Chapter 3 Solutions

***3.1** $x = r\cos\theta = (5.50\text{ m})\cos 240° = (5.50\text{ m})(-0.5) = \boxed{-2.75\text{ m}}$

$y = r\sin\theta = (5.50\text{ m})\sin 240° = (5.50\text{ m})(-0.866) = \boxed{-4.76\text{ m}}$

3.2 (a) $d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} = \sqrt{(2.00 - [-3.00]^2) + (-4.00 - 3.00)^2}$

$d = \sqrt{25.0 + 49.0} = \boxed{8.60\text{ m}}$

(b) $r_1 = \sqrt{(2.00)^2 + (-4.00)^2} = \sqrt{20.0} = \boxed{4.47\text{ m}}$

$\theta_1 = \tan^{-1}\left(-\dfrac{4.00}{2.00}\right) = \boxed{-63.4°}$

$r_2 = \sqrt{(-3.00)^2 + (3.00)^2} = \sqrt{18.0} = \boxed{4.24\text{ m}}$

$\theta_2 = \boxed{135°}$ measured from + x axis.

3.3 We have $2.00 = r\cos 30.0°$

$r = \dfrac{2.00}{\cos 30.0°} = \boxed{2.31}$

and $y = r\sin 30.0° = 2.31\sin 30.0° = \boxed{1.15}$

3.4 (a) $x = r\cos\theta$ and $y = r\sin\theta$, therefore

$x_1 = (2.50\text{ m})\cos 30.0°$, $y_1 = (2.50\text{ m})\sin 30.0°$, and

$(x_1, y_1) = \boxed{(2.17, 1.25)\text{ m}}$

$x_2 = (3.80\text{ m})\cos 120°$, $y_2 = (3.80\text{ m})\sin 120°$, and

$(x_2, y_2) = \boxed{(-1.90, 3.29)\text{ m}}$

(b) $d = \sqrt{(\Delta x)^2 + (\Delta y)^2} = \sqrt{16.6 + 4.16} = \boxed{4.55\text{ m}}$

3.5 The x distance out to the fly is 2.00 m and the y distance up to the fly is 1.00 m.

(a) We can use the Pythagorean theorem to find the distance from the origin to the fly,

$$\text{distance} = \sqrt{x^2 + y^2} = \sqrt{(2.00\text{ m})^2 + (1.00\text{ m})^2} = \sqrt{5.00\text{ m}^2} = \boxed{2.24\text{ m}}$$

(b) $\theta = \text{Arctan}\left(\frac{1}{2}\right) = 26.6°$; $\mathbf{r} = \boxed{2.24\text{ m, }26.6°}$

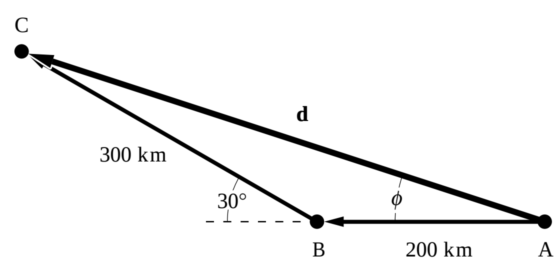
3.6 We have $r = \sqrt{x^2 + y^2}$ and $\theta = \text{Arctan}\left(\frac{y}{x}\right)$

(a) The radius for this new point is $\sqrt{(-x)^2 + y^2} = \sqrt{x^2 + y^2} = \boxed{r}$ and its angle is

$$\text{Arctan}\left(\frac{y}{(-x)}\right) = \boxed{180° - \theta}$$

(b) $\sqrt{(-2x)^2 + (-2y)^2} = \boxed{2r}$ This point is in the third quadrant if (x, y) is in the first quadrant or in the fourth quadrant if (x, y) is in the second quadrant. It is at angle $\boxed{180° + \theta}$.

(c) $\sqrt{(3x)^2 + (-3y)^2} = \boxed{3r}$ This point is in the fourth quadrant if (x, y) is in the first quadrant or in the third quadrant if (x, y) is in the second quadrant. It is at angle $\boxed{-\theta}$.



3.7 (a) The distance d from **A** to **C** is

$$d = \sqrt{x^2 + y^2}$$

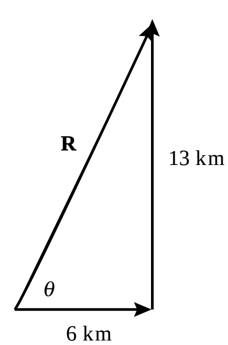
where $x = (200) + (300\cos 30.0°) = 460$ km

and $y = 0 + (300\sin 30.0°) = 150$ km

$$\therefore\ d = \sqrt{(460)^2 + (150)^2} = \boxed{484\text{ km}}$$

(b) $\tan\phi = \frac{y}{x} = \frac{150}{460} = 0.326$

$$\phi = \tan^{-1}(0.326) = \boxed{18.1°\text{ N of W}}$$

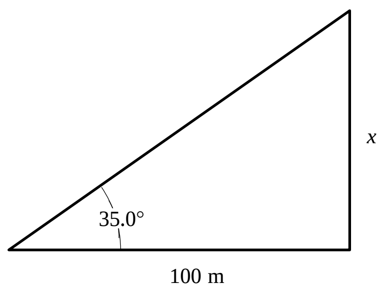


3.8 $R \cong \boxed{14\text{ km}}$

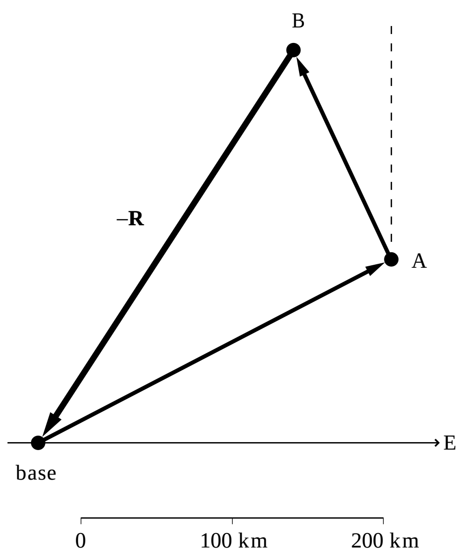
$\theta = \boxed{65°\text{ N of E}}$

3.9 $\tan 35.0° = \dfrac{x}{100\text{ m}}$

$x = (100\text{ m})(\tan 35.0°) = \boxed{70.0\text{ m}}$

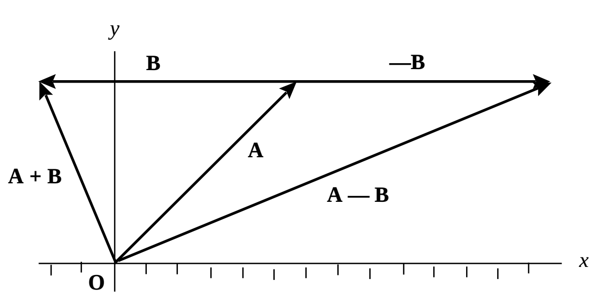


3.10 $-\mathbf{R} = \boxed{310\text{ km at }57°\text{ S of W}}$

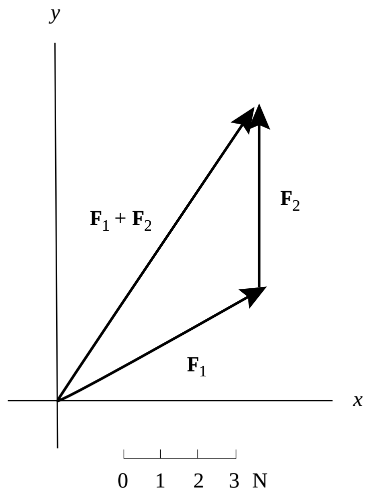


3.11 (a) Using graphical methods, place the tail of vector **B** at the head of vector **A**. The new vector **A** + **B** has a magnitude of <u>6.1</u> <u>at</u> <u>112°</u> from the *x*-axis.

