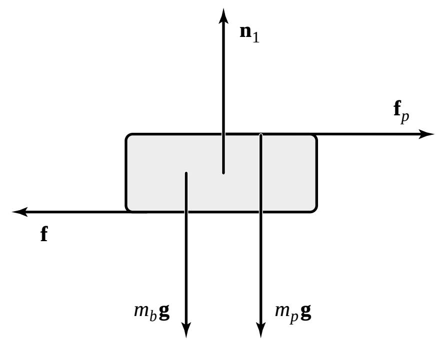
$\boxed{T \propto r^{3/2}}$

6.58 For the block to remain stationary, $\sum F_y = 0$ and $\sum F_x = ma_r$.



$$n_1 = (m_p + m_b)g \quad \text{so} \quad f \le \mu_{s1} n_1 = \mu_{s1}(m_p + m_b)g$$

At the point of slipping, the required centripetal force equals the maximum friction force:



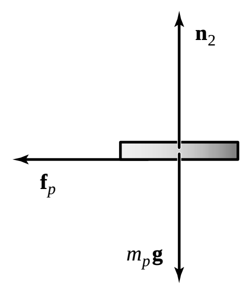
$$\therefore (m_p + m_b)\frac{v_{\max}^2}{r} = \mu_{s1}(m_p + m_b)g$$

or $\quad v_{\max} = \sqrt{\mu_{s1} r g} = \sqrt{(0.750)(0.120)(9.80)} = 0.939 \text{ m/s}$

For the penny to remain stationary on the block:

$$\sum F_y = 0 \Rightarrow n_2 - m_p g = 0 \quad \text{or} \quad n_2 = m_p g$$

and $\quad \sum F_x = ma_r \Rightarrow f_p = m_p \dfrac{v^2}{r}$



When the penny is about to slip on the block, $f_p = f_{p,\max} = \mu_{s2} n_2$

or $\quad \mu_{s2} m_p g = m_p \dfrac{v_{\max}^2}{r}$

$$v_{\max} = \sqrt{\mu_{s2} r g} = \sqrt{(0.520)(0.120)(9.80)} = 0.782 \text{ m/s}$$

This is less than the maximum speed for the block, so the penny slips before the block starts to slip. The maximum rotation frequency is

$$\text{Max rpm} = \frac{v_{\max}}{2\pi r} = (0.782 \text{ m/s})\left[\frac{1 \text{ rev}}{2\pi(0.120 \text{ m})}\right]\left(\frac{60 \text{ s}}{1 \text{ min}}\right) = \boxed{62.2 \text{ rev/min}}$$

6.59 $v = \dfrac{2\pi r}{T} = \dfrac{2\pi(9.00 \text{ m})}{(15.0 \text{ s})} = 3.77 \text{ m/s}$

(a) $a_r = \dfrac{v^2}{r} = \boxed{1.58 \text{ m/s}^2}$

(b) $F_{\text{low}} = m(g + a_r) = \boxed{455 \text{ N}}$

(c) $F_{hi} = m(g - a_r) = \boxed{329 \text{ N}}$

(d) $F_{med} = m\sqrt{g^2 + a_r^2} = \boxed{397 \text{ N}}$ at $\theta = \tan^{-1}\dfrac{a_r}{g} = \dfrac{1.58}{9.80} = \boxed{9.15° \text{ inward}}$

***6.60** Standing on the inner surface of the rim, and moving with it, each person will feel a normal force exerted by the rim. This inward force supplies the centripetal force to cause the 3.00 m / s^2 centripetal acceleration:

$$a_r = \frac{v^2}{r}$$

$$v = \sqrt{a_r r} = \sqrt{(3.00 \text{ m/s}^2)(60.0 \text{ m})} = 13.4 \text{ m/s}$$

The period of rotation comes from $v = \dfrac{2\pi r}{T}$

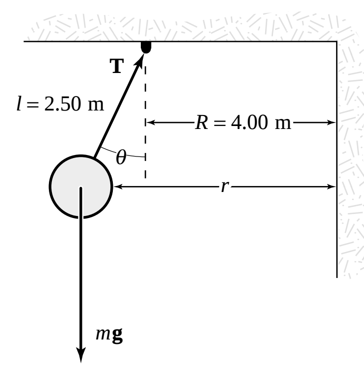
$$T = \frac{2\pi r}{v} = \frac{2\pi(60.0 \text{ m})}{13.4 \text{ m/s}} = 28.1 \text{ s}$$

so the frequency of rotation is

$$f = \frac{1}{T} = \frac{1}{28.1 \text{ s}} = \frac{1}{28.1 \text{ s}}\left(\frac{60 \text{ s}}{1 \text{ min}}\right) = \boxed{2.14 \text{ rev/min}}$$

6.61 (a) The mass at the end of the chain is in vertical equilibrium.

