Chapter 2 Solutions

***2.1** (a) $\bar{v} = \boxed{2.30 \text{ m/s}}$

(b) $\bar{v} = \frac{\Delta x}{\Delta t} = \frac{57.5 \text{ m} - 9.20 \text{ m}}{3.00 \text{ s}} = \boxed{16.1 \text{ m/s}}$

(c) $\bar{v} = \frac{\Delta x}{\Delta t} = \frac{57.5 \text{ m} - 0 \text{ m}}{5.00 \text{ s}} = \boxed{11.5 \text{ m/s}}$

2.2 (a) Displacement $= (8.50 \times 10^4 \text{ m/h})\left(\frac{35.0}{60.0} \text{ h}\right) + 130 \times 10^3 \text{ m}$

$x = (49.6 + 130) \times 10^3 \text{ m} = \boxed{180 \text{ km}}$

(b) Average velocity $= \frac{displacement}{time} = \frac{180 \text{ km}}{\left[\frac{(35.0 + 15.0)}{60.0} + 2.00\right] \text{ h}} = \boxed{63.4 \text{ km/h}}$

2.3 (a) $v_{av} = \frac{\Delta x}{\Delta t} = \frac{10 \text{ m}}{2 \text{ s}} = \boxed{5 \text{ m/s}}$

(b) $v_{av} = \frac{5 \text{ m}}{4 \text{ s}} = \boxed{1.2 \text{ m/s}}$

(c) $v_{av} = \frac{x_2 - x_1}{t_2 - t_1} = \frac{5 \text{ m} - 10 \text{ m}}{4 \text{ s} - 2 \text{ s}} = \boxed{-2.5 \text{ m/s}}$

(d) $v_{av} = \frac{x_2 - x_1}{t_2 - t_1} = \frac{-5 \text{ m} - 5 \text{ m}}{7 \text{ s} - 4 \text{ s}} = \boxed{-3.3 \text{ m/s}}$

(e) $v_{av} = \frac{x_2 - x_1}{t_2 - t_1} = \frac{0 - 0}{8 - 0} = \boxed{0 \text{ m/s}}$

2.4 $x = 10t^2$

For t(s) = 2.0 2.1 3.0

x(m) = 40 44.1 90

(a) $\bar{v} = \frac{\Delta x}{\Delta t} = \frac{50 \text{ m}}{1.0 \text{ s}} = \boxed{50.0 \text{ m/s}}$

(b) $\bar{v} = \frac{\Delta x}{\Delta t} = \frac{4.1 \text{ m}}{0.1 \text{ s}} = \boxed{41.0 \text{ m/s}}$

2.5 (a) Let d represent the distance between A and B. Let t_1 be the time for which the walker has the higher speed in $5.00 \text{ m/s} = \frac{d}{t_1}$. Let t_2 represent the longer time for the return trip in $-3.00 \text{ m/s} = -\frac{d}{t_2}$. Then the times are $t_1 = \frac{d}{(5.00 \text{ m/s})}$ and $t_2 = \frac{d}{(3.00 \text{ m/s})}$. The average speed is:

$$\bar{v} = \frac{\text{Total distance}}{\text{Total time}} = \frac{d + d}{\frac{d}{(5.00 \text{ m/s})} + \frac{d}{(3.00 \text{ m/s})}} = \frac{2d}{\frac{(8.00 \text{ m/s})d}{(15.0 \text{ m}^2/\text{s}^2)}}$$

$$\bar{v} = \frac{2(15.0 \text{ m}^2/\text{s}^2)}{8.00 \text{ m/s}} = \boxed{3.75 \text{ m/s}}$$

(b) She starts and finishes at the same point A.

With total displacement = 0, average velocity = $\boxed{0}$

2.6 (a) $\bar{v} = \frac{\text{Total distance}}{\text{Total time}}$

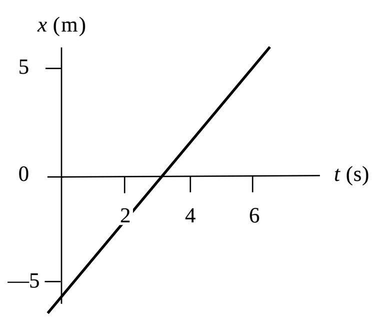
Let d be the distance from A to B.

Then the time required is $\frac{d}{v_1} + \frac{d}{v_2}$.

And the average speed is $\bar{v} = \frac{2d}{\frac{d}{v_1} + \frac{d}{v_2}} = \boxed{\frac{2v_1 v_2}{v_1 + v_2}}$

(b) With total displacement zero, her average velocity is $\boxed{0}$.

2.7 (a)



(b) $v = \text{slope} = \frac{5.00 \text{ m} - (-3.00 \text{ m})}{(6.00 \text{ s} - 1.00 \text{ s})} = \frac{8.00 \text{ m}}{5.00 \text{ s}} = \boxed{1.60 \text{ m/s}}$

2.8 (a) At any time, t, the displacement is given by $x = (3.00 \text{ m/s}^2)t^2$.

Thus, at $t_i = 3.00$ s: $x_i = (3.00 \text{ m/s}^2)(3.00 \text{ s})^2 = \boxed{27.0 \text{ m}}$

(b) At $t_f = 3.00 \text{ s} + \Delta t : x_f = (3.00 \text{ m/s}^2)(3.00 \text{ s} + \Delta t)^2$, or

$$x_f = \boxed{27.0 \text{ m} + (18.0 \text{ m/s})\Delta t + (3.00 \text{ m/s}^2)(\Delta t)^2}$$

(c) The instantaneous velocity at $t = 3.00$ s is:

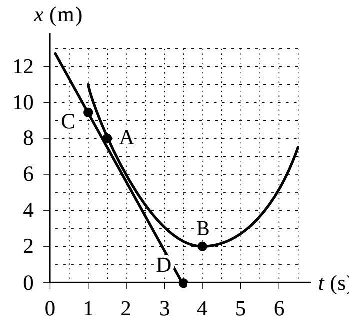
$$v = \lim_{\Delta t \to 0}\left(\frac{x_f - x_i}{\Delta t}\right) = \lim_{\Delta t \to 0}\,[(18.0 \text{ m/s}) + (3.00 \text{ m/s}^2)\Delta t], \text{ or}$$

$$v = \boxed{18.0 \text{ m/s}}$$

2.9 (a) at $t_i = 1.5$ s, $x_i = 8.0$ m (Point A)

at $t_f = 4.0$ s, $x_f = 2.0$ m (Point B)

$$\bar{v} = \frac{x_f - x_i}{t_f - t_i} = \frac{(2.0 - 8.0) \text{ m}}{(4 - 1.5) \text{ s}} = -\frac{6.0 \text{ m}}{2.5 \text{ s}} = \boxed{-2.4 \text{ m/s}}$$



(b) The slope of the tangent line is found from points C and D.

$(t_C = 1.0 \text{ s}, x_C = 9.5 \text{ m})$ and $(t_D = 3.5 \text{ s}, x_D = 0)$,

$$v \cong \boxed{-3.8 \text{ m/s}}$$

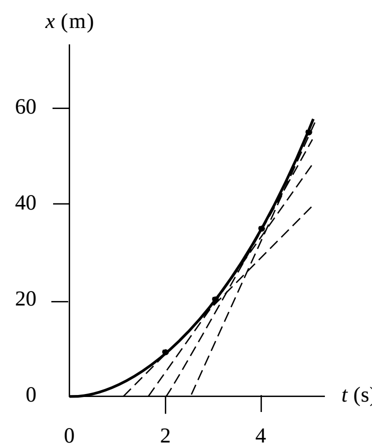
(c) The velocity is zero when x is a minimum. This is at $t \approx \boxed{4 \text{ s}}$.

2.10 (b) At $t = 5.0$ s, the slope is $v \cong \dfrac{58 \text{ m}}{2.5 \text{ s}} \cong \boxed{23 \text{ m/s}}$

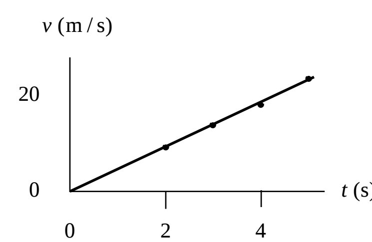
At $t = 4.0$ s, the slope is $v \cong \dfrac{54 \text{ m}}{3 \text{ s}} \cong \boxed{18 \text{ m/s}}$

At $t = 3.0$ s, the slope is $v \cong \dfrac{49 \text{ m}}{3.4 \text{ s}} \cong \boxed{14 \text{ m/s}}$

At $t = 2.0$ s, the slope is $v \cong \dfrac{36 \text{ m}}{4.0 \text{ s}} \cong \boxed{9.0 \text{ m/s}}$

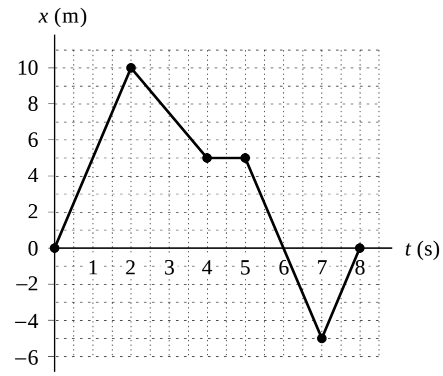


(c) $\bar{a} = \dfrac{\Delta v}{\Delta t} \cong \dfrac{23 \text{ m/s}}{5.0 \text{ s}} \cong \boxed{4.6 \text{ m/s}^2}$



(d) Initial velocity of the car was $\boxed{\text{zero}}$.

2.11 (a) $v = \frac{(5-0)\text{ m}}{(1-0)\text{ s}} = \boxed{5\text{ m/s}}$



(b) $v = \frac{(5-10)\text{ m}}{(4-2)\text{ s}} = \boxed{-2.5\text{ m/s}}$

(c) $v = \frac{(5\text{ m}-5\text{ m})}{(5\text{ s}-4\text{ s})} = \boxed{0}$

(d) $v = \frac{0-(-5\text{ m})}{(8\text{ s}-7\text{ s})} = \boxed{+5\text{ m/s}}$

2.12 $\bar{a} = \frac{v_f - v_i}{t_f - t_i} = \frac{0 - 60.0\text{ m/s}}{15.0\text{ s} - 0} = \boxed{-4.00\text{ m/s}^2}$

The negative sign in the result shows that the acceleration is in the negative x direction.