(b) The vector difference **A** – **B** is found by placing the negative of vector **B** at the head of vector **A**. The resultant vector **A** – **B** has magnitude <u>14.8</u> units at an <u>angle of 22°</u> from the + *x*-axis.



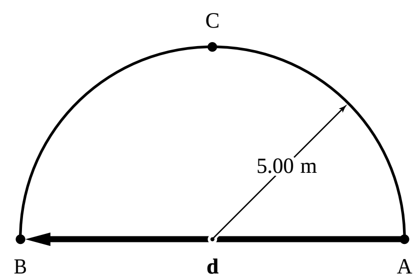
3.12 Find the resultant $\mathbf{F}_1 + \mathbf{F}_2$ graphically by placing the tail of $\mathbf{F}_2$ at the head of $\mathbf{F}_1$. The resultant force vector $\mathbf{F}_1 + \mathbf{F}_2$ is of magnitude $\boxed{9.5\ \text{N}}$ and at an angle of $\boxed{57° \text{ above the } x\text{-axis}}$.



3.13 (a) $|\mathbf{d}| = |-10.0\mathbf{i}| = \boxed{10.0\ \text{m}}$ since the displacement is a straight line from point A to point B.

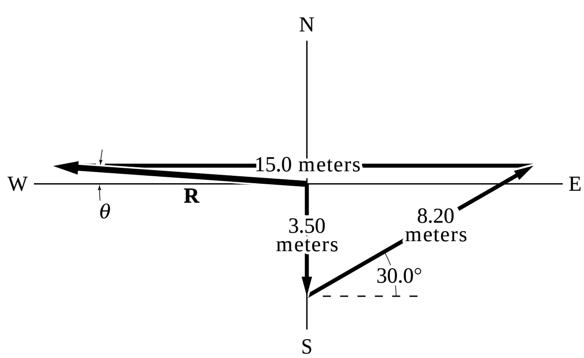
(b) The actual distance walked is not equal to the straight-line displacement. The distance follows the curved path of the semi-circle (ACB).

$$s = \left(\frac{1}{2}\right)(2\pi r) = 5\pi = \boxed{15.7\ \text{m}}$$



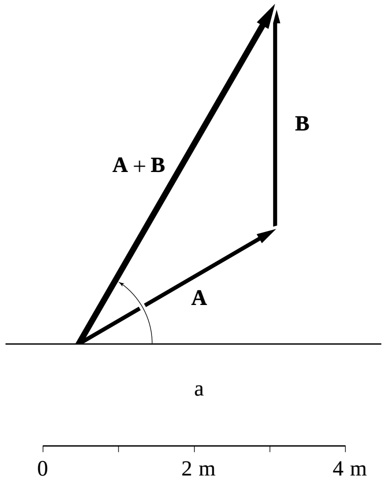
(c) If the circle is complete, $\mathbf{d}$ begins and ends at point A. Hence, $|\mathbf{d}| = \boxed{0}$.

3.14 Your sketch should be drawn to scale, and should look somewhat like that pictured below. The angle from the westward direction, θ, can be measured to be $\boxed{4° \text{ N of W}}$, and the distance R from the sketch can be converted according to the scale to be $\boxed{7.9\ \text{m}}$.

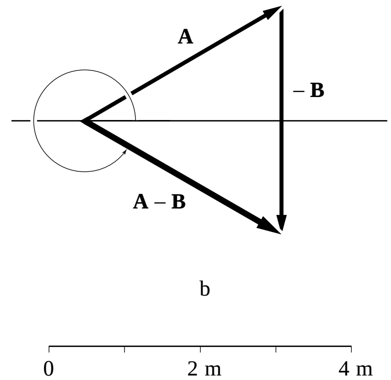


3.15 To find these vector expressions graphically, we draw each set of vectors. Measurements of the results are taken using a ruler and protractor.

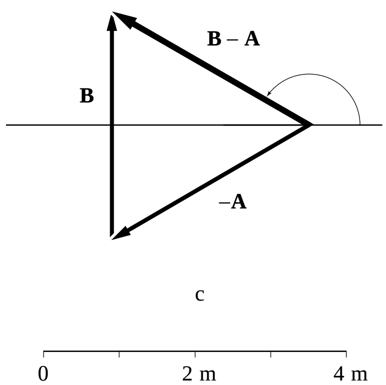
(a) **A** + **B** = 5.2 m at 60°



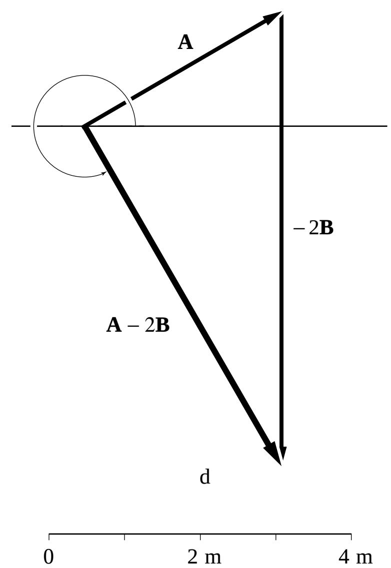
(b) **A** – **B** = 3.0 m at 330°



(c) **B** – **A** = 3.0 m at 150°



(d) **A** – 2**B** = 5.2 m at 300°

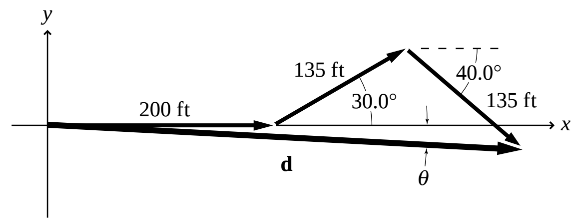


***3.16** (a) The large majority of people are standing or sitting at this hour. Their instantaneous foot-to-head vectors have upward vertical components on the order of 1 m and randomly oriented horizontal components. The citywide sum will be $\boxed{\sim 10^5 \text{ m upward}}$.

(b) Most people are lying in bed early Saturday morning. We suppose their beds are oriented north, south, east, west quite at random. Then the horizontal component of their total vector height is very nearly zero. If their compressed pillows give their height vectors vertical components averaging 3 cm, and if one-tenth of one percent of the population are on-duty nurses or police officers, we estimate the total vector height as

$\sim 10^5(0.03 \text{ m}) + 10^2(1 \text{ m})$ $\boxed{\sim 10^3 \text{ m upward}}$.

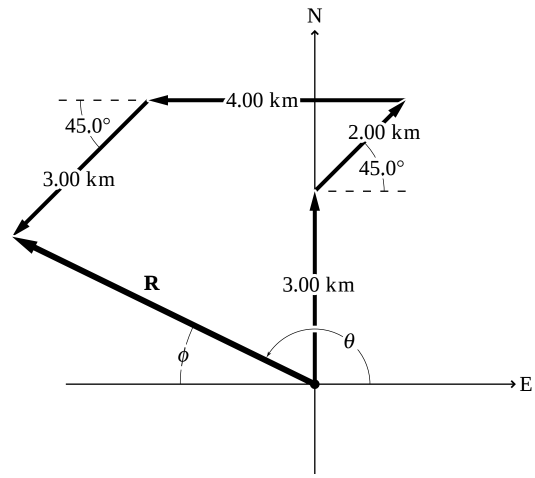
3.17 The scale drawing for the graphical solution should be similar to the figure at the right. The magnitude and direction of the final displacement from the starting point are obtained by measuring d and θ on the drawing and applying the scale factor used in making the drawing. The results should be