Thus $T\cos\theta = mg$



Horizontally $T\sin\theta = ma_r = \dfrac{mv^2}{r}$

$$r = (2.50 \sin\theta + 4.00) \text{ m}$$

$$r = (2.50 \sin 28.0° + 4.00) \text{ m} = 5.17 \text{ m}$$

Then $a_r = \dfrac{v^2}{5.17 \text{ m}}$.

By division $\tan\theta = \dfrac{a_r}{g} = \dfrac{v^2}{5.17g}$

$$v^2 = 5.17g\tan\theta = (5.17)(9.80)(\tan 28.0°) \text{ m}^2/\text{s}^2$$

$$v = \boxed{5.19 \text{ m/s}}$$

(b) $T\cos\theta = mg$

$$T = \frac{mg}{\cos\theta} = \frac{(50.0 \text{ kg})(9.80 \text{ m/s}^2)}{\cos 28.0°} = \boxed{555 \text{ N}}$$

6.62 (a) The putty, when dislodged, rises and returns to the original level in time t. To find t, we use $v_f = v_i + at$: i.e., $-v = +v - gt$ or $t = \dfrac{2v}{g}$ where v is the speed of a point on the rim of the wheel.

If R is the radius of the wheel, $v = \dfrac{2\pi R}{t}$, so $t = \dfrac{2v}{g} = \dfrac{2\pi R}{v}$

Thus, $v^2 = \pi Rg$ and $\boxed{v = \sqrt{\pi Rg}}$

(b) The putty is dislodged when F, the force holding it to the wheel is

$$F = \frac{mv^2}{R} = \boxed{m\pi g}$$

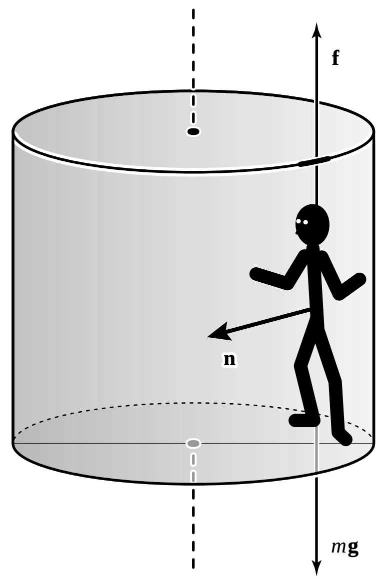
6.63 (a) $n = \dfrac{mv^2}{R}$ $\qquad f - mg = 0$

$f = \mu_s n$ $\qquad v = \dfrac{2\pi R}{T}$

$$T = \boxed{\sqrt{\frac{4\pi^2 R\mu_s}{g}}}$$

(b) $T = \boxed{2.54\text{ s}}$

$$\#\ \frac{\text{rev}}{\text{min}} = \frac{1\text{ rev}}{2.54\text{ s}}\left(\frac{60\text{ s}}{\text{min}}\right) = \boxed{23.6\ \frac{\text{rev}}{\text{min}}}$$



***6.64** Let the x–axis point eastward, the y-axis upward, and the z-axis point southward.

(a) The range is $Z = \dfrac{v_i^2 \sin 2\theta_i}{g}$

The initial speed of the ball is therefore

$$v_i = \sqrt{\frac{gZ}{\sin 2\theta_i}} = \sqrt{\frac{(9.80)(285)}{\sin 96.0°}} = 53.0\text{ m/s}$$

The time the ball is in the air is found from $\Delta y = v_{iy}t + \frac{1}{2}a_y t^2$ as

$$0 = (53.0\text{ m/s})(\sin 48.0°)t - (4.90\text{ m/s}^2)t^2$$

giving $t = \boxed{8.04\text{ s}}$

(b) $v_{ix} = \dfrac{2\pi R_e \cos\phi_i}{86400\text{ s}} = \dfrac{2\pi(6.37\times 10^6\text{ m})\cos 35.0°}{86400\text{ s}} = \boxed{379\text{ m/s}}$

(c) 360° of latitude corresponds to a distance of $2\pi R_e$, so 285 m is a change in latitude of

$$\Delta\phi = \left(\frac{S}{2\pi R_e}\right)(360°) = \left(\frac{285\text{ m}}{2\pi(6.37\times10^6\text{ m})}\right)(360°) = 2.56\times10^{-3}\text{ degrees}$$

The final latitude is then $\phi_f = \phi_i - \Delta\phi = 35.0° - 0.00256° = 34.9974°$.