***2.13** Choose the positive direction to be the outward perpendicular to the wall.

$$v = v_i + at$$

$$a = \frac{\Delta v}{\Delta t} = \frac{22.0\text{ m/s} - (-25.0\text{ m/s})}{3.50 \times 10^{-3}\text{ s}} = \boxed{1.34 \times 10^4\text{ m/s}^2}$$

2.14 (a) Acceleration is constant over the first ten seconds, so at the end

$$v = v_i + at = 0 + (2.00\text{ m/s}^2)(10.0\text{ s}) = \boxed{20.0\text{ m/s}}$$

Then $a = 0$ so v is constant from $t = 10.0$ s to $t = 15.0$ s. And over the last five seconds the velocity changes to

$$v = v_i + at = 20.0\text{ m/s} - (3.00\text{ m/s}^2)(5.00\text{ s}) = \boxed{5.00\text{ m/s}}$$

(b) In the first ten seconds

$$x = x_i + v_i t + \frac{1}{2}at^2 = 0 + 0 + \frac{1}{2}(2.00\text{ m/s}^2)(10.0\text{ s})^2 = 100\text{ m}$$

Over the next five seconds the position changes to

$$x = x_i + v_i t + \frac{1}{2}at^2 = 100\text{ m} + 20.0\text{ m/s}(5.00\text{ s}) + 0 = 200\text{ m}$$

And at $t = 20.0$ s

$$x = x_i + v_i t + \frac{1}{2}at^2 = 200\text{ m} + 20.0\text{ m/s}(5.00\text{ s}) + \frac{1}{2}(-3.00\text{ m/s}^2)(5.00\text{ s})^2 = \boxed{262\text{ m}}$$

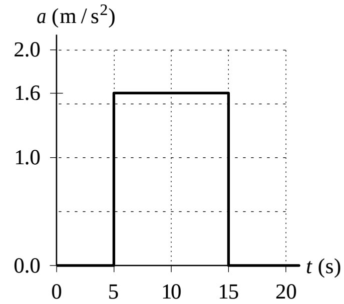
***2.15** (a) Acceleration is the slope of the graph of v vs t.

For $0 < t < 5.00$ s, $a = 0$

For $15.0 \text{ s} < t < 20.0$ s, $a = 0$

For $5.0 \text{ s} < t < 15.0$ s, $a = \frac{v_f - v_i}{t_f - t_i}$

$$a = \frac{8.00 - (-8.00)}{15.0 - 5.00} = 1.60 \text{ m/s}^2$$



We can plot $a(t)$ as shown.

(b) $a = \frac{v_f - v_i}{t_f - t_i}$

(i) For $5.00 \text{ s} < t < 15.0$ s, $t_i = 5.00$ s, $v_i = -8.00$ m/s

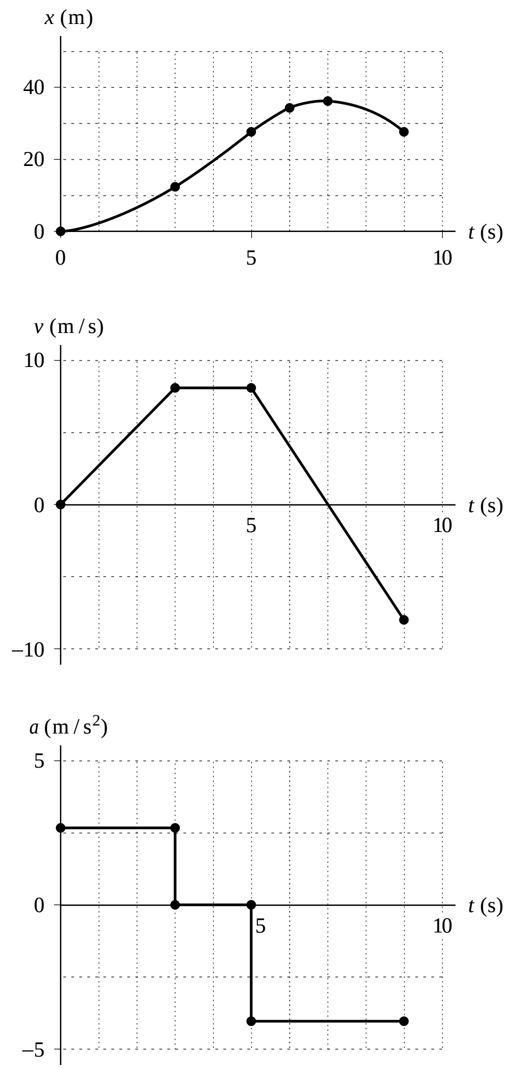
$t_f = 15.0$ s, $v_f = 8.00$ m/s;

$$a = \frac{v_f - v_i}{t_f - t_i} = \frac{8.00 - (-8.00)}{15.0 - 5.00} = \boxed{1.60 \text{ m/s}^2}$$

(ii) $t_i = 0$, $v_i = -8.00$ m/s, $t_f = 20.0$ s, $v_f = 8.00$ m/s

$$a = \frac{v_f - v_i}{t_f - t_i} = \frac{8.00 - (-8.00)}{20.0 - 0} = \boxed{0.800 \text{ m/s}^2}$$

2.16 (a) See the Graphs at the right.



Choose $x = 0$ at $t = 0$

At $t = 3$ s, $x = \frac{1}{2}(8 \text{ m/s})(3 \text{ s}) = 12$ m

At $t = 5$ s, $x = 12 \text{ m} + (8 \text{ m/s})(2 \text{ s}) = 28$ m

At $t = 7$ s, $x = 28 \text{ m} + \frac{1}{2}(8 \text{ m/s})(2 \text{ s}) = 36$ m

(b) For $0 < t < 3$ s, $a = (8 \text{ m/s})/3 \text{ s} = 2.67 \text{ m/s}^2$

For $3 < t < 5$ s, $a = 0$

(c) For $5 \text{ s} < t < 9$ s, $a = -(16 \text{ m/s})/4 \text{ s} = \boxed{-4 \text{ m/s}^2}$

(d) At $t = 6$ s, $x = 28 \text{ m} + (6 \text{ m/s})(1 \text{ s}) = \boxed{34 \text{ m}}$

(e) At $t = 9$ s, $x = 36 \text{ m} + \frac{1}{2}(-8 \text{ m/s})\ 2 \text{ s} = \boxed{28 \text{ m}}$

2.17 $x = 2.00 + 3.00t - t^2$, $v = \frac{dx}{dt} = 3.00 - 2.00t$, $a = \frac{dv}{dt} = -2.00$

At $t = 3.00$ s:

(a) $x = (2.00 + 9.00 - 9.00) \text{ m} = \boxed{2.00 \text{ m}}$

(b) $v = (3.00 - 6.00) \text{ m/s} = \boxed{-3.00 \text{ m/s}}$

(c) $a = \boxed{-2.00 \text{ m/s}^2}$

2.18 (a) At $t = 2.00$ s, $x = [3.00(2.00)^2 - 2.00(2.00) + 3.00] \text{ m} = 11.0$ m

At $t = 3.00$ s, $x = [3.00(9.00)^2 - 2.00(3.00) + 3.00] \text{ m} = 24.0$ m

so $\bar{v} = \frac{\Delta x}{\Delta t} = \frac{24.0 \text{ m} - 11.0 \text{ m}}{3.00 \text{ s} - 2.00 \text{ s}} = \boxed{13.0 \text{ m/s}}$

(b) At all times the instantaneous velocity is

$$v = \frac{d}{dt}(3.00t^2 - 2.00t + 3.00) = (6.00t - 2.00) \text{ m/s}$$

At $t = 2.00$ s, $v = [6.00(2.00) - 2.00]$ m/s = $\boxed{10.0 \text{ m/s}}$

At $t = 3.00$ s, $v = [6.00(3.00) - 2.00]$ m/s = $\boxed{16.0 \text{ m/s}}$

(c) $\bar{a} = \dfrac{\Delta v}{\Delta t} = \dfrac{16.0 \text{ m/s} - 10.0 \text{ m/s}}{3.00 \text{ s} - 2.00 \text{ s}} = \boxed{6.00 \text{ m/s}^2}$

(d) At all times $a = \dfrac{d}{dt}(6.00 - 2.00) = \boxed{6.00 \text{ m/s}^2}$

(This includes both t= 2.00 s and t= 3.00 s).

2.19 (a) $a = \dfrac{\Delta v}{\Delta t} = \dfrac{8.00 \text{ m/s}}{6.00 \text{ s}} = \boxed{\dfrac{4}{3} \text{ m/s}^2}$

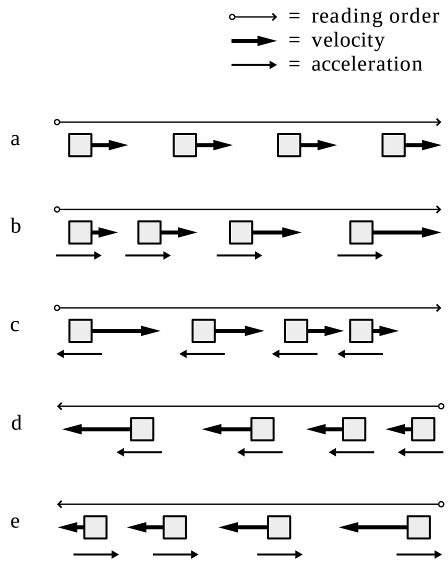
(b) Maximum positive acceleration is at $t = 3$ s, and is approximately $\boxed{2 \text{ m/s}^2}$

(c) $a = 0$, at $\boxed{t = 6 \text{ s}}$, and also for $\boxed{t > 10 \text{ s}}$

(d) Maximum negative acceleration is at $t = 8$ s, and is approximately

$\boxed{-1.5 \text{ m/s}^2}$

***2.20**



f One way of phrasing the answer: The spacing of the successive positions would change with less regularity. Another way: The object would move with some combination of the kinds of motion shown in (a) through (e). Within one drawing, the acceleration vectors would vary in magnitude and direction.