$\boxed{d \approx 420 \text{ ft and } \theta \approx -3°}$

3.18

x	y
0 km	3.00 km
1.41	1.41
–4.00	0
–2.12	–2.12
–4.71	2.29



$R = \sqrt{|x|^2 + |y|^2} = \boxed{5.24 \text{ km}}$

$\theta = \tan^{-1}\dfrac{y}{x} = 154°$ or $\boxed{\phi = 25.9° \text{ N of W}}$

3.19 Call his first direction the x direction.

$$R = 10.0 \text{ m } \mathbf{i} + 5.00 \text{ m}(-\mathbf{j}) + 7.00 \text{ m}(-\mathbf{i})$$

$$= 3.00 \text{ m } \mathbf{i} - 5.00 \text{ m } \mathbf{j}$$

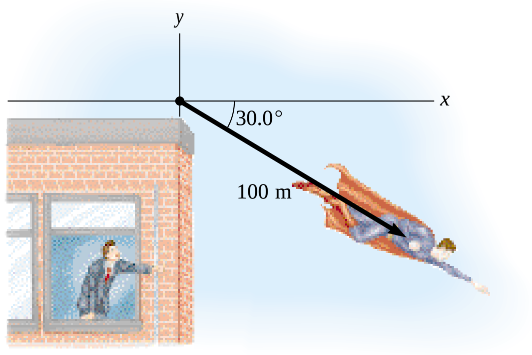
$$= \sqrt{(3.00)^2 + (5.00)^2} \text{ m at Arctan}\left(\frac{5}{3}\right) \text{ to the right}$$

$$R = \boxed{5.83 \text{ m at } 59.0° \text{ to the right from his original motion}}$$

3.20 Coordinates of super-hero are:

$$x = (100 \text{ m}) \cos(-30.0°) = \boxed{86.6 \text{ m}}$$

$$y = (100 \text{ m}) \sin(-30.0°) = \boxed{-50.0 \text{ m}}$$


y
x
30.0°
100 m

3.21 The person would have to walk 3.10 sin(25.0°) = $\boxed{\text{1.31 km north}}$, and 3.10 cos(25.0°) = $\boxed{\text{2.81 km east}}$.

3.22 + x East, + y North

$\Sigma x = 250 + 125 \cos 30° = 358$ m

$\Sigma y = 75 + 125 \sin 30° - 150 = -12.5$ m

$d = \sqrt{(\Sigma x)^2 + (\Sigma y)^2} = \sqrt{(358)^2 + (-12.5)^2} = 358$ m

$\tan \theta = \frac{(\Sigma y)}{(\Sigma x)} = -\frac{12.5}{358} = -0.0349 \quad \theta = -2.00°$

$\boxed{\mathbf{d} = 358 \text{ m at } 2.00° \text{ S of E}}$

***3.23** Let the positive x-direction be eastward, positive y-direction be vertically upward, and the positive z-direction be southward. The total displacement is then

$$\mathbf{d} = (4.80 \text{ cm } \mathbf{i} + 4.80 \text{ cm } \mathbf{j}) + (3.70 \text{ cm } \mathbf{j} - 3.70 \text{ cm } \mathbf{k})$$

or $$\mathbf{d} = 4.80 \text{ cm } \mathbf{i} + 8.50 \text{ cm } \mathbf{j} - 3.70 \text{ cm } \mathbf{k}$$

(a) The magnitude is $d = \sqrt{(4.80)^2 + (8.50)^2 + (-3.70)^2}$ cm = $\boxed{10.4 \text{ cm}}$

(b) Its angle with the y-axis follows from $\cos \theta = \frac{8.50}{10.4}$, giving $\boxed{\theta = 35.5°}$.

3.24 $\mathbf{B} = B_x\mathbf{i} + B_y\mathbf{j} + B_2\mathbf{k}$

$\mathbf{B} = 4.00\mathbf{i} + 6.00\mathbf{j} + 3.00\mathbf{k}$

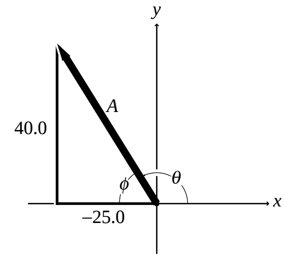
$|\mathbf{B}| = \sqrt{(4.00)^2 + (6.00)^2 + (3.00)^2} = \boxed{7.81}$

$\alpha = \cos^{-1}\left(\frac{4.00}{7.81}\right) = \boxed{59.2°}$

$\beta = \cos^{-1}\left(\frac{6.00}{7.81}\right) = \boxed{39.8°}$

$\gamma = \cos^{-1}\left(\frac{3.00}{7.81}\right) = \boxed{67.4°}$

3.25 $A_x = -25.0 \quad A_y = 40.0$

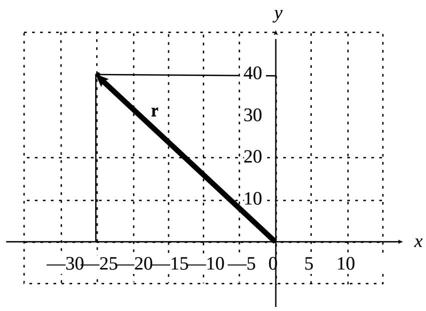


$A = \sqrt{A_x^2 + A_y^2} = \sqrt{(-25.0)^2 + (40.0)^2} = 47.2$ units

From the triangle, we find that $|\phi| = 58.0°$, so that $\theta = 122°$

Goal Solution

A vector has an x component of –25.0 units and a y component of 40.0 units. Find the magnitude and direction of this vector.



G: First we should visualize the vector either in our mind or with a sketch. Since the hypotenuse of the right triangle must be greater than either the x or y components that form the legs, we can estimate the magnitude of the vector to be about 50 units. The direction of the vector appears to be about 120° from the $+x$ axis.

O: The graphical analysis and visual estimates above may be sufficient for some situations, but we can use trigonometry to obtain a more precise result.

A: The magnitude can be found by the Pythagorean theorem: $r = \sqrt{x^2 + y^2}$

$$r = \sqrt{(-25.0\text{ units})^2 + (40\text{ units})^2} = 47.2\text{ units}$$

We observe that $\tan \phi = \frac{y}{x}$ (if we consider x and y to both be positive) .

$$\phi = \tan^{-1}\frac{y}{x} = \tan^{-1}\frac{40.0}{25.0} = \tan^{-1}(1.60) = 58.0°$$

The angle from the $+x$ axis can be found by subtracting from 180.

= 180 – 58 = 122°

L: Our calculated results agree with our graphical estimates. We should always remember to check that our answers are reasonable and make sense, especially for problems like this where it is easy to mistakenly calculate the wrong angle by confusing coordinates or overlooking a minus sign.
Quite often the direction angle of a vector can be specified in more than one way, and we must choose a notation that makes the most sense for the given problem. If compass directions were stated in this question, we could have reported the vector angle to be 32.0° west of north or a compass heading of 328°.

***3.26** The east and north components of the displacement from Dallas (D) to Chicago (C) are the sums of the east and north components of the displacements from Dallas to Atlanta (A) and from Atlanta to Chicago. In equation form:

$d_{\text{DCeast}} = d_{\text{DAeast}} + d_{\text{ACeast}} = 730 \cos 5.00° - 560 \sin 21.0° = 527$ miles.

$d_{\text{DCnorth}} = d_{\text{DAnorth}} + d_{\text{ACnorth}} = 730 \sin 5.00° + 560 \cos 21.0° = 586$ miles.

By the Pythagorean theorem, $d = \sqrt{(d_{\text{DCeast}})^2 + (d_{\text{DCnorth}})^2} = 788$ mi

Then $\tan \theta = \dfrac{d_{\text{DCnorth}}}{d_{\text{DCeast}}} = 1.11$ and $\theta = 48.0°$.

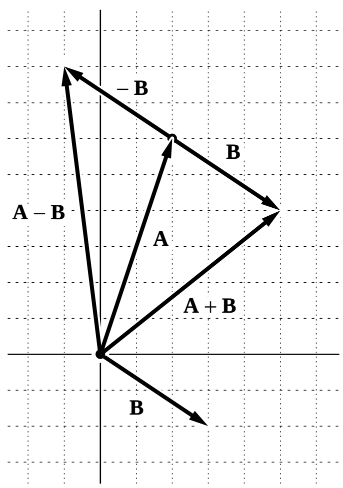
Thus, Chicago is $\boxed{\text{788 miles at 48.0° north east of Dallas}}$.