The cup is moving eastward at a speed $v_{fx} = \dfrac{2\pi R_e \cos\phi_f}{86400\text{ s}}$, which is larger than the eastward velocity of the tee by

$$\Delta v_x = v_{fx} - v_{fi} = \frac{2\pi R_e}{86400\text{ s}}[\cos\phi_f - \cos\phi_i] = \frac{2\pi R_e}{86400\text{ s}}[\cos(\phi_i - \Delta\phi) - \cos\phi_i]$$

$$= \frac{2\pi R_e}{86400\text{ s}}[\cos\phi_i\cos\Delta\phi + \sin\phi_i\sin\Delta\phi - \cos\phi_i]$$

Since $\Delta\phi$ is such a small angle, $\cos\Delta\phi \approx 1$ and $\Delta v_x \approx \dfrac{2\pi R_e}{86400\text{ s}}\sin\phi_i\sin\Delta\phi$.

$$\Delta v_x \approx \frac{2\pi(6.37\times10^6\text{ m})}{86400\text{ s}}\sin 35.0°\sin 0.00256° = \boxed{1.19\times10^{-2}\text{ m/s}}$$

(d) $\Delta x = (\Delta v_x)t = (1.19\times10^{-2}\text{ m/s})(8.04\text{ s}) = 0.0955\text{ m} = \boxed{9.55\text{ cm}}$

***6.65** In $\sum F = m\dfrac{v^2}{r}$, both m and r are unknown but remain constant. Therefore, $\sum F$ is proportional to v^2 and increases by a factor of $\left(\dfrac{18.0}{14.0}\right)^2$ as v increases from 14.0 m/s to 18.0 m/s. The total force at the higher speed is then

$$\sum F_{\text{fast}} = \left(\frac{18.0}{14.0}\right)^2(130\text{ N}) = 215\text{ N}$$

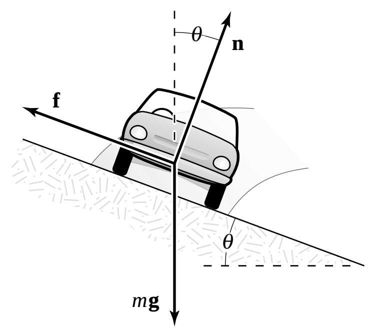
Symbolically, write $\sum F_{\text{slow}} = \left(\dfrac{m}{r}\right)(14.0\text{ m/s})^2$ and $\sum F_{\text{fast}} = \left(\dfrac{m}{r}\right)(18.0\text{ m/s})^2$.

Dividing gives $\dfrac{\sum F_{\text{fast}}}{\sum F_{\text{slow}}} = \left(\dfrac{18.0}{14.0}\right)^2$, or

$$\sum F_{\text{fast}} = \left(\frac{18.0}{14.0}\right)^2\sum F_{\text{slow}} = \left(\frac{18.0}{14.0}\right)^2(130\text{ N}) = \boxed{215\text{ N}}$$

This force must be $\boxed{\text{horizontally inward}}$ to produce the driver's centripetal acceleration.

6.66 (a) If the car is about to slip *down* the incline, f is directed up the incline.

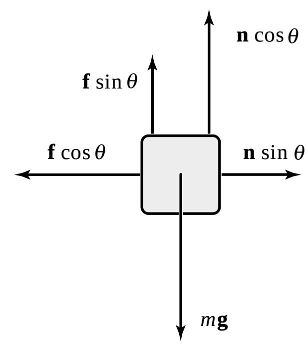


$\sum F_y = n\cos\theta + f\sin\theta - mg = 0$ where $f = \mu_s n$ gives

$$n = \frac{mg}{\cos\theta\,(1+\mu_s\tan\theta)} \quad \text{and} \quad f = \frac{\mu_s mg}{\cos\theta\,(1+\mu_s\tan\theta)}$$