***2.21** From $v_f^2 = v_i^2 + 2ax$, we have $(10.97 \times 10^3 \text{ m/s})^2 = 0 + 2a(220 \text{ m})$, so that

$\boxed{a = 2.74 \times 10^5 \text{ m/s}^2}$ which is $\boxed{2.79 \times 10^4 \text{ times } g}$

2.22 (a) Assuming a constant acceleration:

$$a = \frac{v_f - v_i}{t} = \frac{42.0 \text{ m/s}}{8.00 \text{ s}} = \boxed{5.25 \text{ m/s}^2}$$

(b) Taking the origin at the original position of the car,

$$x = \frac{1}{2}(v_i + v_f)\,t = \frac{1}{2}(42.0 \text{ m/s})(8.00 \text{ s}) = \boxed{168 \text{ m}}$$

(c) From $v_f = v_i + at$, the velocity 10.0 s after the car starts from rest is:

$$v_f = 0 + (5.25 \text{ m/s}^2)(10.0 \text{ s}) = \boxed{52.5 \text{ m/s}}$$

***2.23** (a) $x - x_i = \frac{1}{2}(v_i + v)\,t$ becomes $40\text{ m} = \frac{1}{2}(v_i + 2.80\text{ m/s})(8.50\text{ s})$

which yields $v_i = \boxed{6.61\text{ m/s}}$

(b) $a = \frac{v - v_i}{t} = \frac{2.80\text{ m/s} - 6.61\text{ m/s}}{8.50\text{ s}} = \boxed{-0.448\text{ m/s}^2}$

2.24 Suppose the unknown acceleration is constant as a car moving at $v_i = 35.0$ mi/h comes to a $v = 0$ stop in $x - x_i = 40.0$ ft. We find its acceleration from

$$v^2 = v_i^2 + 2a(x - x_i)$$

$$a = \frac{(v^2 - v_i^2)}{2(x - x_i)}$$

$$= \frac{0 - (35.0\text{ mi/h})^2}{2(40.0\text{ ft})}\left(\frac{5280\text{ ft}}{1\text{ mi}}\right)^2\left(\frac{1\text{ h}}{3600\text{ s}}\right)^2 = -32.9\text{ ft/s}^2$$

Now consider a car moving at $v_i = 70.0$ mi/h and stopping to $v = 0$ with $a = -32.9\text{ ft/s}^2$. From the same equation its stopping distance is

$$x - x_i = \frac{v^2 - v_i^2}{2a}$$

$$= \frac{0 - (70.0\text{ mi/h})^2}{2(-32.9\text{ ft/s}^2)}\left(\frac{5280\text{ ft}}{1\text{ mi}}\right)^2\left(\frac{1\text{ h}}{3600\text{ s}}\right)^2$$

$$= \boxed{160\text{ ft}}$$

2.25 Given $v_i = 12.0$ cm/s when $x_i = 3.00$ cm ($t = 0$), and at $t = 2.00$ s, $x = -5.00$ cm

$$\Delta x = v_i t + \left(\frac{1}{2}\right) a t^2;$$

$$\Rightarrow \quad x - x_i = v_i t + \left(\frac{1}{2}\right) a t^2;$$

$$-5.00 - 3.00 = 12.0(2.00) + \left(\frac{1}{2}\right) a\,(2.00)^2;$$

$$\Rightarrow \quad -8.00 = 24.0 + 2a$$

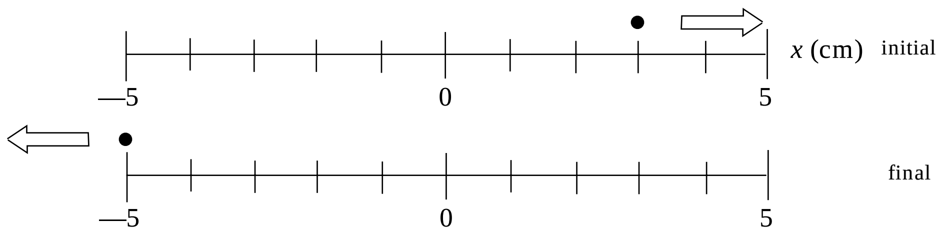
$$a = -\frac{32.0}{2} = \boxed{-16.0\text{ cm/s}^2}$$

Goal Solution

A body moving with uniform acceleration has a velocity of 12.0 cm / s in the positive x direction when its x coordinate is 3.00 cm. If its x coordinate 2.00s later is –5.00 cm, what is the magnitude of its acceleration?

G: Since the object must slow down as it moves to the right and then speeds up to the left, the acceleration must be negative and should have units of cm/s^2.

O: First we should sketch the problem to see what is happening:



Here we can see that the object travels along the x-axis, first to the right, slowing down, and then speeding up as it travels to the left in the negative x direction. We can show the position as a function of time with the notation: $x(t)$

$$x(0) = 3.00\text{ cm},\ x(2.00) = -5.00\text{ cm, and } v(0) = 12.0\text{ cm/s}$$

A: Use the kinematic equation $x - x_i = v_i t + \frac{1}{2}at^2$, and solve for a.

$$a = \frac{2(x - x_i - v_i t)}{t^2}$$

$$a = \frac{2[-5.00\text{ cm} - 3.00\text{ cm} - (12.0\text{ cm/s})(2.00\text{ s})]}{(2.00\text{ s})^2}$$

$$a = -16.0\text{ cm/s}^2$$

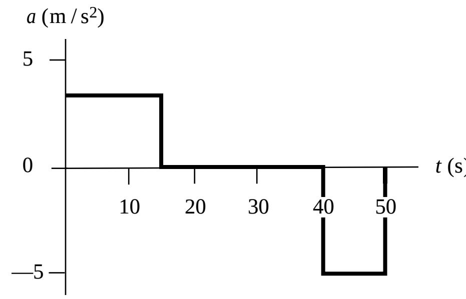
L: The acceleration is negative as expected and it has the correct units of cm/s^2. It also makes sense that the magnitude of the acceleration must be greater than 12 cm/s^2 since this is the acceleration that would cause the object to stop after 1 second and just return the object to its starting point after 2 seconds.

2.26 (a) Total displacement = area under the (v, t) curve from $t = 0$ to 50 s.

$$\Delta x = \frac{1}{2}(50\text{ m/s})(15\text{ s}) + (50\text{ m/s})(40 - 15)\text{s} + \frac{1}{2}(50\text{ m/s})(10\text{ s}) = \boxed{1875\text{ m}}$$

(b) From $t = 10$ s to $t = 40$ s, displacement (area under the curve) is

$$\Delta x = \frac{1}{2}(50\text{ m/s} + 33\text{ m/s})(5\text{ s}) + (50\text{ m/s})(25\text{ s}) = \boxed{1457\text{ m}}$$



(c) $0 \le t \le 15$ s: $\quad a_1 = \frac{\Delta v}{\Delta t} = \frac{(50 - 0)\text{ m/s}}{15\text{ s} - 0} = \boxed{3.3\text{ m/s}^2}$

$15\text{ s} < t < 40$ s: $\boxed{a_2 = 0}$

$40\text{ s} \le t \le 50$ s: $\quad a_3 = \frac{\Delta v}{\Delta t} = \frac{(0 - 50)\text{ m/s}}{50\text{ s} - 40\text{ s}} = \boxed{-5.0\text{ m/s}^2}$

(d) (i) $x_1 = 0 + \frac{1}{2}a_1 t^2 = \frac{1}{2}(3.3\text{ m/s}^2)\,t^2$, or $\boxed{x_1 = (1.67\text{ m/s}^2)t^2}$

(ii) $x_2 = \frac{1}{2}(15\text{ s})\,[50\text{ m/s} - 0] + (50\text{ m/s})(t - 15\text{ s})$, or $\boxed{x_2 = (50\text{ m/s})t - 375\text{ m}}$

(iii) For $40\text{ s} \le t \le 50$ s, $x_3 = \begin{pmatrix}\text{area under } v \text{ vs } t\\ \text{from } t = 0 \text{ to } 40\text{ s}\end{pmatrix} + \frac{1}{2}a_3(t - 40\text{ s})^2 + (50\text{ m/s})(t - 40\text{ s})$

or $x_3 = 375\text{ m} + 1250\text{ m} + \frac{1}{2}(-5.0\text{ m/s}^2)(t - 40\text{ s})^2 + (50\text{ m/s})(t - 40\text{ s})$ which reduces to

$$\boxed{x_3 = (250\text{ m/s})t - (2.5\text{ m/s}^2)t^2 - 4375\text{ m}}$$

(e) $\bar{v} = \frac{\text{total displacement}}{\text{total elapsed time}} = \frac{1875\text{ m}}{50\text{ s}} = \boxed{37.5\text{ m/s}}$

***2.27** (a) Compare the position equation $x = 2.00 + 3.00t - 4.00t^2$ to the general form $x = x_i + v_i t + \frac{1}{2}at^2$ to recognize that:

$$x_i = 2.00\text{ m}, \quad v_i = 3.00\text{ m/s}, \quad \text{and} \quad a = -8.00\text{ m/s}^2$$

The velocity equation, $v = v_i + at$, is then $v = 3.00\text{ m/s} - (8.00\text{ m/s}^2)t$.

The particle changes direction when $v = 0$, which occurs at $t = \frac{3}{8}$ s.