3.27 $x = d \cos \theta = (50.0 \text{ m})\cos(120) = -25.0$ m

$y = d \sin \theta = (50.0 \text{ m})\sin(120) = 43.3$ m

$\mathbf{d} = \boxed{(-25.0 \text{ m})\mathbf{i} + (43.3 \text{ m})\mathbf{j}}$

3.28 (a)



(b) $\mathbf{C} = \mathbf{A} + \mathbf{B} = 2.00\mathbf{i} + 6.00\mathbf{j} + 3.00\mathbf{i} - 2.00\mathbf{j} = \boxed{5.00\mathbf{i} + 4.00\mathbf{j}}$

$\mathbf{C} = \sqrt{25.0 + 16.0}$ at $\text{Arctan}\left(\dfrac{4}{5}\right)$

$\mathbf{C} = \boxed{6.40 \text{ at } 38.7°}$

$\mathbf{D} = \mathbf{A} - \mathbf{B} = 2.00\mathbf{i} + 6.00\mathbf{j} - 3.00\mathbf{i} + 2.00\mathbf{j} = \boxed{-1.00\mathbf{i} + 8.00\mathbf{j}}$

$\mathbf{D} = \sqrt{(-1.00)^2 + (8.00)^2}$ at $\text{Arctan}\left(\dfrac{8.00}{(-1.00)}\right)$

$\mathbf{D}$ = 8.06 at (180° – 82.9°) = $\boxed{8.06 \text{ at } 97.2°}$

3.29 $d = \sqrt{(x_1 + x_2 + x_3)^2 + (y_1 + y_2 + y_3)^2}$

$= \sqrt{(3.00 - 5.00 + 6.00)^2 + (2.00 + 3.00 + 1.00)^2} = \sqrt{52.0} = \boxed{7.21 \text{ m}}$

$\theta = \tan^{-1}\left(\dfrac{6.00}{4.00}\right) = \boxed{56.3°}$

3.30 $\mathbf{A} = -8.70\mathbf{i} + 15.0\mathbf{j}$ $\mathbf{B} = 13.2\mathbf{i} - 6.60\mathbf{j}$

$\mathbf{A} - \mathbf{B} + 3\mathbf{C} = 0$

$3\mathbf{C} = \mathbf{B} - \mathbf{A} = 21.9\mathbf{i} - 21.6\mathbf{j}$

$\mathbf{C} = 7.30\mathbf{i} - 7.20\mathbf{j}$ or

$C_x = \boxed{7.30 \text{ cm}}$

$C_y = \boxed{-7.20 \text{ cm}}$

3.31 (a) $(\mathbf{A} + \mathbf{B}) = (3\mathbf{i} - 2\mathbf{j}) + (-\mathbf{i} - 4\mathbf{j}) = \boxed{2\mathbf{i} - 6\mathbf{j}}$

(b) $(\mathbf{A} - \mathbf{B}) = (3\mathbf{i} - 2\mathbf{j}) - (-\mathbf{i} - 4\mathbf{j}) = \boxed{4\mathbf{i} + 2\mathbf{j}}$

(c) $|\mathbf{A} + \mathbf{B}| = \sqrt{2^2 + 6^2} = \boxed{6.32}$

(d) $|\mathbf{A} - \mathbf{B}| = \sqrt{4^2 + 2^2} = \boxed{4.47}$

(e) $\theta|\mathbf{A} + \mathbf{B}| = \tan^{-1}\left(-\dfrac{6}{2}\right) = -71.6° = \boxed{288°}$

$\theta|\mathbf{A} - \mathbf{B}| = \tan^{-1}\left(\dfrac{2}{4}\right) = \boxed{26.6°}$

3.32 Let $\mathbf{i}$ = east and $\mathbf{j}$ = north.

$\mathbf{R} = 3.00\text{b}\ \mathbf{j} + 4.00\text{b} \cos 45°\ \mathbf{i} + 4.00\text{b} \sin 45°\ \mathbf{j} - 5.00\text{b}\ \mathbf{i}$

$\mathbf{R} = -2.17\text{b}\ \mathbf{i} + 5.83\text{b}\ \mathbf{j}$

$\mathbf{R} = \sqrt{2.17^2 + 5.83^2}$ b at Arctan $\left(\dfrac{5.83}{2.17}\right)$ N of W

= $\boxed{\text{6.22 blocks at 110° counterclockwise from east}}$

3.33 $x = r\cos\theta$ and $y = r\sin\theta$, therefore:

(a) $x = 12.8\cos 150°$, $y = 12.8\sin 150°$, and $(x, y) = (-11.1\mathbf{i} + 6.40\mathbf{j})$ m

(b) $x = 3.30\cos 60.0°$, $y = 3.30\sin 60.0°$, and $(x, y) = (1.65\mathbf{i} + 2.86\mathbf{j})$ cm

(c) $x = 22.0\cos 215°$, $y = 22.0\sin 215°$, and $(x, y) = (-18.0\mathbf{i} - 12.6\mathbf{j})$ in

3.34 (a) $\mathbf{D} = \mathbf{A} + \mathbf{B} + \mathbf{C} = 2\mathbf{i} + 4\mathbf{j}$

$$|\mathbf{D}| = \sqrt{2^2 + 4^2} = \boxed{4.47\text{ m at } \theta = 63.4°}$$

(b) $\mathbf{E} = -\mathbf{A} - \mathbf{B} + \mathbf{C} = -6\mathbf{i} + 6\mathbf{j}$

$$|\mathbf{E}| = \sqrt{6^2 + 6^2} = \boxed{8.49\text{ m at } \theta = 135°}$$

3.35 $d_1 = (-3.50\mathbf{j})$ m

$d_2 = 8.20\cos 45.0°\mathbf{i} + 8.20\sin 45.0°\mathbf{j} = (5.80\mathbf{i} + 5.80\mathbf{j})$ m

$d_3 = (-15.0\mathbf{i})$ m

$\mathbf{R} = d_1 + d_2 + d_3 = (-15.0 + 5.80)\mathbf{i} + (5.80 - 3.50)\mathbf{j} = \boxed{(-9.20\mathbf{i} + 2.30\mathbf{j})\text{ m}}$

(or 9.20 m west and 2.30 m north)

The magnitude of the resultant displacement is

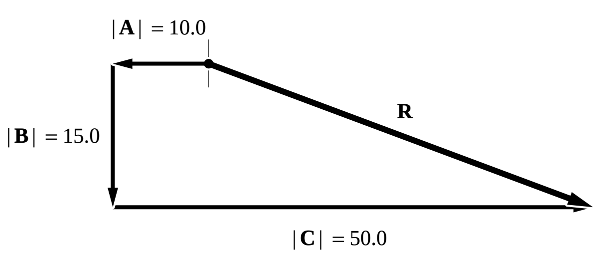
$$|\mathbf{R}| = \sqrt{R_x^2 + R_y^2} = \sqrt{(-9.20)^2 + (2.30)^2} = \boxed{9.48\text{ m}}$$

The direction is $\theta = \text{Arctan}\left(\frac{2.30}{-9.20}\right) = \boxed{166°}$

3.36 Refer to the sketch

$\mathbf{R} = \mathbf{A} + \mathbf{B} + \mathbf{C} = -10.0\mathbf{i} - 15.0\mathbf{j} + 50.0\mathbf{i} = 40.0\mathbf{i} - 15.0\mathbf{j}$

$|\mathbf{R}| = [(40.0)^2 + (-15.0)^2]^{1/2} = \boxed{42.7\text{ yards}}$



3.37 (a) $\mathbf{F} = \mathbf{F}_1 + \mathbf{F}_2$

$\mathbf{F} = 120 \cos(60.0°)\mathbf{i} + 120 \sin(60.0°)\mathbf{j} - 80.0 \cos(75.0°)\mathbf{i} + 80.0 \sin(75.0°)\mathbf{j}$

$\mathbf{F} = 60.0\mathbf{i} + 104\mathbf{j} - 20.7\mathbf{i} + 77.3\mathbf{j} = (39.3\mathbf{i} + 181\mathbf{j})$ N

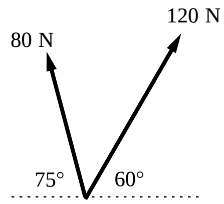
$|\mathbf{F}| = \sqrt{(39.3)^2 + (181)^2} = \boxed{185 \text{ N}}$; $\theta = \tan^{-1}\left(\frac{181}{39.3}\right) = \boxed{77.8°}$

(b) $\mathbf{F}_3 = -\mathbf{F} = \boxed{(-39.3\mathbf{i} - 181\mathbf{j}) \text{ N}}$

Goal Solution

The helicopter view in Figure P3.37 shows two people pulling on a stubborn mule. Find (a) the single force that is equivalent to the two forces shown and (b) the force that a third person would have to exert on the mule to make the resultant force equal to zero. The forces are measured in units of newtons.