Then, $\sum F_x = n\sin\theta - f\cos\theta = m\dfrac{v_{\min}^2}{R}$ yields

$$\boxed{v_{\min} = \sqrt{\frac{Rg(\tan\theta - \mu_s)}{1+\mu_s\tan\theta}}}$$

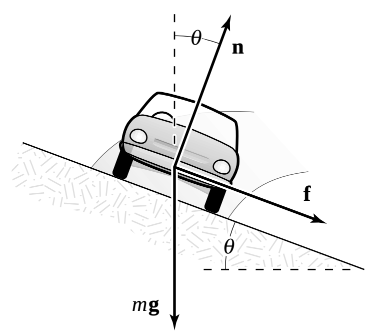


When the car is about to slip *up* the incline, f is directed down the incline. Then,
$\sum F_y = n\cos\theta - f\sin\theta - mg = 0$ with $f = \mu_s n$ yields

$$n = \frac{mg}{\cos\theta(1-\mu_s\tan\theta)} \quad \text{and} \quad f = \frac{\mu_s mg}{\cos\theta(1-\mu_s\tan\theta)}$$

In this case, $\sum F_x = n\sin\theta + f\cos\theta = m\dfrac{v_{\max}^2}{R}$, which gives

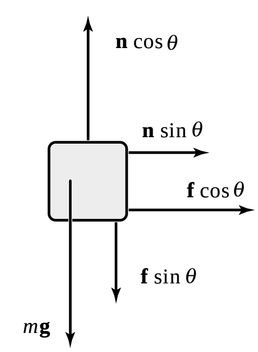
$$\boxed{v_{\max} = \sqrt{\frac{Rg(\tan\theta + \mu_s)}{1-\mu_s\tan\theta}}}$$



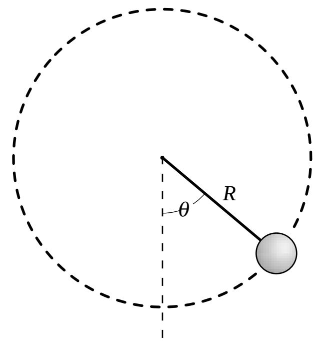
(b) If $v_{\min} = \sqrt{\dfrac{Rg(\tan\theta - \mu_s)}{1+\mu_s\tan\theta}} = 0$, then $\boxed{\mu_s = \tan\theta}$.

(c) $$v_{\min} = \sqrt{\frac{(100\text{ m})(9.80\text{ m/s}^2)(\tan 10.0° - 0.100)}{1 + (0.100)\tan 10.0°}} = \boxed{8.57\text{ m/s}}$$

$$v_{\max} = \sqrt{\frac{(100\text{ m})(9.80\text{ m/s}^2)(\tan 10.0° + 0.100)}{1 - (0.100)\tan 10.0°}} = \boxed{16.6\text{ m/s}}$$



***6.67** (a) The bead moves in a circle with radius $r = R\sin\theta$ at a speed of



$$v = \frac{2\pi r}{T} = \frac{2\pi R\sin\theta}{T}$$

The normal force has an inward radial component of $n\sin\theta$ and upward component of $n\cos\theta$.

$$\sum F_y = ma_y \Rightarrow n\cos\theta - mg = 0 \quad \text{or} \quad n = \frac{mg}{\cos\theta}$$

Then $\sum F_x = n\sin\theta = m\frac{v^2}{r}$ becomes $\left(\frac{mg}{\cos\theta}\right)\sin\theta = \frac{m}{R\sin\theta}\left(\frac{2\pi R\sin\theta}{T}\right)^2$, which reduces to $\frac{g\sin\theta}{\cos\theta} = \frac{4\pi^2 R\sin\theta}{T^2}$. This has two solutions: (1) $\sin\theta = 0 \Rightarrow \theta = 0°$, and (2) $\cos\theta = \frac{gT^2}{4\pi^2 R}$.

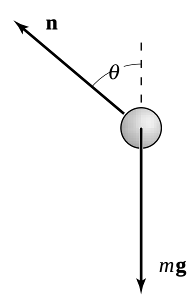
If $R = 15.0$ cm and $T = 0.450$ s, the second solution yields

$$\cos\theta = \frac{(9.80\text{ m/s}^2)(0.450\text{ s})^2}{4\pi^2(0.150\text{ m})} = 0.335 \quad \text{and} \quad \theta = 70.4°$$

Thus, in this case, the bead can ride at two positions $\boxed{\theta = 70.4°}$ and $\boxed{\theta = 0°}$.

(b) At this slower rotation, solution (2) above becomes

$$\cos\theta = \frac{(9.80\text{ m/s}^2)(0.850\text{ s})^2}{4\pi^2(0.150\text{ m})} = 1.20, \text{ which is impossible.}$$



In this case, the bead can ride only at the bottom of the loop, $\boxed{\theta = 0°}$. The loop's rotation must be faster than a certain threshold value in order for the bead to move away from the lowest position.