The position at this time is:

$x = 2.00$ m + (3.00 m/s)**Error!** s) – (4.00 m/s²)**Error!** s)² = **Error!**

(b) From $x = x_i + v_i t + \frac{1}{2} at^2$, observe that when $x = x_i$, the time is given by $t = -\frac{2v_i}{a}$. Thus, when the particle returns to its initial position, the time is

$$t = \frac{-2(3.00 \text{ m/s})}{-8.00 \text{ m/s}^2} = \frac{3}{4} \text{ s and the velocity is}$$

$$v = 3.00 \text{ m/s} - (8.00 \text{ m/s}^2)\left(\frac{3}{4} \text{ s}\right) = \boxed{-3.00 \text{ m/s}}$$

2.28 $v_i = 5.20$ m/s

(a) $v(t = 2.50 \text{ s}) = v_i + at = 5.20 \text{ m/s} + (3.00 \text{ m/s}^2)(2.50 \text{ s}) = \boxed{12.7 \text{ m/s}}$

(b) $v(t = 2.50 \text{ s}) = v_i + at = 5.20 \text{ m/s} + (-3.00 \text{ m/s}^2)(2.50 \text{ s}) = \boxed{-2.30 \text{ m/s}}$

2.29 (a) $x = \frac{1}{2} at^2$ (Eq 2.11)

$$400 \text{ m} = \frac{1}{2}(10.0 \text{ m/s}^2)\, t^2$$

$$t = \boxed{8.94 \text{ s}}$$

(b) $v = at$ (Eq 2.8)

$$v = (10.0 \text{ m/s}^2)(8.94 \text{ s}) = \boxed{89.4 \text{ m/s}}$$

2.30 (a) Take $t_i = 0$ at the bottom of the hill where $x_i = 0$, $v_i = 30.0$ m/s, and $a = -2.00$ m/s^2. Use these values in the general equation

$$x = x_i + v_i t + \frac{1}{2} at^2$$

to find $x = 0 + 30.0t \text{ m/s} + \frac{1}{2}(-2.00 \text{ m/s}^2)\, t^2$

when t is in seconds $x = \boxed{(30.0t - t^2) \text{ m}}$

To find an equation for the velocity, use

$$v = v_i + at = 30.0 \text{ m/s} + (-2.00 \text{ m/s}^2)t$$

$$v = \boxed{(30.0 - 2.00t) \text{ m/s}}$$

(b) The distance of travel x becomes a maximum, x_{max}, when $v = 0$ (turning point in the motion). Use the expressions found in part (a) for v to find the value of t when x has its maximum value:

From $v = (30.0 - 2.00t)$ m/s,

$$v = 0 \quad \text{when} \quad t = 15.0 \text{ s}$$

Then $x_{max} = (30.0t - t^2)\text{ m} = (30.0)(15.0) - (15.0)^2 = \boxed{225 \text{ m}}$

2.31 (a) $v_i = 100$ m/s, $a = -5.00$ m/s^2

$$v^2 = v_i^2 + 2ax \qquad 0 = (100)^2 - 2(5.00)x$$

$x = \boxed{1000 \text{ m}}$ and $t = \boxed{20.0 \text{ s}}$

(b) No, at this acceleration the plane would overshoot the runway.

***2.32** In the simultaneous equations

$$\begin{cases} v_x = v_{xi} + a_x t \\ x - x_i = \frac{1}{2}(v_{xi} + v_x)t \end{cases}$$

we have

$$\begin{cases} v_x = v_{xi} - (5.60 \text{ m/s}^2)(4.20 \text{ s}) \\ 62.4 \text{ m} = \frac{1}{2}(v_{xi} + v_x)4.20 \text{ s} \end{cases}$$

So substituting for v_{xi} gives

$$62.4 \text{ m} = \frac{1}{2}[v_x + (5.60 \text{ m/s}^2)(4.20 \text{ s}) + v_x]4.20 \text{ s}$$

$$14.9 \text{ m/s} = v_x + \frac{1}{2}(5.60 \text{ m/s}^2)(4.20 \text{ s})$$

$$v_x = \boxed{3.10 \text{ m/s}}$$

***2.33** Take any two of the standard four equations, such as

$$\begin{cases} v_x = v_{xi} + a_x t \\ x - x_i = \frac{1}{2}(v_{xi} + v_x)t \end{cases}$$

solve one for v_{xi}, and substitute into the other:

$$v_{xi} = v_x - a_x t$$

$$x - x_i = \frac{1}{2}(v_x - a_x t + v_x)\,t$$

Thus $\boxed{x - x_i = v_x t - \frac{1}{2}a_x t^2}$

Back in problem 32,

$$62.4\text{ m} = v_x(4.20\text{ s}) - \frac{1}{2}(-5.60\text{ m/s}^2)(4.20\text{ s})^2$$

$$v_x = \frac{62.4\text{ m} - 49.4\text{ m}}{4.20\text{ s}} = \boxed{3.10\text{ m/s}}$$

2.34 We assume the bullet is a cylinder. It slows down just as its front end pushes apart wood fibers.

(a) $a = \dfrac{v^2 - v_i^2}{2x} = \dfrac{(280\text{ m/s})^2 - (420\text{ m/s})^2}{2(0.100\text{ m})} = \boxed{-4.90 \times 10^5\text{ m/s}^2}$

(b) $t = \dfrac{0.100}{350} + \dfrac{0.020}{280} = \boxed{3.57 \times 10^{-4}\text{ s}}$

(c) $v_i = 420\text{ m/s}$, $v = 0$; $a = -4.90 \times 10^5\text{ m/s}^2$; $v^2 = v_i^2 + 2ax$

$$x = \frac{v^2 - v_i^2}{2a} = \frac{v_i^2}{2a} = -\frac{(420\text{ m/s})^2}{(-2 \times 4.90 \times 10^5\text{ m/s}^2)}$$

$$x = \boxed{0.180\text{ m}}$$

***2.35** (a) The time it takes the truck to reach 20.0 m/s is found from $v = v_i + at$,

solving for t yields $t = \dfrac{v - v_i}{a} = \dfrac{20.0\text{ m/s} - 0\text{ m/s}}{2.00\text{ m/s}^2} = 10.0\text{ s}$

The total time is thus 10.0 s + 20.0 + 5.00 s = $\boxed{35.0\text{ s}}$

(b) The average velocity is the total distance traveled divided by the total time taken. The distance traveled during the first 10.0 s is

$$x_1 = \bar{v}\,t = \left(\frac{0 + 20.0}{2}\right)(10.0) = 100 \text{ m}$$

The distance traveled during the next 20.0 s is

$$x_2 = v_i t + \frac{1}{2}at^2 = (20.0)(20.0) + 0 = 400 \text{ m}, \ a \text{ being 0 for this interval.}$$

The distance traveled in the last 5.00 s is

$$x_3 = \bar{v}\,t = \left(\frac{20.0 + 0}{2}\right)(5.00) = 50.0 \text{ m}$$

The total distance $x = x_1 + x_2 + x_3 = 100 + 400 + 50.0 = 550$ m, and the average velocity is given by

$$\bar{v} = \frac{x}{t} = \frac{550}{35.0} = \boxed{15.7 \text{ m/s}}$$

***2.36** Using the equation $x = v_i t + \frac{1}{2}at^2$ yields $x = 20.0(40.0) - 1.00(40.0)^2/2 = 0$, which is obviously wrong. The error occurs because the equation used is for uniformly accelerated motion, which this is not. The acceleration is -1.00 m/s^2 for the first 20.0 s and 0 for the last 20.0 s. The distance traveled in the first 20.0 s is:

$$x = v_i t + \frac{1}{2}at^2 = (20.0)(20.0) - 1.00(20.0^2)/2 = 200 \text{ m}$$

During the last 20.0 s, the train is at rest. Thus, the total distance traveled in the 40.0 s interval is $\boxed{200 \text{ m}}$.

2.37 (a) $a = \dfrac{v - v_i}{t} = \dfrac{632(5280/3600)}{1.40} = \boxed{-662 \text{ ft/s}^2} = -202 \text{ m/s}^2$

(b) $x = v_i t + \frac{1}{2}at^2 = (632)(5280/3600)(1.40) - \frac{1}{2}662(1.40)^2 = \boxed{649 \text{ ft}} = \boxed{198 \text{ m}}$

2.38 We have $v_i = 2.00 \times 10^4$ m/s, $v = 6.00 \times 10^6$ m/s,

$$x - x_i = 1.50 \times 10^{-2} \text{ m}$$

(a) $x - x_i = \frac{1}{2}(v_i + v)\,t$

$$t = \frac{2(x - x_i)}{v_i + v} = \frac{2(1.50 \times 10^{-2} \text{ m})}{2.00 \times 10^4 \text{ m/s} + 6.00 \times 10^6 \text{ m/s}} = \boxed{4.98 \times 10^{-9} \text{ s}}$$

(b) $v^2 = v_i^2 + 2a(x - x_i)$

$$a = \frac{v^2 - v_i^2}{2(x - x_i)} = \frac{(6.00 \times 10^6 \text{ m/s})^2 - (2.00 \times 10^4 \text{ m/s})^2}{2(1.50 \times 10^{-2} \text{ m})} = \boxed{1.20 \times 10^{15} \text{ m/s}^2}$$

2.39 (a) Take initial and final points at top and bottom of the incline.

If the ball starts from rest, $v_i = 0$, $a = 0.500 \text{ m/s}^2$, $x - x_i = 9.00 \text{ m}$

Then $v^2 = v_i^2 + 2a(x - x_i) = 0^2 + 2(0.5\,00 \text{ m/s}^2)\,9.00 \text{ m}$

$$v = \boxed{3.00 \text{ m/s}}$$

(b) $x - x_i = v_i t + \frac{1}{2} a t^2$

$$9.00 \text{ m} = 0 + \frac{1}{2}(0.500 \text{ m/s}^2)\, t^2$$

$$t = \boxed{6.00 \text{ s}}$$

(c) Take initial and final points at the bottom of the planes and the top of the second plane, respectively. $v_i = 3.00 \text{ m/s}$ $v = 0$ $x - x_i = 15.00 \text{ m}$

$$v^2 = v_i^2 + 2a(x - x_i)$$

gives

$$a = \frac{(v^2 - v_i^2)}{2(x - x_i)} = \frac{[0 - (3.00 \text{ m/s})^2]}{2(15.0 \text{ m})}$$

$$= \boxed{-0.300 \text{ m/s}^2}$$

(d) Take initial point at the bottom of the planes and final point 8.00 m along the second:
$v_i = 3.00 \text{ m/s}$ $x - x_i = 8.00 \text{ m}$ $a = -0.300 \text{ m/s}^2$

$$v^2 = v_i^2 + 2a(x - x_i)$$

$$= (3.00 \text{ m/s})^2 + 2(-0.300 \text{ m/s}^2)(8.00 \text{ m}) = 4.20 \text{ m}^2/\text{s}^2$$

$$v = \boxed{2.05 \text{ m/s}}$$

2.40 Take the original point to be when Sue notices the van. Choose the origin of the x-axis at Sue's car. For her we have

$$x_{is} = 0 \qquad v_{is} = 30.0 \text{ m/s} \qquad a_S = -2.00 \text{ m/s}^2$$

so her position is given by

$$x_s(t) = x_{is} + v_{is}\,t + \frac{1}{2}\,a_s t^2$$

$$= (30.0 \text{ m/s})t + \frac{1}{2}(-2.00 \text{ m/s}^2)\,t^2$$

For the van, $x_{iv} = 155$ m $\quad v_{iv} = 5.00$ m/s $\quad a_v = 0$ and

$$x_v(t) = x_{iv} + v_{iv}t + \frac{1}{2}\,a_v t^2 = 155 \text{ m} + (5.00 \text{ m/s})t + 0$$

To test for a collision, we look for an instant t_c when both are at the same place:

$$30.0t_c - t_c^2 = 155 + 5.00t_c$$

$$0 = t_c^2 - 25.0t_c + 155$$

From the quadratic formula

$$t_c = \frac{25.0 \pm \sqrt{(25.0)^2 - 4(155)}}{2} = 13.6 \text{ s or } \boxed{11.4 \text{ s}}$$

The smaller value is the collision time. (The larger value tells when the van would pull ahead again if the vehicles could move through each other). The wreck happens at position 155 m + $(5.00 \text{ m/s})(11.4 \text{ s}) = \boxed{212 \text{ m}}$.