G: The resultant force will be larger than either of the two individual forces, and since the two people are not pulling in exactly the same direction, the magnitude of the resultant should be less than the sum of the magnitudes of the two forces. Therefore, we should expect 120 N < R < 200 N. The angle of the resultant force appears to be straight ahead and perhaps slightly to the right. If the stubborn mule remains at rest, the ground must be exerting on the animal a force equal to the resultant **R** but in the opposite direction.



O: We can find **R** by adding the components of the two force vectors.

A: $\mathbf{F}_1 = (120 \cos 60)\mathbf{i}$ N $+ (120 \sin 60)\mathbf{j}$ N $= 60.0\mathbf{i}$ N $+ 103.9\mathbf{j}$ N

$\mathbf{F}_2 = -(80 \cos 75)\mathbf{i}$ N $+ (80 \sin 75)\mathbf{j}$ N $= -20.7\mathbf{i}$ N $+ 77.3\mathbf{j}$ N

$\mathbf{R} = \mathbf{F}_1 + \mathbf{F}_2 = 39.3\mathbf{i}$ N $+ 181.2\mathbf{j}$ N

$R = |\mathbf{R}| = \sqrt{(39.3)^2 + (181.2)^2} = 185$ N

The angle can be found from the arctan of the resultant components.

$$\theta = \tan^{-1}\frac{y}{x} = \tan^{-1}\frac{181.2}{39.3} = \tan^{-1}(4.61) = 77.8° \text{ counterclockwise from the } +x \text{ axis}$$

The opposing force that the either the ground or a third person must exert on the mule, in order for the overall resultant to be zero, is 185 N at 258° counterclockwise from $+x$.

L: The resulting force is indeed between 120 N and 200 N as we expected, and the angle seems reasonable as well. The process applied to solve this problem can be used for other "statics" problems encountered in physics and engineering. If another force is added to act on a system that is already in equilibrium (sum of the forces is equal to zero), then the system may accelerate. Such a system is now a "dynamic" one and will be the topic of Chapter 5.

3.38

East x	North y
0 m	4.00 m
1.41	1.41
–0.500	–0.866
+0.914	4.55

$|\mathbf{R}| = \sqrt{|x|^2 + |y|^2}$ = **4.64 m at 78.6° N of E**

3.39 $\mathbf{A} = 3.00$ m, $\theta_A = 30.0°$, $\mathbf{B} = 3.00$ m, $\theta_B = 90.0°$

$A_x = A\cos\theta_A = 3.00\cos 30.0° = 2.60$ m, $A_y = A\sin\theta_A = 3.00\sin 30.0° = 1.50$ m

so, $\mathbf{A} = A_x\mathbf{i} + A_y\mathbf{j} = (2.60\mathbf{i} + 1.50\mathbf{j})$ m

$B_x = 0$, $B_y = 3.00$ m so $\mathbf{B} = 3.00\mathbf{j}$ m

$\mathbf{A} + \mathbf{B} = (2.60\mathbf{i} + 1.50\mathbf{j}) + 3.00\mathbf{j} =$ $\boxed{(2.60\mathbf{i} + 4.50\mathbf{j})\text{ m}}$

***3.40** The y coordinate of the airplane is constant and equal to 7.60×10^3 m whereas the x coordinate is given by $x = v_i t$ where v_i is the constant speed in the horizontal direction.

At $t = 30.0$ s we have $x = 8.04 \times 10^3$, so $v_i = 268$ m/s. The position vector as a function of time is $\mathbf{P} = (268\text{ m/s})t\,\mathbf{i} + (7.60 \times 10^3\text{ m})\mathbf{j}$.

At $t = 45.0$ s, $\mathbf{P} = [1.21 \times 10^4\,\mathbf{i} + 7.60 \times 10^3\,\mathbf{j}]$ m. The magnitude is

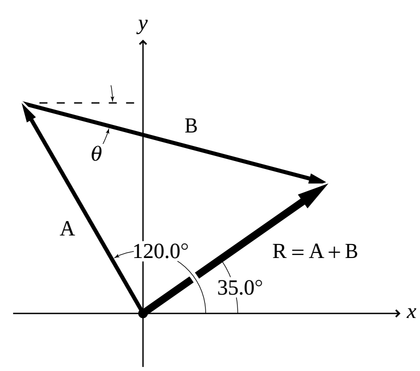
$$\mathbf{P} = \sqrt{(1.21 \times 10^4)^2 + (7.60 \times 10^3)^2}\text{ m} = \boxed{1.43 \times 10^4\text{ m}}$$

and the direction is

$$\theta = \text{Arctan}\left(\frac{7.60 \times 10^3}{1.21 \times 10^4}\right) =$$

$\boxed{32.2° \text{ above the horizontal}}$

3.41 We have $\mathbf{B} = \mathbf{R} - \mathbf{A}$



$A_x = 150\cos 120° = -75.0$ cm

$A_y = 150\sin 120° = 130$ cm

$R_x = 140\cos 35.0° = 115$ cm

$R_y = 140\sin 35.0° = 80.3$ cm

Therefore,

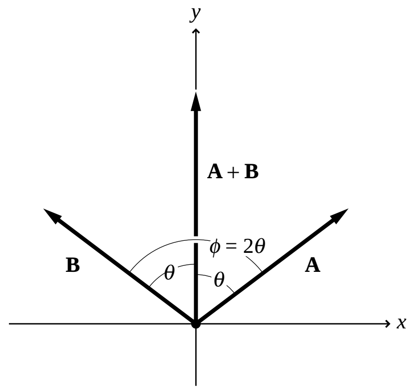
$\mathbf{B} = [115 - (-75)]\mathbf{i} + [80.3 - 130]\mathbf{j} = (190\mathbf{i} - 49.7\mathbf{j})$ cm

$$\left|\mathbf{B}\right| = [190^2 + (49.7)^2]^{1/2} = \boxed{196\text{ cm}}, \quad \theta = \tan^{-1}\left(-\frac{49.7}{190}\right) = \boxed{-14.7°}$$

***3.42** Since **A** + **B** = 6.00**j**, we have $(A_x + B_x)\mathbf{i} + (A_y + B_y)\mathbf{j} = 0\mathbf{i} + 6.00\,\mathbf{j}$ giving

$A_x + B_x = 0$, or $A_x = -B_x$ (1)

and $A_y + B_y = 6.00$ (2)



Since both vectors have a magnitude of 5.00, we also have:

$$A_x^2 + A_y^2 = B_x^2 + B_y^2 = (5.00)^2$$

From $A_x = -B_x$, it is seen that $A_x^2 = B_x^2$. Therefore $A_x^2 + A_y^2 = B_x^2 + B_y^2$ gives $A_y^2 = B_y^2$. Then $A_y = B_y$, and Equation (2) gives $A_y = B_y = 3.00$.