6.68 At terminal velocity, the accelerating force of gravity is balanced by frictional drag:

$$mg = arv + br^2v^2$$

(a) $mg = 3.10 \times 10^{-9}\, v + 0.870 \times 10^{-10}\, v^2$

For water, $m = \rho V = (1000\text{ kg/m}^8)\left(\frac{4}{3}\pi\right)(10^{-5})^{\,3}$

$$4.11 \times 10^{-11} = (3.10 \times 10^{-9})v + (0.870 \times 10^{-10})v^2$$

Assuming v is small, ignore the second term: $\boxed{v = 0.0132\text{ m/s}}$

(b) $mg = 3.10 \times 10^{-8}\, v + 0.870 \times 10^{-8}\, v^2$

Here we cannot ignore the second term because the coefficients are of nearly equal magnitude.

$$4.11 \times 10^{-8} = (3.10 \times 10^{-8})v + (0.870 \times 10^{-8})v^2$$

$$v = \frac{-3.10 \pm \sqrt{(3.10)^2 + 4(0.870)(4.11)}}{2(0.870)} = \boxed{1.03 \text{ m/s}}$$

(c) $mg = 3.10 \times 10^{-7} v + 0.870 \times 10^{-6}\, v^2$

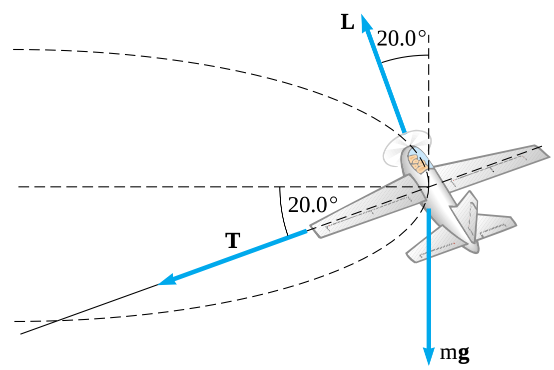
Assuming $v > 1$ m/s, and ignoring the first term:

$$4.11 \times 10^{-5} = 0.870 \times 10^{-6}\, v^2$$

$$v = \boxed{6.87 \text{ m/s}}$$

6.69 $\Sigma F_y = L_y - T_y - mg = L \cos 20.0° - T \sin 20.0° - 7.35 \text{ N} = ma_y = 0$

$$\Sigma F_x = L_x + T_x = L \sin 20.0° + T \cos 20.0° = m\frac{v^2}{r}$$



$$m\frac{v^2}{r} = (0.750 \text{ kg})\frac{(35.0 \text{ m/s})^2}{[(60.0 \text{ m}) \cos 20.0°]} = 16.3 \text{ N}$$

$$\therefore L \sin 20.0° + T \cos 20.0° = 16.3 \text{ N}$$

$$L \cos 20.0° - T \sin 20.0° = 7.35 \text{ N}$$

$$L + T\frac{\cos 20.0°}{\sin 20.0°} = \frac{16.3 \text{ N}}{\sin 20.0°}$$

$$L - T\frac{\sin 20.0°}{\cos 20.0°} = \frac{7.35 \text{ N}}{\cos 20.0°}$$

$$T(\cot 20.0° + \tan 20.0°) = \frac{16.3 \text{ N}}{\sin 20.0°} - \frac{7.35 \text{ N}}{\cos 20.0°}$$

$$T(3.11) = 39.8 \text{ N}$$

$$T = \boxed{12.8 \text{ N}}$$

6.70 $v = \left(\frac{mg}{b}\right)\left[1 - \exp\left(\frac{-bt}{m}\right)\right]$

At $t \to \infty$, $v \to v_T = \frac{mg}{b}$

At $t = 5.54$ s,

$$0.500v_t = v_t\left[1 - \exp\left(\frac{-b(5.54\text{ s})}{9.00\text{ kg}}\right)\right]$$

$$\exp\left(\frac{-b(5.54\text{ s})}{9.00\text{ kg}}\right) = 0.500$$

$$\frac{-b(5.54\text{ s})}{9.00\text{ kg}} = \ln 0.500 = -0.693$$

$$b = \frac{9.00\text{ kg}(0.693)}{5.54\text{ s}} = 1.13\text{ m/s}$$

(a) $v_t = \frac{mg}{b} = \frac{9.00\text{ kg}(9.80\text{ m/s}^2)}{1.13\text{ kg/s}} = \boxed{78.3\text{ m/s}}$