2.41 Choose the origin ($y = 0$, $t = 0$) at the starting point of the ball and take upward as positive. Then, $y_i = 0$, $v_i = 0$, and $a = -g = -9.80 \text{ m/s}^2$. The position and the velocity at time t become:

$$y - y_i = v_i t + \frac{1}{2}\,at^2 \Rightarrow y = -\frac{1}{2}\,gt^2 = -\frac{1}{2}(9.80 \text{ m/s}^2)\,t^2$$

and $\quad v = v_i + at \Rightarrow v = -gt = -(9.80 \text{ m/s}^2)t$

(a) at $t = 1.00$ s: $y = -\frac{1}{2}(9.80 \text{ m/s}^2)(1.00 \text{ s})^2 = \boxed{-4.90 \text{ m}}$

at $t = 2.00$ s: $y = -\frac{1}{2}(9.80 \text{ m/s}^2)(2.00 \text{ s})^2 = \boxed{-19.6 \text{ m}}$

at $t = 3.00$ s: $y = -\frac{1}{2}(9.80 \text{ m/s}^2)(3.00 \text{ s})^2 = \boxed{-44.1 \text{ m}}$

(b) at t = 1.00 s: $v = -(9.80 \text{ m/s}^2)(1.00 \text{ s}) = \boxed{-9.80 \text{ m/s}}$

at t = 2.00 s: $v = -(9.80 \text{ m/s}^2)(2.00 \text{ s}) = \boxed{-19.6 \text{ m/s}}$

at t = 3.00 s: $v = -(9.80 \text{ m/s}^2)(3.00 \text{ s}) = \boxed{-29.4 \text{ m/s}}$

***2.42** Assume that air resistance may be neglected. Then, the acceleration at all times during the flight is that due to gravity, $a = -g = -9.80 \text{ m/s}^2$. During the flight, Goff went 1 mile (1609 m) up and then 1 mile back down. Determine his speed just after launch by considering his upward flight:

$$v^2 = v_i^2 + 2a(y - y_i) \Rightarrow 0 = v_i^2 - 2(9.80 \text{ m/s}^2)(1609 \text{ m}) \Rightarrow v_i = 178 \text{ m/s}$$

His time in the air may be found by considering his motion from just after launch to just before impact:

$$y - y_i = v_i t + \frac{1}{2} at^2 \Rightarrow 0 = (178 \text{ m/s})t - \frac{1}{2}(-9.80 \text{ m/s}^2)\, t^2$$

The root $t = 0$ describes launch; the other root, $t = 36.2$ s, describes his flight time. His rate of pay may then be found from

$$pay\ \ rate = \frac{\$1.00}{36.2 \text{ s}} = \left(0.0276 \frac{\$}{\text{s}}\right)\left(\frac{3600 \text{ s}}{1 \text{ h}}\right) = \boxed{\$99.4/\text{h}}$$

2.43 (a) $y = v_i t + \frac{1}{2} at^2$

$4.00 = (1.50)v_i - (4.90)(1.50)^2$ and $\boxed{v_i = 10.0 \text{ m/s upward}}$

(b) $v = v_i + at = 10.0 - (9.80)(1.50) = -4.68 \text{ m/s}$

$v = \boxed{4.68 \text{ m/s downward}}$

2.44 We have

$$y = -\frac{1}{2} gt^2 + v_i t + y_i$$

$$0 = -(4.90 \text{ m/s}^2)t^2 - (8.00 \text{ m/s})t + 30.0 \text{ m}$$

Solving for t, $t = \dfrac{8.00 \pm \sqrt{64.0 + 588}}{-9.80}$

Using only the positive value for t, we find $t = \boxed{1.79 \text{ s}}$

***2.45** The bill starts from rest $v_i = 0$ and falls with a downward acceleration of $9.80\ \text{m/s}^2$ (due to gravity). Thus, in 0.20 s it will fall a distance of

$$\Delta y = v_i t - \frac{1}{2} gt^2 = 0 - (4.90\ \text{m/s}^2)(0.20\ \text{s})^2 = -0.20\ \text{m}$$

This distance is about twice the distance between the center of the bill and its top edge (≅ 8 cm). Thus, David will be unsuccessful .

Goal Solution

Emily challenges her friend David to catch a dollar bill as follows. She holds the bill vertically, as in Figure P2.45, with the center of the bill between David's index finger and thumb. David must catch the bill after Emily releases it without moving his hand downward. If his reaction time is 0.2 s, will he succeed? Explain your reasoning.

G: David will be successful if his reaction time is short enough that he can catch the bill before it falls half of its length (about 8 cm). Anyone who has tried this challenge knows that this is a difficult task unless the catcher "cheats" by anticipating the moment the bill is released. Since David's reaction time of 0.2 s is typical of most people, we should suspect that he will not succeed in meeting Emily's challenge.

O: Since the bill is released from rest and experiences free fall, we can use the equation $y = \frac{1}{2} gt^2$ to find the distance y the bill falls in $t = 0.2$ s

A: $y = \frac{1}{2}(9.80\ \text{m/s}^2)(0.2\ \text{s})^2 = 0.196\ \text{m} > 0.08\ \text{m}$

Since the bill falls below David's fingers before he reacts, he will not catch it.

L: It appears that even if David held his fingers at the bottom of the bill (about 16 cm below the top edge), he still would not catch the bill unless he reduced his reaction time by tensing his arm muscles or anticipating the drop.

***2.46** At any time t, the position of the ball released from rest is given by $y_1 = h - \frac{1}{2} gt^2$. At time t, the position of the ball thrown vertically upward is described by $y_2 = v_i t - \frac{1}{2} gt^2$.

The time at which the first ball has a position of $y_1 = h/2$ is found from the first equation as $h/2 = h - \frac{1}{2} gt^2$, which yields $t = \sqrt{h/g}$. To require that the second ball have a position of $y_2 = h/2$ at this time, use the second equation to obtain $h/2 = v_i\sqrt{h/g} - \frac{1}{2} g(h/g)$. This gives the required initial upward velocity of the second ball as $\boxed{v_i = \sqrt{gh}}$.

2.47 (a) $v = v_i - gt$ (Eq. 2.8)

$v = 0$ when $t = 3.00$ s, $g = 9.80$ m/s^2,

$\therefore\ v_i = gt = (9.80\ \text{m/s}^2)(3.00\ \text{s}) = \boxed{29.4\ \text{m/s}}$

(b) $y = \frac{1}{2}(v + v_i)\,t = \frac{1}{2}(29.4\ \text{m/s})(3.00\ \text{s}) = \boxed{44.1\ \text{m}}$

Goal Solution

A baseball is hit such that it travels straight upward after being struck by the bat. A fan observes that it requires 3.00 s for the ball to reach its maximum height. Find (a) its initial velocity and (b) the maximum height it reaches.

G: We can expect the initial speed of the ball to be somewhat greater than the speed of the pitch, which might be about 60 mph (~30 m/s), so an initial upward velocity off the bat of somewhere between 20 and 100 m/s would be reasonable. We also know that the length of a ball field is about 300 ft. (~100m), and a pop-fly usually does not go higher than this distance, so a maximum height of 10 to 100 m would be reasonable for the situation described in this problem.

O: Since the ball's motion is entirely vertical, we can use the equation for free fall to find the initial velocity and maximum height from the elapsed time.

A: Choose the "initial" point when the ball has just left contact with the bat. Choose the "final" point at the top of its flight. In between, the ball is in free fall for $t = 3.00$ s and has constant acceleration, $a = -g = -9.80$ m/s^2. Solve the equation $v_{yf} = v_{yi} - gt$ for v_{yi} when $v_{yf} = 0$ (when the ball reaches its maximum height).

(a) $v_{yi} = v_{yf} + gt = 0 + (9.80\ \text{m/s}^2)(3.00\ \text{s}) = 29.4\ \text{m/s}$ (upward)

(b) The maximum height in the vertical direction is

$$y_f = v_{yi}\,t + \frac{1}{2}\,at^2 = (29.4\ \text{m/s})(3.00\ \text{s}) + \frac{1}{2}(-9.80\ \text{m/s}^2)(3.00\ \text{s})^2 = 44.1\ \text{m}$$

L: The calculated answers seem reasonable since they lie within our expected ranges, and they have the correct units and direction. We must remember that it is possible to solve a problem like this correctly, yet the answers may not seem reasonable simply because the conditions stated in the problem may not be physically possible (e.g. a time of 10 seconds for a pop fly would not be realistic).

2.48 Take downward as the positive y direction.

(a) While the woman was in free fall,

$$\Delta y = 144 \text{ ft}, \; v_i = 0, \text{ and } a = g = 32.0 \text{ ft/s}^2$$

Thus,

$$\Delta y = v_i t + \frac{1}{2} a t^2 \rightarrow 144 \text{ ft} = 0 + (16.0 \text{ ft/s}^2) t^2$$

giving $t_{\text{fall}} = 3.00$ s.

Her velocity just before impact is:

$$v = v_i + gt = 0 + (32.0 \text{ ft/s}^2)(3.00 \text{ s}) = \boxed{96.0 \text{ ft/s}} \; .$$

(b) While crushing the box, $v_i = 96.0$ ft/s, $v = 0$, and $\Delta y = 18.0$ in $= 1.50$ ft.