Defining θ as the angle between either **A** or **B** and the *y* axis, it is seen that

$$\cos\theta = \frac{A_y}{A} = \frac{B_y}{B} = \frac{3.00}{5.00} = 0.600$$

and $\theta = 53.1°$

The angle between **A** and **B** is then $\boxed{\phi = 2\theta = 106°}$.

3.43 (a) $\mathbf{A} = \boxed{8.00\mathbf{i} + 12.0\mathbf{j} - 4.00\,\mathbf{k}}$

(b) $\mathbf{B} = \mathbf{A}/4 = \boxed{2.00\mathbf{i} + 3.00\mathbf{j} - 1.00\mathbf{k}}$

(c) $\mathbf{C} = -3\mathbf{A} = \boxed{-24.0\mathbf{i} - 36.0\mathbf{j} + 12.0\mathbf{k}}$

3.44 $\mathbf{R} = 75.0\cos 240°\mathbf{i} + 75.0\sin 240°\mathbf{j} + 125\cos 135°\mathbf{i} + 125\sin 135°\mathbf{j} + 100\cos 160°\mathbf{i} + 100\sin 160°\mathbf{j}$

$\mathbf{R} = -37.5\mathbf{i} - 65.0\mathbf{j} - 88.4\mathbf{i} + 88.4\mathbf{j} - 94.0\mathbf{i} + 34.2\mathbf{j}$

$\mathbf{R} = \boxed{-220\mathbf{i} + 57.6\mathbf{j}}$

$\mathbf{R} = \sqrt{(-220)^2 + 57.6^2}$ at $\text{Arctan}\left(\frac{57.6}{220}\right)$ above the –*x*-axis

$\mathbf{R} = \boxed{\text{227 paces at 165°}}$

3.45 (a) $\mathbf{C} = \mathbf{A} + \mathbf{B} = \boxed{(5.00\mathbf{i} - 1.00\mathbf{j} - 3.00\mathbf{k})\text{ m}}$

$|\mathbf{C}| = \sqrt{(5.00)^2 + (1.00)^2 + (3.00)^2}\text{ m} = \boxed{5.92\text{ m}}$

(b) $\mathbf{D} = 2\mathbf{A} - \mathbf{B} = \boxed{(4.00\mathbf{i} - 11.0\mathbf{j} + 15.0\mathbf{k})\text{ m}}$

$$|\mathbf{D}| = \sqrt{(4.00)^2 + (11.0)^2 + (15.0)^2}\text{ m} = \boxed{19.0\text{ m}}$$

***3.46** The displacement from radar station to ship is

$$\mathbf{S} = (17.3 \sin 136°\mathbf{i} + 17.3 \cos 136°\mathbf{j})\text{ km} = (12.0\mathbf{i} - 12.4\mathbf{j})\text{ km}$$

From station to plane, the displacement is

$$\mathbf{P} = (19.6 \sin 153°\mathbf{i} + 19.6 \cos 153°\mathbf{j} + 2.20\mathbf{k})\text{ km, or}$$

$$\mathbf{P} = (8.90\mathbf{i} - 17.5\mathbf{j} + 2.20\mathbf{k})\text{ km.}$$

(a) From plane to ship the displacement is

$$\mathbf{D} = \mathbf{S} - \mathbf{P} = \boxed{(3.12\mathbf{i} + 5.02\mathbf{j} - 2.20\mathbf{k})\text{ km}}$$

(b) The distance the plane must travel is

$$D = |\mathbf{D}| = \sqrt{(3.12)^2 + (5.02)^2 + (2.20)^2}\text{ km} = \boxed{6.31\text{ km}}$$

3.47 The hurricane's first displacement is $\left(\frac{41.0\text{ km}}{\text{h}}\right)(3.00\text{ h})$ at 60.0° N of W, and its second displacement is $\left(\frac{25.0\text{ km}}{\text{h}}\right)(1.50\text{ h})$ due North. With **i** representing east and **j** representing north, its total displacement is:

$$\left(41.0\frac{\text{km}}{\text{h}}\cos 60.0°\right)(3.00\text{ h})(-\mathbf{i}) + \left(41.0\frac{\text{km}}{\text{h}}\sin 60.0°\right)(3.00\text{ h})\,\mathbf{j}$$

$$+\left(25.0\frac{\text{km}}{\text{h}}\right)(1.50\text{ h})\,\mathbf{j} = 61.5\text{ km }(-\mathbf{i}) + 144\text{ km }\mathbf{j}$$

with magnitude $\sqrt{(61.5\text{ km})^2 + (144\text{ km})^2} = \boxed{157\text{ km}}$

***3.48** (a) $\mathbf{E} = (17.0\text{ cm})\cos 27.0°\mathbf{i} + (17.0\text{ cm})\sin 27.0°\mathbf{j}$

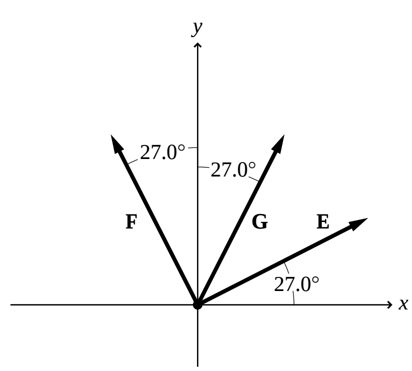
$$\mathbf{E} = \boxed{(15.1\mathbf{i} + 7.72\mathbf{j})\text{ cm}}$$

(b) $\mathbf{F} = -(17.0\text{ cm})\sin 27.0°\mathbf{i} + (17.0\text{ cm})\cos 27.0°\mathbf{j}$

$$\mathbf{F} = \boxed{(-7.72\mathbf{i} + 15.1\mathbf{j})\text{ cm}}$$

(c) $\mathbf{G} = +(17.0\text{ cm})\sin 27.0°\mathbf{i} + (17.0\text{ cm})\cos 27.0°\mathbf{j}$

$$\mathbf{G} = \boxed{(+7.72\mathbf{i} + 15.1\mathbf{j})\text{ cm}}$$



3.49 $A_x = -3.00$, $A_y = 2.00$

(a) $\mathbf{A} = A_x\mathbf{i} + A_y\mathbf{j} = \boxed{-3.00\mathbf{i} + 2.00\mathbf{j}}$

(b) $|\mathbf{A}| = \sqrt{A_x^2 + A_y^2} = \sqrt{(-3.00)^2 + (2.00)^2} = \boxed{3.61}$

$\tan\theta = \dfrac{A_y}{A_x} = \dfrac{2.00}{(-3.00)} = -0.667$, $\tan^{-1}(-0.667) = -33.7°$

θ is in the 2nd quadrant, so $\theta = 180° + (-33.7°) = \boxed{146°}$

(c) $\mathbf{R}_x = 0$, $\mathbf{R}_y = -4.00$, $\mathbf{R} = \mathbf{A} + \mathbf{B}$ thus $\mathbf{B} = \mathbf{R} - \mathbf{A}$ and

$B_x = R_x - A_x = 0 - (-3.00) = 3.00$, $B_y = R_y - A_y = -4.00 - 2.00 = -6.00$

Therefore, $\mathbf{B} = \boxed{3.00\mathbf{i} - 6.00\mathbf{j}}$

3.50 Let $+x$ = East, $+y$ = North,

x	y
300	0
–175	303
0	150
125	453

(a) $\theta = \tan^{-1}\dfrac{y}{x} = \boxed{74.6° \text{ N of E}}$

(b) $|\mathbf{R}| = \sqrt{x^2 + y^2} = \boxed{470 \text{ km}}$

3.51 Refer to Figure P3.51 in the textbook.

(a) $R_x = 40.0 \cos 45.0° + 30.0 \cos 45.0° = 49.5$

$R_y = 40.0 \sin 45.0° - 30.0 \sin 45.0° + 20.0 = 27.1$

$\mathbf{R} = \boxed{49.5\mathbf{i} + 27.1\mathbf{j}}$

(b) $|\mathbf{R}| = \sqrt{(49.4)^2 + (27.1)^2} = \boxed{56.4}$

$\theta = \tan^{-1}\left(\dfrac{27.1}{49.5}\right) = \boxed{28.7°}$

3.52 Taking components along **i** and **j**, we get two equations:

$$6.00a - 8.00b + 26.0 = 0$$

$$-8.00a + 3.00b + 19.0 = 0$$

Solving simultaneously, $\boxed{a = 5.00,\ b = 7.00}$

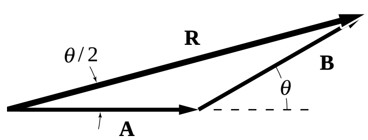
Therefore, $5.00\mathbf{A} + 7.00\mathbf{B} + \mathbf{C} = 0$