(b) $0.750v_t = v_t\left[1 - \exp\left(\frac{-1.13t}{9.00\text{ s}}\right)\right]$

$$\exp\left(\frac{-1.13t}{9.00\text{ s}}\right) = 0.250$$

$$t = \frac{9.00\ (\ln 0.250)}{-1.13}\text{ s} = \boxed{11.1\text{ s}}$$

(c) $\frac{dx}{dt} = \left(\frac{mg}{b}\right)\left[1 - \exp\left(-\frac{bt}{m}\right)\right]$

$$\int_{x_0}^{x} dx = \int_0^t \left(\frac{mg}{b}\right)\left[1 - \exp\left(\frac{-bt}{m}\right)\right] dt$$

$$x - x_0 = \frac{mgt}{b} + \left(\frac{m^2g}{b^2}\right)\exp\left(\frac{-bt}{m}\right)\Bigg|_0^t$$

$$= \frac{mgt}{b} + \left(\frac{m^2g}{b^2}\right)\left[\exp\left(\frac{-bt}{m}\right) - 1\right]$$

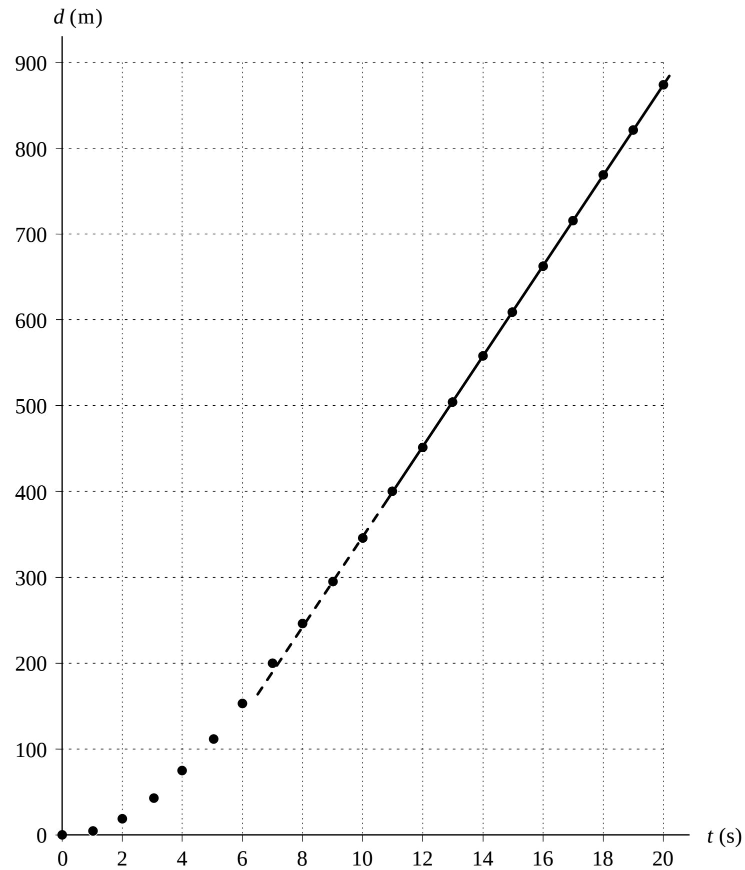
At t = 5.54 s,

$$x = (9.00\text{ kg})(9.80\text{ m/s}^2)\frac{5.54\text{ s}}{1.13\text{ kg/s}} + \left(\frac{(9.00\text{ kg})^2(9.80\text{ m/s}^2)}{(1.13\text{ kg/s})^2}\right)[\exp(-0.693) - 1]$$

$$x = 434\text{ m} + (626\text{ m})(-0.500) = \boxed{121\text{ m}}$$

6.71 (a)

t (s)	d (m)
1.00	4.88
2.00	18.9
3.00	42.1
4.00	73.8
5.00	112
6.00	154
7.00	199
8.00	246
9.00	296
10.0	347
11.0	399
12.0	452
13.0	505
14.0	558
15.0	611
16.0	664
17.0	717
18.0	770
19.0	823
20.0	876



(c) A straight line fits the points from t = 11.0 s to 20.0 s quite precisely. Its slope is the terminal speed.

$$v_t = slope = \frac{876\text{ m} - 399\text{ m}}{20.0\text{ s} - 11.0\text{ s}} = \boxed{53.0\text{ m/s}}$$