Therefore, $a = \dfrac{v^2 - v_i^2}{2(\Delta y)} = \dfrac{0 - (96.0 \text{ ft/s})^2}{2(1.50 \text{ ft})} = -3.07 \times 10^3 \text{ ft/s}^2$,

or $\boxed{a = 3.07 \times 10^3 \text{ ft/s}^2 \text{ upward}}$

(c) Time to crush box:

$$\Delta t = \frac{\Delta y}{\bar{v}} = \frac{\Delta y}{(v + v_i)/2} = \frac{2(1.50 \text{ ft})}{0 + 96.0 \text{ ft/s}}$$

or $\boxed{\Delta t = 3.13 \times 10^{-2} \text{ s}}$

2.49 Time to fall 3.00 m is found from Eq. 2.11 with $v_i = 0$,

$$3.00 \text{ m} = \frac{1}{2}(9.80 \text{ m/s}^2) \; t^2; \qquad t = 0.782 \text{ s}$$

(a) With the horse galloping at 10.0 m/s, the horizontal distance is

$$vt = \boxed{7.82 \text{ m}}$$

(b) $t = \boxed{0.782 \text{ s}}$

2.50 Time to top = 10.0 s. $v = v_i - gt$

(a) At the top, $v = 0$. Then, $t = \frac{v_i}{g} = 10.0 \text{ s}$ $v_i = \boxed{98.0 \text{ m/s}}$

(b) $h = v_i t - \frac{1}{2} g t^2$

At $t = 10.0$ s, $h = (98.0)(10.0) - \frac{1}{2}(9.80)(10.0)^2 = \boxed{490 \text{ m}}$

2.51 $v_i = 15.0 \text{ m/s}$

(a) $v = v_i - gt = 0$

$t = \frac{v_i}{g} = \frac{15.0 \text{ m/s}}{9.80 \text{ m/s}^2} = \boxed{1.53 \text{ s}}$

(b) $h = v_i t - \frac{1}{2} g t^2 = \frac{v_i^2}{2g} = \frac{225}{19.6} \text{ m} = \boxed{11.5 \text{ m}}$

(c) At $t = 2.00$ s

$v = v_i - gt = 15.0 - 19.6 = \boxed{-4.60 \text{ m/s}}$

$a = -g = \boxed{-9.80 \text{ m/s}^2}$

2.52 $y = 3.00t^3$

At $t = 2.00$ s, $y = 3.00(2.00)^3 = 24.0$ m, and $v_y = \frac{dy}{dt} = 9.00t^2 = 36.0 \text{ m/s} \uparrow$

If the helicopter releases a small mailbag at this time, the equation of motion of the mailbag is

$$y_b = y_{bi} + v_i t - \frac{1}{2} g t^2 = 24.0 + 36.0t - \frac{1}{2}(9.80)\, t^2$$

Setting $y_b = 0$, $0 = 24.0 + 36.0t - 4.90t^2$

Solving for t, (only positive values of t count), $\boxed{t = 7.96 \text{ s}}$

2.53 (a) $J = \frac{da}{dt}$ = constant

$da = Jdt \qquad a = J\int dt = Jt + c_2$

but $a = a_i$ when $t = 0$ so $c_1 = a_i$,

Therefore, $\boxed{a = Jt + a_i}$

$a = \frac{dv}{dt}$; $dv = adt$

$v = \int adt = \int (Jt + a_i)dt = \frac{1}{2}Jt^2 + a_i t + c_2$

but $v = v_i$ when $t = 0$, so $c_2 = v_i$ and $\boxed{v = \frac{1}{2}Jt^2 + a_i t + v_i}$

$v = \frac{dx}{dt}$; $dx = vdt$

$x = \int vdt = \int\left(\frac{1}{2}Jt^2 + a_i t + v_i\right) dt$

$x = \frac{1}{6}Jt^3 + \frac{1}{2}a_i t^2 + v_i t + c_3$

$x = x_i$ when $t = 0$, so $c_3 = x_i$

Therefore, $\boxed{x = \frac{1}{6}Jt^3 + \frac{1}{2}a_i t^2 + v_i t + x_i}$

(b) $a^2 = (Jt + a_i)^2 = J^2t^2 + a_i^2 + 2Ja_i t$

$a^2 = a_i^2 + (J^2t^2 + 2Ja_i t)$

$a^2 = a_i^2 + 2J\left(\frac{1}{2}Jt^2 + a_i t\right)$

Recall the expression for v: $v = \frac{1}{2}Jt^2 + a_i t + v_i$

So $(v - v_i) = \frac{1}{2}Jt^2 + a_i t$

Therefore, $\boxed{a^2 = a_i^2 + 2J(v - v_i)}$

2.54 (a) $a = \frac{dv}{dt} = \frac{d}{dt}[-5.00 \times 10^7\, t^2 + 3.00 \times 10^5\, t]$

Error! $t + 3.00 \times 10^5$ m / s^2)

Take $x_i = 0$ at $t = 0$. Then $v = \frac{dx}{dt}$

$$x - 0 = \int_0^t v\,dt = \int_0^t (-5.00 \times 10^7\, t^2 + 3.00 \times 10^5 t)\, dt$$

$$x = -5.00 \times 10^7 \frac{t^3}{3} + 3.00 \times 10^5 \frac{t^2}{2}$$

$$\boxed{x = -(1.67 \times 10^7 \text{ m / s}^3)t^3 + (1.50 \times 10^5 \text{ m / s}^2)t^2}$$

(b) The bullet escapes when $a = 0$, at $-(10.0 \times 10^7 \text{ m / s}^3)t + 3.00 \times 10^5 \text{ m / s}^2 = 0$

$$t = \frac{3.00 \times 10^5 \text{ s}}{10.0 \times 10^7} = \boxed{3.00 \times 10^{-3} \text{ s}}$$

(c) New $v = (-5.00 \times 10^7)(3.00 \times 10^{-3})^2 + (3.00 \times 10^5)(3.00 \times 10^{-3})$

$$v = -450 \text{ m / s} + 900 \text{ m / s} = \boxed{450 \text{ m / s}}$$

(d) $x = -(1.67 \times 10^7)(3.00 \times 10^{-3})^3 + (1.50 \times 10^5)(3.00 \times 10^{-3})^2$

$$x = -0.450 \text{ m} + 1.35 \text{ m} = \boxed{0.900 \text{ m}}$$

2.55 $a = \frac{dv}{dt} = -3.00v^2$, $v_i = 1.50$ m / s

Solving for v, $\frac{dv}{dt} = -3.00v^2$

$$\int_{v = v_i}^{v} v^{-2} dv = -3.00 \int_{t=0}^{0} dt$$

$$-\frac{1}{v} + \frac{1}{v_i} = -3.00t \quad \text{or} \quad 3.00t = \frac{1}{v} - \frac{1}{v_i}$$

When $v = \frac{v_i}{2}$, $t = \frac{1}{3.00\, v_i} = \boxed{0.222 \text{ s}}$

2.56 (a) The minimum distance required for the motorist to stop, from an initial speed of 18.0 m/s, is

$$\Delta x = \frac{v^2 - v_i^2}{2a} = \frac{0 - (18.0\ \text{m/s})^2}{2(-4.50\ \text{m/s}^2)} = 36.0\ \text{m}$$

Thus, the motorist can travel at most (38.0 m – 36.0 m) = 2.00 m before putting on the brakes if he is to avoid hitting the deer. The maximum acceptable reaction time is then

$$t_{\text{max}} = \frac{2.00\ \text{m}}{v_i} = \frac{2.00\ \text{m}}{18.0\ \text{m/s}} = \boxed{0.111\ \text{s}}$$

(b) In 0.300 s, the distance traveled at 18.0 m/s is

$$x = v_i t_1 = (18.0\ \text{m/s})(0.300) = 5.40\ \text{m}$$

∴ The displacement for an acceleration – 4.50 m/s² is 38.0 – 5.40 = 32.6 m.

$$v^2 = v_i^2 + 2ax = (18.0\ \text{m/s})^2 - 2(4.50\ \text{m/s}^2)(32.6\ \text{m}) = 30.6\ \text{m}^2/\text{s}^2$$

$$v = \sqrt{30.6} = \boxed{5.53\ \text{m/s}}$$

2.57 The total time to reach the ground is given by

$$y - y_i = v_i t + \frac{1}{2} a t^2$$

$$0 - 25.0\ \text{m} = 0 + \frac{1}{2}(-9.80\ \text{m/s}^2)\ t^2$$

$$t = \sqrt{\frac{2(25.0\ \text{m})}{9.80\ \text{m/s}^2}} = 2.26\ \text{s}$$

The time to fall the first fifteen meters is found similarly:

$$-15.0\ \text{m} = 0 - \frac{1}{2}(9.80\ \text{m/s}^2)\ t_1^2$$

$$t_1 = 1.75\ \text{s}$$

So $t - t_1 = 2.26\ \text{s} - 1.75\ \text{s} = \boxed{0.509\ \text{s}}$ suffices for the last ten meters.

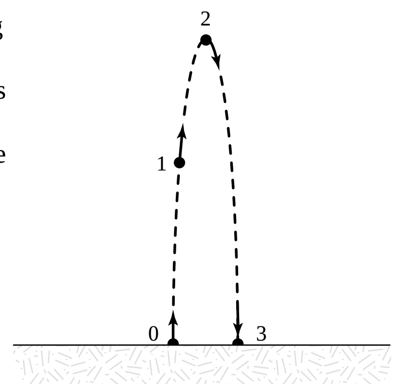
***2.58** The rate of hair growth is a velocity and the rate of its increase is an acceleration. Then $v_i = 1.04 \text{ mm/d}$ and $a = 0.132\left(\frac{\text{mm/d}}{\text{w}}\right)$. The increase in the length of the hair (i.e., displacement) during a time of $t = 5.00 \text{ w} = 35.0 \text{ d}$ is

$$\Delta x = v_i t + \frac{1}{2} a t^2$$

$$\Delta x = (1.04 \text{ mm/d})(35.0 \text{ d}) + \frac{1}{2}(0.132 \text{ mm/d} \cdot \text{w})(35.0 \text{ d})(5.00 \text{ w})$$

or $\boxed{\Delta x = 48.0 \text{ mm}}$

2.59 Let path (#1) correspond to the motion of the rocket accelerating under its own power. Path (#2) is the motion of the rocket under the influence of gravity with the rocket still rising. Path (#3) is the motion of the rocket under the influence of gravity, but with the rocket falling. The data in the table is found for each phase of the rocket's motion.