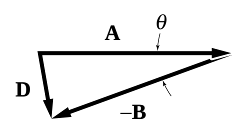
***3.53** Let θ represent the angle between the directions of **A** and **B**. Since **A** and **B** have the same magnitudes, **A**, **B**, and $\mathbf{R} = \mathbf{A} + \mathbf{B}$ form an isosceles triangle in which the angles are $180° - \theta$, $\theta/2$, and $\theta/2$. The magnitude of **R** is then $R = 2A\cos(\theta/2)$. [*Hint:* apply the law of cosines to the isosceles triangle and use the fact that $B = A$.]

Again, **A**, –**B**, and $\mathbf{D} = \mathbf{A} - \mathbf{B}$ form an isosceles triangle with apex angle θ. Applying the law of cosines and the identity $(1 - \cos\theta) = 2\sin^2(\theta/2)$ gives the magnitude of **D** as $D = 2A\sin(\theta/2)$.

The problem requires that $R = 100D$.

Thus, $2A\cos(\theta/2) = 200A\sin(\theta/2)$. This gives $\tan(\theta/2) = 0.010$ and $\boxed{\theta = 1.15°}$.

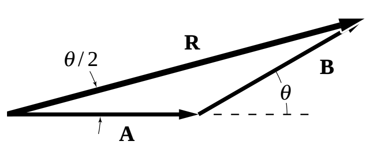


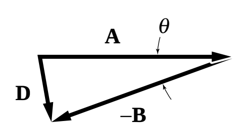


***3.54** Let θ represent the angle between the directions of **A** and **B**. Since **A** and **B** have the same magnitudes, **A**, **B**, and $\mathbf{R} = \mathbf{A} + \mathbf{B}$ form an isosceles triangle in which the angles are $180° - \theta$, $\theta/2$, and $\theta/2$. The magnitude of **R** is then $R = 2A\cos(\theta/2)$. [*Hint:* apply the law of cosines to the isosceles triangle and use the fact that $B = A$.]

Again, **A**, –**B**, and $\mathbf{D} = \mathbf{A} - \mathbf{B}$ form an isosceles triangle with apex angle θ. Applying the law of cosines and the identity $(1 - \cos\theta) = 2\sin^2(\theta/2)$ gives the magnitude of **D** as $D = 2A\sin(\theta/2)$.

The problem requires that $R = nD$, or $\cos(\theta/2) = n\sin(\theta/2)$, giving $\boxed{\theta = 2\tan^{-1}(1/n)}$.





3.55 (a) $R_x = \boxed{2.00}$, $R_y = \boxed{1.00}$, $R_z = \boxed{3.00}$

(b) $|\mathbf{R}| = \sqrt{R_x^2 + R_y^2 + R_Z^2} = \sqrt{4.00 + 1.00 + 9.00} = \sqrt{14.0} = \boxed{3.74}$

(c) $\cos\theta_x = \dfrac{R_x}{|\mathbf{R}|} \Rightarrow \theta_x = \cos^{-1}\left(\dfrac{R_x}{|\mathbf{R}|}\right) = \boxed{57.7° \text{ from } +x}$

$\cos\theta_y = \dfrac{R_y}{|\mathbf{R}|} \Rightarrow \theta_y = \cos^{-1}\left(\dfrac{R_y}{|\mathbf{R}|}\right) = \boxed{74.5° \text{ from } +y}$

$\cos\theta_z = \dfrac{R_z}{|\mathbf{R}|} \Rightarrow \theta_z = \cos^{-1}\left(\dfrac{R_z}{|\mathbf{R}|}\right) = \boxed{36.7° \text{ from } +z}$

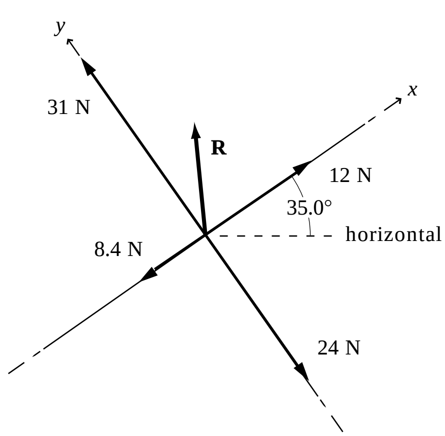
***3.56** Choose the $+x$-axis in the direction of the first force. The total force, in newtons, is then

$$12.0\mathbf{i} + 31.0\mathbf{j} - 8.40\mathbf{i} - 24.0\mathbf{j} = \boxed{(3.60\mathbf{i}) + (7.00\mathbf{j})\text{ N}}$$

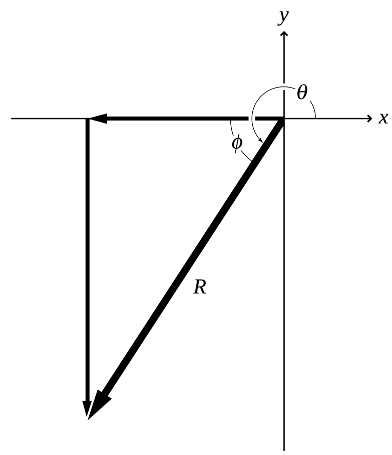
The magnitude of the total force is

$$\sqrt{(3.60)^2 + (7.00)^2}\text{ N} = \boxed{7.87\text{ N}}$$

and the angle it makes with our $+x$-axis is given by $\tan\theta = \dfrac{(7.00)}{(3.60)}$, $\theta = 62.8°$. Thus, its angle counterclockwise from the horizontal is $35.0° + 62.8° = \boxed{97.8°}$



3.57 $\mathbf{d}_1 = 100\mathbf{i}$ $\mathbf{d}_2 = -300\mathbf{j}$



$\mathbf{d}_3 = -150\cos(30.0^\circ)\mathbf{i} - 150\sin(30.0^\circ)\mathbf{j} = -130\mathbf{i} - 75.0\mathbf{j}$

$\mathbf{d}_4 = -200\cos(60.0^\circ)\mathbf{i} + 200\sin(60.0^\circ)\mathbf{j} = -100\mathbf{i} + 173\mathbf{j}$

$\mathbf{R} = \mathbf{d}_1 + \mathbf{d}_2 + \mathbf{d}_3 + \mathbf{d}_4 = -130\mathbf{i} - 202\mathbf{j}$

$\left|\mathbf{R}\right| = [(-130)^2 + (-202)^2]^{1/2} = \boxed{240\text{ m}}$

$\phi = \tan^{-1}\left(\frac{202}{130}\right) = 57.2^\circ$

$\theta = 180 + \phi = \boxed{237^\circ}$

***3.58** $d\mathbf{P}/dt = d(4\mathbf{i} + 3\mathbf{j} - 2t\,\mathbf{j})/dt = 0 + 0 - 2\mathbf{j} = \boxed{-(2.00\text{ m/s})\mathbf{j}}$

The position vector at $t = 0$ is $4\mathbf{i} + 3\mathbf{j}$. At $t = 1$ s, the position is $4\mathbf{i} + 1\mathbf{j}$, and so on. The object is moving straight downward at 2 m/s, so

$d\mathbf{P}/dt$ represents $\boxed{\text{its velocity vector}}$.