(#1): $v^2 - (80.0)^2 = 2(4.00)(1000)$; therefore $v = 120 \text{ m/s}$

$120 = 80.0 + (4.00)t$ giving $t = 10.0 \text{ s}$

(#2): $0 - (120)^2 = 2(-9.80)\Delta x$ giving $\Delta x = 735 \text{ m}$

$0 - 120 = -9.80t$ giving $t = 12.2 \text{ s}$

This is the time of maximum height of the rocket.

(#3): $v^2 - 0 = 2(-9.80)(-1735)$

$v = -184 = (-9.80)t$ giving $t = 18.8 \text{ s}$

(a) $t_{\text{total}} = 10 + 12.2 + 18.8 = \boxed{41.0 \text{ s}}$

(b) $\Delta x_{\text{total}} = \boxed{1.73 \text{ km}}$

(c) $v_{\text{final}} = \boxed{-184 \text{ m/s}}$

		t	x	v	a
0	Launch	0	0	80	+4.00
#1	End Thrust	10.0	1000	120	+4.00
#2	Rise Upwards	22.2	1735	0	–9.80
#3	Fall to Earth	41.0	0	–184	–9.80

2.60 Distance traveled by motorist = $(15.0\ \text{m/s})t$

Distance traveled by policeman = $\frac{1}{2}(2.00\ \text{m/s}^2)\,t^2$

(a) intercept occurs when $15.0t = t^2$ $\quad t = \boxed{15.0\ \text{s}}$

(b) v (officer) = $(2.00\ \text{m/s}^2)t = \boxed{30.0\ \text{m/s}}$

(c) x (officer) = $\frac{1}{2}(2.00\ \text{m/s}^2)\,t^2 = \boxed{225\ \text{m}}$

***2.61** Area A_1 is a rectangle. Thus, $A_1 = hw = v_i t$.

Area A_2 is triangular.

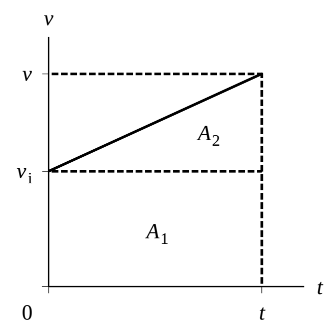
Therefore $A_2 = \frac{1}{2}\,bh = \frac{1}{2}\,t(v - v_i)$.

The total area under the curve is

$$A = A_1 + A_2 = v_i t + (v - v_i)t/2$$

and since $v - v_i = at$

$$\boxed{A = v_i t + \frac{1}{2}at^2}$$



The displacement given by the equation is:

$x = v_i t + \frac{1}{2}\,at^2$, the same result as above for the total area.

2.62 $a_1 = 0.100\ \text{m/s}^2$, $a_2 = -0.500\ \text{m/s}^2$

$$x = 1000\ \text{m} = \frac{1}{2}\,a_1 t_1^2 + v_1 t_2 + \frac{1}{2}\,a_2 t_2^2$$

$t = t_1 + t_2$ and $v_1 = a_1 t_1 = -a_2 t_2$

$$1000 = \frac{1}{2}\,a_1 t_1^2 + a_1 t_1\left(-\frac{a_1 t_1}{a_2}\right) + \frac{1}{2}\,a_2\left(\frac{a_1 t_1}{a_2}\right)^2$$

$$1000 = \frac{1}{2}\, a_1 \left(1 - \frac{a_1}{a_2}\right) t_1^2$$

$$t_1 = \sqrt{\frac{20{,}000}{1.20}} = \boxed{129\text{ s}}$$

$$t_2 = \frac{a_1 t_1}{-a_2} = \frac{12.9}{0.500} \approx 26\text{ s}$$

Total time = t = $\boxed{155\text{ s}}$

2.63 (a) Let x be the distance traveled at acceleration a until maximum speed v is reached. If this achieved in time t_1 we can use the following three equations:

$$x = \frac{(v + v_i)}{2}\, t_1, \qquad 100 - x = v(10.2 - t_1) \quad \text{and} \quad v = v_i + at_1$$

The first two give $100 = \left(10.2 - \frac{1}{2}\, t_1\right) v = \left(10.2 - \frac{1}{2}\, t_1\right) a t_1$

$$a = \frac{200}{(20.4 - t_1)t_1} \;.$$

For Maggie $a = \dfrac{200}{(18.4)(2.00)} = \boxed{5.43\text{ m/s}^2}$

For Judy $a = \dfrac{200}{(17.4)(3.00)} = \boxed{3.83\text{ m/s}^2}$

(b) $v = at_1$

Maggie: $v = (5.43)(2.00) = \boxed{10.9\text{ m/s}}$

Judy: $v = (3.83)(3.00) = \boxed{11.5\text{ m/s}}$

(c) At the six-second mark $x = \frac{1}{2}\; at_1^2 \; + v(6.00 - t_1)$

Maggie: $x = \frac{1}{2}(5.43)(2.00)^2 + (10.9)(4.00) = 54.3\text{ m}$

Judy: $x = \frac{1}{2}(3.83)(3.00)^2 + (11.5)(3.00) = 51.7\text{ m}$

Maggie is ahead by $\boxed{2.62\text{ m}}$.

***2.64** Let the ball fall 1.50 m. It strikes at speed given by:

$$v_x^2 = v_{xi}^2 + 2a(x - x_i)$$

$$v_x^2 = 0 + 2(-9.80 \text{ m/s}^2)(-1.50 \text{ m})$$

$$v_x = -5.42 \text{ m/s}$$

and its stopping is described by

$$v_x^2 = v_{xi}^2 + 2a_x(x - x_i)$$

$$0 = (-5.42 \text{ m/s})^2 + 2a_x(-10^{-2} \text{ m})$$

$$a_x = \frac{-29.4 \text{ m}^2/\text{s}^2}{-2.00 \times 10^{-2} \text{ m}} = +1.47 \times 10^3 \text{ m/s}^2$$

Its maximum acceleration will be larger than the average acceleration we estimate by imagining constant acceleration, but will still be of order of magnitude $\boxed{\sim 10^3 \text{ m/s}^2}$.

2.65 Acceleration $a = 3.00 \text{ m/s}^2$ Deceleration $a' = -4.50 \text{ m/s}^2$

(a) Keeping track of speed and time for each phase of motion,

$v_0 = 0$, $v_1 = 12.0$ m/s $\Delta t_{01} = 4.00$ s

$v_1 = 12.0$ m/s $t_1 = 5.00$ s

$v_1 = 12.0$ m/s, $v_2 = 0$ $\Delta t_{12} = 2.67$ s

$v_2 = 0$ m/s, $v_3 = 18.0$ m/s $\Delta t_{23} = 6.00$ s

$v_3 = 18.0$ m/s $t_3 = 20.0$ s

$v_3 = 18.0$ m/s, $v_4 = 6.00$ m/s $\Delta t_{34} = 2.67$ s

$v_4 = 6.00$ m/s $t_4 = 4.00$ s

$v_4 = 6.00$ m/s, $v_5 = 0$ $\Delta t_{45} = 1.33$ s

$$\boxed{\Sigma t = 45.7 \text{ s}}$$

(b) $x = \Sigma \bar{v}_i t_i = 6.00(4.00) + 12.0(5.00) + 6.00(2.67) + 9.00(6.00) + 18.0(20.0) + 12.0(2.67)$

$+ 6.00(4.00) + 3.00(1.33) = \boxed{574 \text{ m}}$

(c) $\bar{v} = \dfrac{\Delta x}{\Delta t} = \dfrac{574 \text{ m}}{45.7 \text{ s}} = \boxed{12.6 \text{ m/s}}$

(d) $t_{\text{WALK}} = \dfrac{2\Delta x}{v_{\text{WALK}}} = \dfrac{2(574\text{ m})}{(1.50\text{ m/s})} = \boxed{765\text{ s}}$

(about 13 minutes, and better exercise!)

2.66 (a) $d = \frac{1}{2}(9.80)\ t_1^2 \qquad d = 336t_2$

$t_1 + t_2 = 2.40$

$336t_2 = 4.90(2.40 - t_2)^2$

$4.90t_2^2 - 359.5t_2 + 28.22 = 0$

$$t_2 = \frac{359.5 \pm \sqrt{(359.5)^2 - 4(4.90)(28.22)}}{9.80}$$

$$t_2 = \frac{359.5 \pm 358.75}{9.80} = 0.0765\text{ s}$$

$\therefore d = 336t_2 = \boxed{26.4\text{ m}}$

(b) Ignoring the sound travel time, $d = \frac{1}{2}(9.80)(2.40)^2 = 28.2$ m, an error of $\boxed{6.82\%}$.

2.67 (a) $y = v_{i1}t + \frac{1}{2}at^2 = 50.0 = 2.00t + \frac{1}{2}(9.80)\ t^2$

$t = \boxed{2.99\text{ s}}$ after the first stone is thrown.

(b) $y = v_{i2}t + \frac{1}{2}at^2$ and $t = 2.99 - 1.00 = 1.99$ s

substitute $50.0 = v_{i2}(1.99) + \frac{1}{2}(9.80)(1.99)^2$

$v_{i2} = \boxed{15.4\text{ m/s}}$ downward

(c) $v_1 = v_{i1} + at = -2.00 + (-9.80)(2.99) = \boxed{-31.3\text{ m/s}}$

$v_2 = v_{i2} + at = -15.3 + (-9.80)(1.99) = \boxed{-34.9\text{ m/s}}$

2.68 The time required for the car to come to rest and the time required to regain its original speed of 25.0 m/s are both given by $\Delta t = \dfrac{|\Delta v|}{|a|} = \dfrac{25.0\text{ m/s}}{2.50\text{ m/s}^2}$. The total distance the car travels in these two intervals is

$$x_{\text{car}} = \Delta x_1 + \Delta x_2 = \frac{(25.0\text{ m/s} + 0)}{2}(10.0\text{ s}) + \frac{(0 + 25.0\text{ m/s})}{2}(10.0\text{ s}) = 250\text{ m}$$

The total elapsed time when the car regains its original speed is

$$\Delta t_{\text{total}} = 10.0 \text{ s} + 45.0 \text{ s} + 10.0 \text{ s} = 65.0 \text{ s}$$

The distance the train has traveled in this time is

$$x_{\text{train}} = (25.0 \text{ m/s})(65.0 \text{ s}) = 1.63 \times 10^3 \text{ m}$$

Thus, the train is $1.63 \times 10^3 \text{ m} - 250 \text{ m} = \boxed{1.38 \times 10^3 \text{ m}}$ ahead of the car.

2.69 (a) We require $x_s = x_k$ when $t_s = t_k + 1.00$

$$x_s = \frac{1}{2}(3.50 \text{ m/s}^2)(t_k + 1.00)^2 = \frac{1}{2}(4.90 \text{ m/s}^2)(t_k)^2 = x_k$$

$$t_k + 1.00 = 1.183 t_k$$

$$t_k = \boxed{5.46 \text{ s}}$$

(b) $x_k = \frac{1}{2}(4.90 \text{ m/s}^2)(5.46 \text{ s})^2 = \boxed{73.0 \text{ m}}$

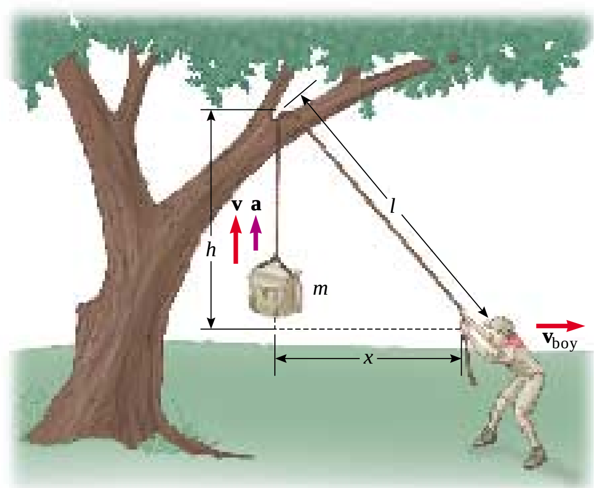
(c) $v_k = (4.90 \text{ m/s}^2)(5.46 \text{ s}) = \boxed{26.7 \text{ m/s}}$

$v_s = (3.50 \text{ m/s}^2)(6.46 \text{ s}) = \boxed{22.6 \text{ m/s}}$

2.70 (a) In walking a distance Δx, in a time Δt, the length of rope l is only increased by $\Delta x \sin\theta$.

$\therefore$ The pack lifts at a rate $\frac{\Delta x}{\Delta t} \sin\theta$.

$$v = \frac{\Delta x}{\Delta t} \sin\theta = v_{\text{boy}} \frac{x}{l} = \boxed{v_{\text{boy}} \frac{x}{\sqrt{x^2 + h^2}}}$$



(b) $a = \frac{dv}{dt} = \frac{v_{\text{boy}}}{l}\frac{dx}{dt} + v_{\text{boy}} x \frac{d}{dt}\left(\frac{1}{l}\right)$

$$a = v_{\text{boy}} \frac{v_{\text{boy}}}{l} - \frac{v_{\text{boy}} x}{l^2}\frac{dl}{dt}, \text{ but } \frac{dl}{dt} = v$$

$$\therefore a = \frac{v_{\text{boy}}^2}{l}\left(1 - \frac{x^2}{l^2}\right) = \frac{v_{\text{boy}}^2}{l}\frac{h^2}{l^2} = \boxed{\frac{h^2 v_{\text{boy}}^2}{(x^2 + h^2)^{3/2}}}$$

(c) $\frac{v_{\text{boy}}^2}{h}$, 0

(d) v_{boy}, 0

2.71 $h = 6.00\text{ m}$, $v_{\text{boy}} = 2.00\text{ m/s}$

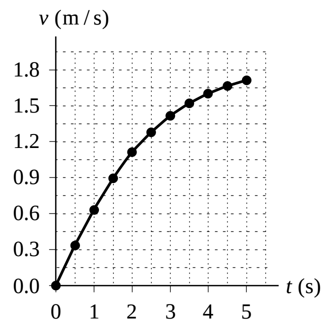
$$v = \frac{\Delta x}{\Delta t} \quad \sin\theta = v_{\text{boy}}\frac{x}{l} = \frac{v_{\text{boy}}x}{(x^2+h^2)^{1/2}}$$

However $x = v_{\text{boy}}t$

$$\therefore v = \frac{v_{\text{boy}}^2 t}{(v_{\text{boy}}^2 t^2 + h^2)^{1/2}} = \frac{4t}{(4t^2+36)^{1/2}}$$

(a)

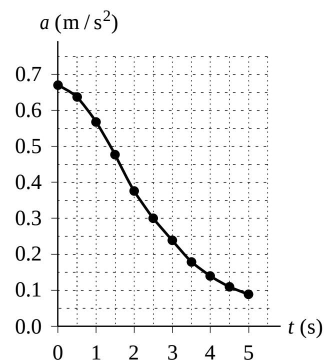
t(s)	v(m/s)
0	0
0.5	0.32
1	0.63
1.5	0.89
2	1.11
2.5	1.28
3	1.41
3.5	1.52
4	1.60
4.5	1.66
5	1.71



(b) From problem 2.70 above,

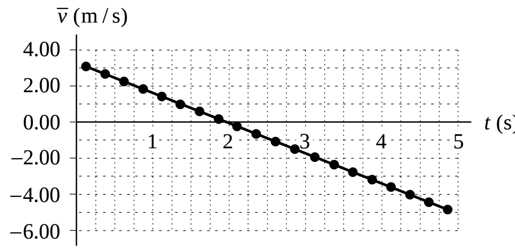
$$a = \frac{h^2 v_{\text{boy}}^2}{(x^2+h^2)^{3/2}} = \frac{h^2 v_{\text{boy}}^2}{(v_{\text{boy}}^2 t^2 + h^2)^{3/2}} = \frac{144}{(4t^2+36)^{3/2}}$$

t(s)	a(m/s^2)
0	0.67
0.5	0.64
1	0.57
1.5	0.48
2	0.38
2.5	0.30
3	0.24
3.5	0.18
4	0.14
4.5	0.11
5	0.09



2.72

Time t(s)	Height h(m)	Δh (m)	Δt (s)	$\bar{v}$ (m/s)	midpt time t (s)
0.00	5.00				
		0.75	0.25	3.00	0.13
0.25	5.75				
		0.65	0.25	2.60	0.38
0.50	6.40				
		0.54	0.25	2.16	0.63
0.75	6.94				
		0.44	0.25	1.76	0.88
1.00	7.38				
		0.34	0.25	1.36	1.13
1.25	7.72				
		0.24	0.25	0.96	1.38
1.50	7.96				
		0.14	0.25	0.56	1.63
1.75	8.10				
		0.03	0.25	0.12	1.88
2.00	8.13				
		–0.06	0.25	–0.24	2.13
2.25	8.07				
		–0.17	0.25	–0.68	2.38
2.50	7.90				
		–0.28	0.25	–1.12	2.63
2.75	7.62				
		–0.37	0.25	–1.48	2.88
3.00	7.25				
		–0.48	0.25	–1.92	3.13
3.25	6.77				
		–0.57	0.25	–2.28	3.38
3.50	6.20				
		–0.68	0.25	–2.72	3.63
3.75	5.52				
		–0.79	0.25	–3.16	3.88
4.00	4.73				
		–0.88	0.25	–3.52	4.13
4.25	3.85				
		–0.99	0.25	–3.96	4.38
4.50	2.86				
		–1.09	0.25	–4.36	4.63
4.75	1.77				
		–1.19	0.25	–4.76	4.88
5.00	0.58				

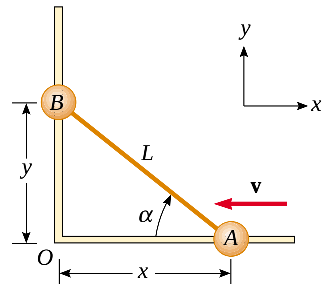


acceleration = slope of line is constant.

$\bar{a} = -1.63\ \text{m/s}^2 =$ **[1.63 m/s² downward]**

2.73 The distance x and y are always related by $x^2 + y^2 = L^2$. Differentiating this equation with respect to time, we have

$$2x\frac{dx}{dt} + 2y\frac{dy}{dt} = 0$$



Now $\frac{dy}{dt}$ is v_B, the unknown velocity of B; and $\frac{dx}{dt} = -v$.

From the equation resulting from differentiation, we have

$$\frac{dy}{dt} = -\frac{x}{y}\left(\frac{dx}{dt}\right) = -\frac{x}{y}(-v)$$

But $\frac{y}{x} = \tan\alpha$ so $v_B = \left(\frac{1}{\tan\alpha}\right)v$

When $\alpha = 60.0°$, $v_B = \frac{v}{\tan 60.0°} = \frac{v\sqrt{3}}{3} = \boxed{0.577v}$

Goal Solution

Two objects, A and B, are connected by a rigid rod that has a length L. The objects slide along perpendicular guide rails, as shown in Figure P2.73. If A slides to the left with a constant speed v, find the velocity of B when $\alpha = 60.0°$.

G: The solution to this problem may not seem obvious, but if we consider the range of motion of the two objects, we realize that B will have the same speed as A when $\alpha = 45$, and when $\alpha = 90$, then $v_B = 0$. Therefore when $\alpha = 60$, we should expect v_B to be between 0 and v.

O: Since we know a distance relationship and we are looking for a velocity, we might try differentiating with respect to time to go from what we know to what we want. We can express the fact that the distance between A and B is always L, with the relation: $x^2 + y^2 = L^2$. By differentiating this equation with respect to time, we can find $v_B = dy/dt$ in terms of $dx/dt = v_A = -v$.

A: Differentiating $x^2 + y^2 = L^2$ gives us $2x\frac{dx}{dt} + 2y\frac{dy}{dt} = 0$

Substituting and solving for the speed of B: $v_B = \frac{dy}{dt} = -\frac{x}{y}\left(\frac{dx}{dt}\right) = -\frac{x}{y}(-v)$

Now from the geometry of the figure, we notice that $\frac{y}{x} = \tan\alpha$, so $v_B = \frac{v}{\tan\alpha}$

When $\alpha = 60.0°$, $v_B = \frac{v}{\tan 60} = \frac{v}{\sqrt{3}} = 0.577v$ (B is moving up)

L: Our answer seems reasonable since we have specified both a magnitude and direction for the velocity of B, and the speed is between 0 and v in agreement with our earlier prediction. In this and many other physics problems, we can find it helpful to examine the limiting cases that define boundaries for the answer.