3.59 $\mathbf{v} = v_x\mathbf{i} + v_y\mathbf{j} = (300 + 100\cos 30.0^\circ)\mathbf{i} + (100\sin 30.0^\circ)\mathbf{j}$

$\mathbf{v} = (387\mathbf{i} + 50.0\mathbf{j})$ mi/h

$\left|\mathbf{v}\right| = \boxed{390\text{ mi/h at }7.37^\circ\text{ N of E}}$

3.60 (a) You start at $\mathbf{r}_1 = \mathbf{r}_A = 30.0\text{ m }\mathbf{i} - 20.0\text{ m }\mathbf{j}$.

The displacement to **B** is

$$\mathbf{r}_B - \mathbf{r}_A = 60.0\mathbf{i} + 80.0\mathbf{j} - 30.0\mathbf{i} + 20.0\mathbf{j} = 30.0\mathbf{i} + 100\mathbf{j}$$

You cover one–half of this, $15.0\mathbf{i} + 50.0\mathbf{j}$, to move to

$$\mathbf{r}_2 = 30.0\mathbf{i} - 20.0\mathbf{j} + 15.0\mathbf{i} + 50.0\mathbf{j} = 45.0\mathbf{i} + 30.0\mathbf{j}$$

Now the displacement from your current position to **C** is

$$\mathbf{r}_C - \mathbf{r}_2 = -10.0\mathbf{i} - 10.0\mathbf{j} - 45.0\mathbf{i} - 30.0\mathbf{j} = -55.0\mathbf{i} - 40.0\mathbf{j}$$

You cover one–third, moving to

$$\mathbf{r}_3 = \mathbf{r}_2 + \Delta\mathbf{r}_{23} = 45.0\mathbf{i} + 30.0\mathbf{j} + \frac{1}{3}(-55.0\mathbf{i} - 40.0\mathbf{j}) = 26.7\mathbf{i} + 16.7\mathbf{j}$$

The displacement from where you are to **D** is

$$\mathbf{r}_D - \mathbf{r}_3 = 40.0\mathbf{i} - 30.0\mathbf{j} - 26.7\mathbf{i} - 16.7\mathbf{j} = 13.3\mathbf{i} - 46.7\mathbf{j}$$

You traverse one–quarter of it, moving to

$$\mathbf{r}_4 = \mathbf{r}_3 + \frac{1}{4}(\mathbf{r}_D - \mathbf{r}_3) = 26.7\mathbf{i} + 16.7\mathbf{j} + \frac{1}{4}(13.3\mathbf{i} - 46.7\mathbf{j}) = 30.0\mathbf{i} + 5.00\mathbf{j}$$

The displacement from your new location to **E** is

$$\mathbf{r}_E - r_4 = -70.0\mathbf{i} + 60.0\mathbf{j} - 30.0\mathbf{i} - 5.00\mathbf{j} = -100\mathbf{i} + 55.0\mathbf{j}$$

of which you cover one–fifth, –20.0**i** + 11.0**j**, moving to

$$\mathbf{r}_4 + \Delta\mathbf{r}_{45} = 30.0\mathbf{i} + 5.00\mathbf{j} - 20.0\mathbf{i} + 11.0\mathbf{j} = 10.0\mathbf{i} + 16.0\mathbf{j}.$$

The treasure is at $\boxed{(10.0 \text{ m}, 16.0 \text{ m})}$

(b) Following the directions brings you to the average position of the trees. The steps we took numerically in part (a) bring you to

$$\mathbf{r}_A + \frac{1}{2}(\mathbf{r}_B - \mathbf{r}_A) = \left(\frac{\mathbf{r}_A + \mathbf{r}_B}{2}\right)$$

then to

$$\frac{(\mathbf{r}_A + \mathbf{r}_B)}{2} + \frac{\mathbf{r}_C - (\mathbf{r}_A + \mathbf{r}_B)/2}{3} = \frac{(\mathbf{r}_A + \mathbf{r}_B + \mathbf{r}_C)}{3}$$

then to

$$\frac{\mathbf{r}_A + \mathbf{r}_B + \mathbf{r}_C}{3} + \frac{\mathbf{r}_D - (\mathbf{r}_A + \mathbf{r}_B + \mathbf{r}_C)/3}{4} = \frac{(\mathbf{r}_A + \mathbf{r}_B + \mathbf{r}_C + \mathbf{r}_D)}{4}$$

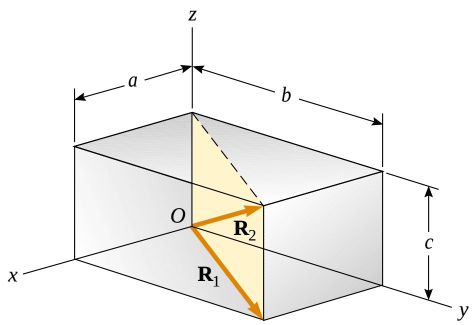
and at last to

$$\frac{\mathbf{r}_A + \mathbf{r}_B + \mathbf{r}_C + \mathbf{r}_D}{4} + \frac{\mathbf{r}_E - (\mathbf{r}_A + \mathbf{r}_B + \mathbf{r}_C + \mathbf{r}_D)/4}{5} = \frac{\mathbf{r}_A + \mathbf{r}_B + \mathbf{r}_C + \mathbf{r}_D + \mathbf{r}_E}{5}$$

This center of mass of the tree distribution is in the same location whatever order we take the trees in.

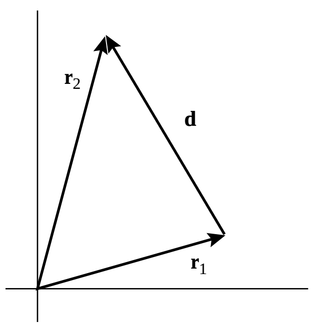
3.61 (a) From the picture $\mathbf{R}_1 = a\mathbf{i} + b\mathbf{j}$ and $\left|\mathbf{R}_1\right| = \sqrt{a^2 + b^2}$

(b) $\mathbf{R}_2 = a\mathbf{i} + b\mathbf{j} + c\mathbf{k}$. Its magnitude is $\sqrt{\left|\mathbf{R}_1\right|^2 + c^2} = \sqrt{a^2 + b^2 + c^2}$



3.62 (a) $\mathbf{r}_1 + \mathbf{d} = \mathbf{r}_2$ defines the displacement $\mathbf{d}$, so $\mathbf{d} = \mathbf{r}_2 - \mathbf{r}_1$.

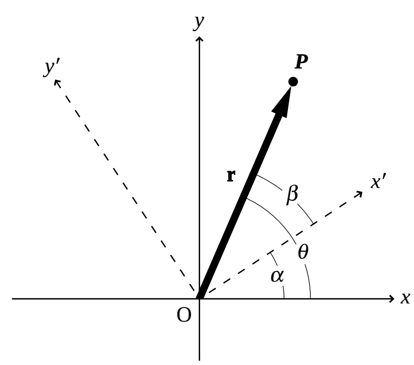
(b)



3.63 The displacement of point P is invariant under rotation of the coordinates.

Therefore, $r = r'$ and $r^2 = (r')^2$ or, $x^2 + y^2 = (x')^2 + (y')^2$

Also, from the figure, $\beta = \theta - \alpha$



$$\therefore \tan^{-1}\left(\frac{y'}{x'}\right) = \tan^{-1}\left(\frac{y}{x}\right) - \alpha$$

$$\frac{y'}{x'} = \frac{\left(\frac{y}{x}\right) - \tan\alpha}{1 + \left(\frac{y}{x}\right)\tan\alpha}$$

Which we simplify by multiplying top and bottom by $x\cos\alpha$. Then,

$x' = x\cos\alpha + y\sin\alpha$, $y' = -x\sin\alpha + y\cos\alpha$